Equilibrium Refinement through Negotiation in Binary Voting

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[Joint work with Davide Grossi (University of Groningen)
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Motivational Example 1

Consider a multiple referendum with three questions asked simultaneously:

- Voter 1 wants question 1 to be accepted, indifferent over the other two
- Voter 2 wants question 2 to be accepted, indifferent over the other two
- Voter 3 wants question 3 to be accepted, indifferent over the other two

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If they vote according to their goals (which is a weakly dominant strategy), they might be surprised to see that the result might not satisfy any of their goals:

	$Q1$	$Q2$	$Q3$
voter 1	YES	NO	NO
voter 2	NO	YES	NO
voter 3	NO	NO	YES
maj	NO	NO	NO

Outline

1. Related work and context: binary aggregation, equilibrium refinement, bribery and control
2. Aggregation games and their (bad) equilibria
   - Games with no NE
   - Efficient equilibria and winning coalitions
   - Truthful equilibria can be totally inefficient
3. Equilibrium selection via pre-play negotiations
   - Endogenous aggregation games
   - Surviving Nash equilibria
   - Main results: "good" equilibria survive, and only those
4. Conclusions and perspectives
5. Extra: monotonic goals, judgment aggregation

Binary aggregation

Anything that looks like this:

	p	q	r
voter 1	1	0	1
voter 2	1	1	0
voter 3	0	0	0
Aggregator	1	1	0

A simple setting, where voters express 0/1 answers to a set of questions/issues:

- **Voting in multiple referenda** (Lacy and Niou, 2000)
- **Voting in binary combinatorial domains** (survey by Lang and Xia, 2016)
- ***Binary aggregation with constraints** (Wilson, 1975, Dokow and Holzman, 2010, Grandi and Endriss, 2013)
- ***Judgment aggregation** (see eg. Grossi and Pigozzi, 2014)

A mixture of Economics and CS, as in this audience!

Equilibrium refinement

Game-theoretic models of voting are known to allow for too many (bad) equilibria. Suppose three voters all wants a candidate A to be elected in a plurality election, then the following is a Nash-equilibria:

	candidate voted
voter 1	B
voter 2	B
voter 3	B
Plurality	B is winner

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Nash equilibrium refinement can take multiple forms:

- Many examples from Economics (starting from Myerson, 1978)
- In iterative voting the elections are repeated until the voting dynamics converges (survey by Meir, 2017)
- Other proposals from CS: lazy voters, truth-biased voters...

Bribery and Pre-Play Negotiations

We study a form of equilibrium selection which is akin to bribery:

- Most works assume the presence of an external agent
- Payments to agents (modify the game) to reach desired equilibria

Borrowing from Jackson and Wilkie (2005), we study pre-play negotiations:

- Each voter can bribe each other voter (endogenous actions)
- Unlimited resources to do so
- Binding contracts for endogenous payments

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We show that if all agents have conjunctive goals (=they are interested in certains questions being accepted or rejected) then pre-play negotiations leave out undesirable equilibria, and sustain desirable equilibria.

Binary Aggregation

Ingredients:

- A finite set $\mathcal{N}$ of individuals, $|\mathcal{N}| > 3$
- A finite set $\mathcal{I} = \{1, \dots, m\}$ of issues
- Defining a boolean combinatorial domain: $\mathcal{D} = \{0,1\}^{\mathcal{I}}$

Definition

An aggregation procedure is a function $F : \mathcal{D}^{\mathcal{N}} \to \mathcal{D}$ mapping each profile of ballots $\boldsymbol{B} = (B_1, \dots, B_n)$ to an element of the domain $\mathcal{D}$.

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- Focus on non-manipulable rules: there is no profile $\boldsymbol{B}$ s.t. for some $j \in \mathcal{I}$ and $i \in \mathcal{N}$, $F(\boldsymbol{B})(j) \neq B_i(j)$ and for some B_i' we have that $F(B_i', \boldsymbol{B}_{-i}) = B_i(j)$.

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- Dietrich and List (2007) proved that rules are non-manipulable iff they are independent and monotonic
- Examples: quota rules, oligarchies
- Non-examples: distance-based rules (if different than majority)

Winning and Veto Coalitions

Independent rules can be characterised by winning coalitions:

$C \subseteq \mathcal{N}$ is a winning coalition for issue $j \in \mathcal{I}$ if for every profile $\boldsymbol{B}$ we have that *if* $B_i(j) = 1$ for all $i \in C$ and $B_i(j) = 0$ for all $i \notin C$ *then* $F(\boldsymbol{B})(j) = 1$.

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Clearly, C is winning iff $\mathcal{N} \setminus C$ is veto. More notation: C a resilient winning coalition if for all $j \in C$ we have that $C \setminus \{j\}$ is also a winning coalition.

Examples:

- **Strict majority**: veto and winning are $\left\{ C \subseteq \mathcal{N} \;\middle|\; |C| \geq \frac{|\mathcal{N}|+1}{2} \right\}$
- **Constant rule** accepting always issue j: all coalitions are winning, no coalition is losing

Aggregation Games

Consider the following object:

Definition [Aggregation Game]

An aggregation game *for* $\mathcal{I}$ *and* $\mathcal{N}$ *is a tuple* $\mathcal{A} = \langle \mathcal{N}, \mathcal{I}, F, \gamma_i, \pi_i \rangle$ *where:*

- F *is an aggregator*
- *each goal* γ_i *is a conjunction of literals from* $\mathcal{I}$
- $\pi_i : \mathcal{D}^{\mathcal{N}} \to \mathbb{R}$ *is a payoff function on profiles*

Interpretation (quasi-dichotomous preferences):

- goals γ_i represent uncompromising positions
- any two profiles which both satisfy or both falsify the goal γ_i can be compared by looking at the payoff function π_i

Aggregation Games with no Nash Equilibria

Consider the following game:

- One issue, two voters, F is the majority
- $\gamma_1 = \gamma_2 = \top$
- $\pi_1(\boldsymbol{B}) = 1$ iff $B_1 = B_2$, 0 otherwise
- $\pi_2(\boldsymbol{B}) = 1$ iff $B_1 \neq B_2$, 0 otherwise

This is a matching penny game which has no Nash equilibria:

	vote 1	vote 0
vote 1	$1, 0$	$0, 1$
vote 0	$0, 1$	$1, 0$

Truthful Strategies and Equilibria

	Issue 1	Issue 2	Issue 3
Voter 1	1	0	1
Voter 2	0	1	1
Voter 3	0	0	0
Majority	0	0	1

- F is strict majority, 3 issues and 3 voters
- $\gamma_1 = 1$, $\gamma_2 = 2$, $\gamma_3 = \neg 3$ (game is $\mathcal{N}$-consistent)
- π_i are constant on all profiles
- Voters are truthful since $B_i \models \gamma_i$ for all i

Proposition

If $\mathcal{A}$ is a constant aggregation game for a non-manipulable F, then every truthful strategy is weakly dominant.

The converse is true if F is a quota rule. Proposition not true if goals are not conjunctions or payoffs are not constant.

C-efficient equilibria

	Issue 1	Issue 2
Voter 1	1	0
Voter 2	1	0
Voter 3	1	0
Voter 4	0	1
Voter 5	0	1
Majority	1	0

- F is strict majority, 2 issues and 5 voters. Coalition $C = \{1, 2, 3\}$.
- $\gamma_1 = 1$, $\gamma_2 = \gamma_3 = 1 \wedge \neg 2$, $\gamma_4 = \gamma_5 = 2$ (game is C-efficient)
- π_i are uniform: they are defined on the outcome of aggregation
- C is a resilient winning coalition for $1 \wedge \neg 2$
- The result is C-efficient: for all $i \in C$ we have $F(\boldsymbol{B}) \models \gamma_i$

Proposition

Every C-consistent uniform aggregation game for non-manipulable F has a NE that is C-truthful and C-efficient if C is a resilient winning coalition for $\bigwedge_{i \in C} \gamma_i$

Totally Inefficient Equilibria

	Issue 1	Issue 2	Issue 3
Voter 1	1	0	1
Voter 2	0	1	1
Voter 3	0	0	0
Majority	0	0	1

- F is strict majority, 3 issues and 3 voters
- $\gamma_1 = 1$, $\gamma_2 = 2$, $\gamma_3 = \neg 3$
- π_i are constant on all profiles
- The result is totally inefficient: for all i we have $F(\boldsymbol{B}) \not\models \gamma_i$

Proposition

For every uniform quota rule, there exists constant aggregation games with truthful and totally inefficient NE.

Intermezzo: Boolean Games

Boolean games are a well-studied model in knowledge representation:

- Each agent i has exclusive control over a number of binary variables p_i
- Each agent has a propositional goal γ_i (and possibly a payoff function π_i)
- Typically i cannot make γ_i true by itself

Boolean games are aggregation games where the aggregator F is a dictatorship of agent i over its variables ($\Rightarrow$ hardness complexity results can be imported).

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Boolean games are aggregation games where the aggregator F is a dictatorship of agent i over its variables ($\Rightarrow$ hardness complexity results can be imported).

The strategic structure of the two models is very different. However from a formal verification standpoint (ATL-formulas) the complexity is the same:

Belardinelli, Grandi, Herzig, Longin, Lorini, Novaro, and Perrussel. Relaxing Exclusive Control in Boolean Games. In *Proceedings of TARK-2017*.

Pre-play negotiations

The main message of the (still incomplete) analysis of aggregation games with conjunctive goals is that truthful equilibria maybe totally inefficient (=they do not satisfy the goal of any of the voters) even if goals are mutually consistent.

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Let us augment the aggregation model with a pre-play negotiation phase:

- A pre-vote phase: starting from a uniform aggregation game $\mathcal{A}$ players make simultaneous transfers of payoff to their fellow players
- A vote phase: players play the original game $\mathcal{A}$ updated with transfers

An original idea by Jackson and Wilkie (2005), to overcome prisoner dilemmas using endogenous transfers. However we do not want to use mixed strategies (somehow controversial in voting?), as we have goal+utility kind of preferences.

Endogenous Aggregation Games

Definition [Endogenous aggregation games]

An endogenous aggregation game is defined as a tuple $\langle \mathcal{A}, \{T_i\}_{i \in N} \rangle$ *where* $\mathcal{A}$ *is a uniform aggregation game, and each* T_i *is the set of all transfer functions* $\tau_i : \mathcal{D}^{\mathcal{N}} \times \mathcal{N} \to \mathbb{R}_+$

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What is a strategy σ_i:

- the choice of a transfer τ_i, specifying how much is given to which player if a certain profile of votes occur
- the choice of a ballot B_i for each game that may result from transfers

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- the choice of a ballot B_i for each game that may result from transfers

What is a solution? A strategy profile such that:

- The profile of transfers $(\tau_1, \ldots, \tau_n)$ is a NE of a suitably defined transfer game incorporating the agents' ballot strategies
- The ballot profile played at the game $\mathcal{A}$ modified by τ is a NE, or a profile of maximin ballots

Surviving NE in EAG

Definition

$\boldsymbol{B}$ is a surviving Nash equilibrium (SNE) of $\mathcal{A}$ if there exists a transfer profile τ such that $(\tau, \boldsymbol{B})$ is a solution outcome of the endogenous agg. game $\mathcal{A}^{\mathcal{T}}$.

Equilibrium refinement: restrict to those equilibria in aggregation game that are surviving (=can be sustained by endogenous transfers).

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Equilibrium refinement: restrict to those equilibria in aggregation game that are surviving (=can be sustained by endogenous transfers).

Lemma

For every $\mathcal{N}$-efficient and $\mathcal{N}$-truthful ballot profile $\boldsymbol{B}$ of a uniform aggregation game, there exists a transfer profile τ such that $\boldsymbol{B}$ is a weakly dominant strategy equilibrium in $\tau(\mathcal{A})$.

Proof idea. Every voter commits to pay a sum higher than any other attainable payoff to all other players to implement the truthful and efficient equilibrium.

Main results

Theorem [good equilibria survive]

Let $\mathcal{A}$ be a uniform aggregation game for a non-manipulable aggregator F. Every $\mathcal{N}$-efficient and $\mathcal{N}$-truthful NE of $\mathcal{A}$ is a SNE.

Difficulty of proof is showing that τ of previous lemma is a solution of the endogenous aggregation game. Non-manipulability is a necessary condition.

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Let $\mathcal{A}$ be a uniform aggregation game for a non-manipulable aggregator F. Every $\mathcal{N}$-efficient and $\mathcal{N}$-truthful NE of $\mathcal{A}$ is a SNE.

Difficulty of proof is showing that τ of previous lemma is a solution of the endogenous aggregation game. Non-manipulability is a necessary condition.

Theorem [only good equilibria survive]

Let $\mathcal{A}$ be a C-consistent uniform aggregation game for a non-manipulable aggregator F, and such that C is a winning coalition for $\bigwedge_{i \in C} \gamma_i$. Then, every SNE of $\mathcal{A}$ is C-efficient.

Consequence: if there are two winning coalitions C_1 and C_2 with conflicting goals then there exists no SNE.

Conclusions

1. Aggregation games: voters have conjunctive goals on binary issues, decision taken by non-manipulable aggregator
2. Disturbing observation: even if all agents have consistent goals, there are truthful and totally inefficient Nash equilibria of the game
3. Pre-vote negotiation can be used to rule out undesirable equilibria
   - The model: two phase-games (first negotiate then vote), surviving equilibria as solution concept
   - Main result 1: truthful and efficient equilibria for all agents are surviving
   - Main result 2: surviving equilibria must satisfy all winning coalitions that have mutually consistent goals

If you want to know more:

U. Grandi, D. Grossi, P. Turrini. Negotiable Votes – Pre-Vote Negotiations in Binary Voting with Non-Manipulable Rules. To appear in *Journal of Artificial Intelligence Research (JAIR)*.

Final remarks and perspectives

Many open problems and directions for future work:

1. The study of aggregation games is far from complete (only conjunctions considered!). Characterisation of equilibria? Connexion between syntax of the goals and type/existence of NE?
2. Avoid using utility and focus on games with propositional goals (for computational reasons, more plausible model...)
3. Transfer functions are a powerful mathematical objects but computationally unfriendly. Can strategies and transfers be represented compactly?
4. Relax the assumption the all transfers are possible. How about restricting transfers via a trust network? Connections with recent ANR "Social Choice and Social Networks": *www.irit.fr/~Umberto.Grandi/scone/*
5. Voters' behavious in multi-issue voting/multiple referenda has not yet been assessed experimentally (to the best of my knowledge): ongoing work with J. Lang, A. Ozkes, S. Airiau (Paris Dauphine)

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Thank you for your attention!

Extra: Non-Conjunctive Goals

	Issue 1	Issue 2	Issue 3
Voter 1	1	0	0
Voter 2	0	1	0
Voter 3	0	0	1
Majority	0	0	0

	Issue 1	Issue 2	Issue 3
Voter 1	1	0	**1**
Voter 2	0	1	0
Voter 3	0	0	1
Majority	0	0	**1**

- F is strict majority, 3 issues and 3 voters
- $\gamma_1 =$ "I want an odd number of accepted issues", $\gamma_2 = \gamma_3 = \top$
- π_i are constant on all profiles
- Voting against its goal is a better strategy for 1!

Extra: Monotonic Goals

Definition [Monotonic goals]

A goal γ is called:

- **positively monotonic in issue** j *if when B satisfies γ and $B(j) = 0$ then changing to $B'(j) = 1$ still satisfies γ;*
- **negatively monotonic in issue** j *if when B satisfies γ and $B(j) = 1$ then changing to $B'(j) = 0$ still satisfies γ;*

A formula ϕ is **monotonic** *if for every p_j occurring in ϕ, it is either positively or negatively monotonic in p_j.*

Monotonic goals are also known as *unate boolean functions*.

With a similar proof as the conjunctive case, we can show that if all individual goals are aligned, and $\mathcal{N}$ is a winning coalition for the conjunction of the goals, then there exists a surviving NE that is $\mathcal{N}$-truthful and $\mathcal{N}$-efficient.

Extra: Integrity Constraints

Can voting paradoxes and discursive dilemmas be avoided by equilibrium refinement? Take the following example:

- $\gamma_1 = p$, $\gamma_2 = q$, $\gamma_3 = \neg r$
- Constant payoff
- Integrity constraint: $p \to (q \vee r)$

Observe that goals are consistent with the integrity constraint. The following is a Nash equilibrium and a discursive dilemma:

	p	q	r
voter 1	1	0	1
voter 2	1	1	0
voter 3	0	0	0
maj	1	0	0

However it is not surviving! Voter 2 could transfer payoff to voter 3 to vote for issue q and change the outcome to (110)

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