# Pre-Vote Negotiations and Binary Voting with Constraints

Umberto Grandi

Department of Mathematics
University of Padova

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Joint work with Davide Grossi (University of Liverpool)
and Paolo Turrini (Imperial College London)

# Outline

1. Majority rule over interconnected issues: paradoxes cannot be avoided
   - Binary aggregation with constraints / Judgment aggregation
   - Condorcet paradox and characterisation results
2. Dynamic social choice: collective decisions as games
   - Judgment aggregation games
   - Bad Nash equilibria cannot be avoided
3. Pre-play negotiations can be used to avoid bad equilibria
   - Endogenous games
   - No paradox realized at equilibrium

# Everything Starts From Paradoxical Situations

Three parties $A, B, C$ need to decide on building nuclear power plants (P), buying nuclear energy from the foreign market (F), anddeveloping atomic weapons (W). If importing nuclear technology from abroad is not an option, the development of atomic weapons involves the construction of a power plant.

| | $W$ | $F$ | $P$ |
|---|---|---|---|
| Party A | Yes | No | Yes |
| Party B | Yes | Yes | No |
| Party C | No | No | No |
| Majority | Yes | No | No |

What is the collective choice on building a nuclear power plant?

- A majority think atomic weapons should be developed.
- A majority think energy should not be bought from the foreign market.
- But: a majority is against building a power plant!

# Binary Aggregation

Ingredients:

- A finite set $\mathcal{N}$ of individuals
- A finite set $\mathcal{I} = \{1, \ldots, m\}$ of issues
- A boolean combinatorial domain: $\mathcal{D} = \{0, 1\}^{\mathcal{I}}$

**Definition**

*An aggregation procedure is a function $F : \mathcal{D}^{\mathcal{N}} \to \mathcal{D}$ mapping each profile of ballots $\boldsymbol{B} = (B_1, \ldots, B_n)$ to an element of the domain $\mathcal{D}$.*

*Wilson (1975), Dokow and Holzman (2010)*

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**Example: Nuclear power plant?**

- $\mathcal{N} = \{1, 2, 3\}$
- $\mathcal{I} = \{W, F, P\}$
- Individuals submit ballots in $\mathcal{D} = \{0,1\}^3$

# Integrity Constraints

A propositional language $\mathcal{L}$ to express integrity constraints on $\mathcal{D} = \{0,1\}^m$

- One propositional symbol for every issue: $PS = \{p_1, \ldots, p_m\}$
- $\mathcal{L}_{PS}$ closing under connectives $\wedge$, $\vee$ ,$\neg$, $\rightarrow$ the set of atoms $PS$

Given an integrity constraint $\mathrm{IC} \in \mathcal{L}_{PS}$, a rational ballot is $B \in \mathrm{Mod}(\mathrm{IC})$

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**Example: Nuclear power plant?**

Build a power plant if you do not want to import technology.
Propositional constraint: $\mathrm{IC} = (p_W \wedge \neg p_F) \rightarrow p_P$

| | |
|---|---|
| Individual 1 submits $B_1 = (1,0,1)$: | $B_1$ satisfies IC ✓ |
| Individual 2 submits $B_2 = (1,1,0)$: | $B_2 \models \mathrm{IC}$ ✓ |
| Individual 3 submits $B_3 = (0,0,0)$: | $B_3 \models \mathrm{IC}$ ✓ |

Majority aggregation outputs $(1,0,0)$: IC not satisfied.

# Paradoxical situations cannot be avoided

The notorious Condorcet paradox in binary aggregation...

$$\begin{array}{c} \triangle \succ_1 \bigcirc \succ_1 \square \\ \square \succ_2 \triangle \succ_2 \bigcirc \\ \bigcirc \succ_3 \square \succ_3 \triangle \\ \hline \triangle \succ \bigcirc \succ \square \succ \triangle \end{array} \quad \Rightarrow$$

| | $\triangle\bigcirc$ | $\bigcirc\square$ | $\triangle\square$ |
|---|---|---|---|
| Agent 1 | 1 | 1 | 1 |
| Agent 2 | 0 | 1 | 0 |
| Agent 3 | 1 | 0 | 0 |
| *Maj* | 1 | 1 | 0 |

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$$\bigcirc \succ_3 \square \succ_3 \triangle$$

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$$\triangle \succ \bigcirc \succ \square \succ \triangle$$

$\Rightarrow$

| | $\triangle\bigcirc$ | $\bigcirc\square$ | $\triangle\square$ |
|---|---|---|---|
| Agent 1 | 1 | 1 | 1 |
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...is not an isolated case:

**Proposition**

*The majority rule does not generate a paradox with respect to* $\mathrm{IC}$ *if and only if* $\mathrm{IC}$ *is equivalent to a conjunction of clauses of size* $\leq 2$ *(i.e.,* $2$-$\mathrm{CNF}$*)*

Similar characterisations can be obtained for various aggregation procedures:

Grandi and Endriss, Lifting Integrity Constraints in Binary Aggregation. *AIJ*, 2013.

# Where are decisions actually taken?

Ok, paradoxes cannot be avoided... ...but we do take decisions!

How? Offline (Italian expression: "in the corridors")

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### Vision

Pre-vote dynamics
create consensus and avoid paradoxes!

*– deliberation, negotiation, bribery, power and authority, opinion polls –*

Dryzek and List, Social Choice Theory and Deliberative Democracy: A reconciliation. *BJPS*, 2003.
List, Group Communication and the Transformation of Judgment. *JPP*, 2011.

# Structure of the results

We model collective decision making as a game:

- players have preferences/goals over collective outcomes
- strategies are voting ballots
- framework of boolean games and endogenous games

**Question**

Can a paradoxical situation be a Nash equilibrium?

Harrenstein, van der Hoek, Meyer, Witteveen. Boolean games. *TARK-2001*.

Jackson and Wilkie. Endogenous Games and Mechanisms: Side Payments among Players. *Review of Economic Studies*, 2005

# Aggregation Games

To model the dynamic aspect of social choice we first model it as a game:

**Definition [Aggregation game]**

*An aggregation game is a tuple* $\mathcal{A} = \langle \mathcal{N}, \mathcal{I}, \mathrm{IC}, maj, \{\gamma_i\}_{i \in \mathcal{N}} \rangle$ *such that:*

- $\langle \mathcal{N}, \mathcal{I}, \mathrm{IC} \rangle$ *is a binary aggregation structure;*
- $maj$ *is the majority rule;*
- *the individual goals* $\gamma_i$ *are* $\mathrm{IC}$*-consistent propositional formulas in* $\mathcal{L}_{PS}$*;*

Focus on consistent game: $\bigwedge_i \gamma_i \wedge \mathrm{IC}$ is consistent.

# How do equilibria look like?

**Good news**: there always exists an $\mathrm{IC}$-consistent equilibrium, i.e, a profile $\boldsymbol{B}$ in which no individual has incentive to deviate and $maj(\boldsymbol{B}) \models \mathrm{IC}$ (truthful strategies are weakly dominant in aggregation games for majority)

**Bad news**: there are games with inefficient NE (no individual goal satisfied)

**Very bad news**: there are games with inefficient and $\mathrm{IC}$-inconsistent NE

# Discursive dilemma as an inconsistent NE

- Three parties $A, B, C$
- Three issues $W, F, P$
- Integrity constraint $\mathrm{IC} = (W \wedge \neg F) \rightarrow P$

Individual goals:

$$\gamma_A = W$$
$$\gamma_B = F$$
$$\gamma_C = \neg P$$

The discursive dilemma/docrinal paradox is an IC-inconsistent NE:

| | $W$ | $F$ | $P$ |
|---|---|---|---|
| Party A | 1 | 0 | 1 |
| Party B | 1 | 1 | 0 |
| Party C | 0 | 0 | 0 |
| Majority | 1 | 0 | 0 |

# Aggregation Games with Payoff

Let us give individuals more expressive preferences:

- goals $\gamma_i$ represent uncompromising positions
- any two states which both satisfy or both falsify the goal $\gamma_i$ can be compared by looking at the payoff function $\pi_i$

**Definition[$A^\pi$ games]**

*An aggregation game with payoff is a tuple* $\langle \mathcal{A}, \{\pi_i\}_{i \in \mathcal{N}} \rangle$ *where* $\mathcal{A}$ *is an aggregation game and* $\pi_i : \mathrm{Mod}(\mathrm{IC})^{\mathcal{N}} \to \mathbb{R}$ *is a payoff function.*

## How do equilibria look like?

**Bad news**: inefficient and $\mathrm{IC}$ inconsistent equilibria still exists...

**Good news**: for every consistent aggregation game $\mathcal{A}$ we can build payoff functions such that $\langle \mathcal{A}, \{\pi_i\}_{i \in \mathcal{N}} \rangle$ has no inefficient NE!

# Endogenous Aggregation Games

We model pre-vote negotiations in two phases:

- A pre-vote phase: starting from a uniform $A^{\pi}$-game players make simultaneous transfers of payoff to their fellow players
- A vote phase: players play the original $A^{\pi}$-game, updated with transfers

**Definition [$\mathcal{A}^{T}$-games]**

*An endogenous aggregation game is defined as a tuple* $\langle \mathcal{A}, \{\pi_i\}_{i\in\mathcal{N}}, \{T_i\}_{i\in N}\rangle$ *where* $\langle \mathcal{A}, \{\pi_i\}_{i\in\mathcal{N}}\rangle$ *is a uniform* $A^{\pi}$ *game, and each* $T_i$ *is the set of all transfer functions* $\tau_i : \mathrm{Mod}(\mathrm{IC})^{\mathcal{N}} \times \mathcal{N} \rightarrow \mathbb{R}_+$

Paolo Turrini. Endogenous Boolean Games. *Proceedings of IJCAI-2013.*

# Pre-vote Negotiation avoids Inefficient Equilibria

A profile $\boldsymbol{B}$ is a surviving NE if there are transfer functions $\tau_i$ that sustain it: $(\tau, \boldsymbol{B}) = (\tau_1, \ldots, \tau_n, B_1, \ldots, B_n)$ is an equilibrium of the two-phase game

**Proposition**

*Every goal-efficient NE of an $\mathcal{A}^T$-games is a surviving NE.*

**Proposition**

*Every surviving NE of a consistent $\mathcal{A}^T$-games is goal-efficient.*

**Good news**: individuals can redistribute payoff to reach goal-efficient NE!

# Are we avoiding paradoxes?

It is possible to avoid paradoxes:

**Corollary**

*Every consistent $\mathcal{A}^T$-game has an $\mathrm{IC}$-consistent NE that is surviving.*

If at least one individuals has coherence as a goal it is also guaranteed:

**Corollary**

*Every surviving NE of an $\mathcal{A}^T$-game such that $\bigwedge_i \gamma_i \models \mathrm{IC}$ is $\mathrm{IC}$-consistent.*

# Preplay negotiations avoid paradoxical situations

Individual goals:

$$\gamma_A = W$$
$$\gamma_B = F$$
$$\gamma_C = \neg P$$

The discursive dilemma/docrinal paradox is not surviving!:

| | $W$ | $F$ | $P$ |
|---|---|---|---|
| Party A | 1 | 0 | 1 |
| Party B | 1 | 1 | 0 |
| Party C | 0 | 0 | 0 |
| Majority | 1 | 0 | 0 |

Party B can transfer utility to either Party A or Party B to get issue F accepted!

# Conclusions

1. Collective decisions on logically interconnected issues can be paradoxical
   Notorious examples: Condorcet paradox, discursive dilemma...
2. When voters are strategic (read self-interested) the collective decision at equilibrium may be undesirable or irrealistic
3. Pre-vote negotiation rules out undesirable equilibria!
   - The model: two phase-games, first negotiate then vote
   - Main result: surviving equilibria iff they satisfy the goals of each voter
   - Corollary: there always exists a consistent equilibria which is surviving

If you want to know more:

U. Grandi, D. Grossi, P. Turrini. Pre-vote Negotiations and Binary Voting with Constraints. *11th Conference on Logic and the Foundations of Game and Decision Theory*, 2014.

ESSLLI 2014 course on "Logical Frameworks for Multiagent Aggregation"
Tübingen, 11–15 August — Student grants available!