A simple and tractable extension of Situation Calculus to Epistemic Logic

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1 Introduction

The frame problem and the representation of knowledge change have deserved a lot of works. In particular, at the Cognitive Robotics Group, at Toronto, several researchers in the last ten years have produced quite interesting papers in a uniform logical framework based on Situation Calulus [Rei91, SL93, LR94, LL98]. In [Rei91] Reiter has proposed a simple solution to the frame problem. Scherl and Levesque in [SL93] have defined an extension to Epistemic Logic to represent knowledge dynamics in contexts where some actions may produce knowledge, like, for instance, sensing actions for a robot. This approach has been extended by Lakemeyer and Levesque in [LL98] to modal operators of the kind "I know and only know". Also, they have given a formal semantics and axiomatics, and they proved soundness and completeness of the axiomatics.

These extensions to Epistemic Logic offer a large expressive power. Indeed, there is no restriction on formulas in the scope of modal operators. However, they have lost the simplicity of the solution to the frame problem initially proposed in [Rei91], and the possibility to find a tractable implementation of these extensions is far to be obvious. As far as we know, at the present time there is no such implementation.

In this paper a simple extension to Epistemic Logic of Reiter's initial solution is presented that could easily be implemented. In exchange we have to accept strong restrictions on the expressive power of the epistemic part of the logical framework. However, we believe that for a large class of applications these restrictions are not real limitations. In the following intuitive ideas of the proposed solution are presented with a simple example. Then, we give the general logical framework, and, finally a comparison is made with the solutions that we have mentioned before.

2 The frame problem in the context of extended situation calculus: an example

Situation Calculus [McC68, Rei99] is a sort of classical first order logic where predicates may have an argument (the last argument) of a particular sort, which

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is called a "situation"; these predicates are called "fluents". This argument is intended to represent the sequence of actions which have been performed from the initial state to the current state. A situation is syntactically represented by a term of the form $do(a, s)$ where a denotes an action, and s denotes a situation. The initial situation is denoted by $S0$.

For instance, $position(x, s)$ represents the fact that a given object is at the position x in the situation s. Action variables and situation variables can be quantified. For instance, $\neg\exists s(position(2, s))$ represents the fact that in no situation a given object is at the position 2. Action quantification is an essential feature in the solution to the frame problem proposed by Reiter. Indeed, the fact that, for example, there is no other possibility to change the position of an object than to perform the action $move$ can be represented by: $\forall s \forall a \forall x(position(x, s) \wedge \neg position(x, do(a, s)) \rightarrow a = move)$. To intuitively present how the solution to the frame problem can be extended to epistemic logic, we use the following scenario.

Let's consider a simple robot that can move forward (action adv) or backward (action rev) along a railtrack. Performance of actions adv or rev changes his position of one distance unit. There may be obstacles on the railtrack, like branches of trees that have fallen. Suppose the robot is moving during the night and there is a pilot in the robot. The pilot can recognise obstacles, provided he has switched on a spotlight (action $obs.obstacle$), and the obstacle is not beyond the visibility distance d. The spotlight is not always on because it consumes battery ressources, which are limited. When the robot moves he computes his new position, and this position is indicated on a screen which can be seen by the pilot (action $inf.position(x)$). The pilot performs the action $inf.position(x)$ before the action $obs.obstacle$ in order to know his position and to determine the position of visible obstacles, if there are. The pilot can inform the robot about the existence of an obstacle at x (action $inf.obstacle(x)$), and the robot stops if he knows that there is an obstacle in a short distance sd.

We see that the description of this scenario involves evolution of the world and evolution of what the pilot and the robot believe [2]. We first show how the frame problem can be solved if we only consider evolution of the world.

For each fluent, two axioms define the positive effects or the negative effects of the actions. For instance, for the fluent $position(x, s)$, the effect of performing the action adv (respectively rev) when the robot is at the position $x-1$ (respectively $x+1$) in the situation s, is that it is at the position x in the situation $do(a, s)$ [3]:

(1) $(a = adv \wedge position(x-1, s) \vee a = rev \wedge position(x+1, s)) \rightarrow position(x, do(a, s))$

The negative effect axiom expresses that if the robot is at the position x in the situation s and he performs either the action adv or the action rev, then in the situation $do(a, s)$ he is no more at the position x:

[2] We have no room here to give a complete formal description of this scenario. Also, some assumptions are not perfectly realistic, but we mainly want to show how such scenarios can be formalised.

[3] All the variables are implicitly universally quantified.

(2) $(a = adv \lor a = rev) \land position(x, s) \rightarrow \neg position(x, do(a, s))$

One of the most important features to solve the frame problem in the approach presented in [Rei99] is the "causal completeness assumption". This assumption expresses that the positive effect axioms and the negative effect axioms "characterize **all** the conditions underwhich action a can cause the fluent *position* to become true (respectively false) in the successor situation". If, in addition to (1) and (2), we accept this assumption, then we have (see axiom (G2) for the general form):

(3) $position(x, do(a, s)) \leftrightarrow [a = adv \land position(x-1, s) \lor a = rev \land position(x+1, s)] \lor position(x, s) \land \neg[(a = adv \lor a = rev) \land position(x, s)]$

This axiom defines the objective representation of the evolution of the world. If we want to define the subjective representation of the evolution of the world, we can extend the language with epistemic modal operators. For that purpose, we introduce modal operators like B_r, such that $B_r\phi$ is intended to mean that the robot r believes that ϕ holds in the present situation. To represent, in a similar approach, the evolution of what the robot believes, we have to consider four effect axioms for each fluent. For example, for the fluent $position(x, s)$, there are four distinct possible attitudes of the robot which are formally represented by: $B_r position(x, s)$, $\neg B_r position(x, s)$, $B_r \neg position(x, s)$ and $\neg B_r \neg position(x, s)$. The corresponding axioms (4), (5), (6) and (7) are given below.

The effect of performing action adv (respectively rev) when the robot believes that he is at the position $x - 1$ (respectively $x + 1$) in the situation s is that he believes that he is at the position x in the situation $do(a, s)$:

(4) $(a = adv \land B_r position(x - 1, s) \lor a = rev \land B_r position(x + 1, s)) \rightarrow B_r position(x, do(a, s))$

The effect of performing either action adv or rev when the robot believes that he is at the position x in the situation s is that he does not believe that he is at the position x in the situation $do(a, s)$:

(5) $(a = adv \lor a = rev) \land B_r position(x, s) \rightarrow \neg B_r position(x, do(a, s))$

We have two similar axioms to define the attitude of the robot with respect to the fact that he believes that he is not at the position x in the situation $do(a, s)$:

(6) $(a = adv \lor a = rev) \land B_r position(x, s) \rightarrow B_r \neg position(x, do(a, s))$

(7) $(a = adv \land B_r position(x - 1, s) \lor a = rev \land B_r position(x + 1, s)) \rightarrow \neg B_r \neg position(x, do(a, s))$

If we extend the causal completeness assumptions to the robot's beliefs, we get, after some simplifications, the two axioms (8) and (9) (see axioms (G3) and (G4) for the general form):

(8) $B_r position(x, do(a, s)) \leftrightarrow [a = adv \land B_r position(x - 1, s) \lor a = rev \land B_r position(x+1, s)] \lor B_r position(x, s) \land \neg[(a = adv \lor a = rev) \land B_r position(x, s)]$

(9) $B_r \neg position(x, do(a, s)) \leftrightarrow [(a = adv \lor a = rev) \land B_r position(x, s)] \lor B_r \neg position(x, s) \land \neg[a = adv \land B_r position(x - 1, s) \lor a = rev \land B_r position(x + 1, s)]$

Notice that in the definition of these axioms we have implicitly assumed that if the robot performs either the action adv or the action rev, he believes that he has performed these actions. However, if some action is performed by the pilot, like the action $obs.obstacle$, the robot is not necessarily informed about this fact.

It is interesting to see with this example how the pilot's beliefs and the robot's beliefs about the fluent *obstacle* may evolve in two different way. We have the following effect axioms (10), (11), (12) and (13) for this fluent.

If the pilot has switched on the spot light, and there is an obstacle at some position x which is visible by the pilot, then the pilot believes that there is an obstacle at x [4] :

(10) $a = obs.obstacle \wedge obstacle(x,s) \wedge position(y,s) \wedge y \leq x \leq y+d \rightarrow B_p obstacle(x, do(a,s))$

If the pilot has switched on the spot light, and there is no obstacle at some position x which is visible by the pilot, then the pilot does not believe that there is an obstacle at x:

(11) $a = obs.obstacle \wedge \neg obstacle(x,s) \wedge position(y,s) \wedge y \leq x \leq y+d \rightarrow \neg B_p obstacle(x, do(a,s))$

We have two similar effect axioms for $\neg obstacle(x, do(a,s))$.

(12) $a = obs.obstacle \wedge \neg obstacle(x,s) \wedge position(y,s) \wedge y \leq x \leq y+d \rightarrow B_p \neg obstacle(x, do(a,s))$

(13) $a = obs.obstacle \wedge obstacle(x,s) \wedge position(y,s) \wedge y \leq x \leq y+d \rightarrow \neg B_p \neg obstacle(x, do(a,s))$

Then, from the causal completion assumption we have the axioms (14) and (15).

(14) $B_p obstacle(x, do(a,s)) \leftrightarrow [a = obs.obstacle \wedge obstacle(x,s) \wedge position(y,s) \wedge y \leq x \leq y+d] \vee B_p obstacle(x,s) \wedge \neg[a = obs.obstacle \wedge \neg obstacle(x,s) \wedge position(y,s) \wedge y \leq x \leq y+d]$

(15) $B_p \neg obstacle(x, do(a,s)) \leftrightarrow [a = obs.obstacle \wedge \neg obstacle(x,s) \wedge position(y,s) \wedge y \leq x \leq y+d] \vee B_p \neg obstacle(x,s) \wedge \neg[a = obs.obstacle \wedge obstacle(x,s) \wedge position(y,s) \wedge y \leq x \leq y+d]$

If the only way for the robot to be informed about the fact that there is an obstacle at x is to perform the action $inf.obstacle(x)$, then we have the axioms (16) and (17) below.

(16) $B_r obstacle(x, do(a,s)) \leftrightarrow a = inf.obstacle(x) \vee B_r obstacle(x,s)$

(17) $B_r \neg obstacle(x, do(a,s)) \leftrightarrow B_r \neg obstacle(x,s) \wedge \neg(a = inf.obstacle(x))$

Let's assume that in the initial situation $S0$ the pilot and the robot both ignore whether there are obstacles in any places. This is formally represented by: $\neg \exists x B_r obstacle(x, S0)$, $\neg \exists x B_r \neg obstacle(x, S0)$, $\neg \exists x B_p obstacle(x, S0)$ and $\neg \exists x B_p \neg obstacle(x, S0)$. If in the situation $S0$ there is an obstacle at the position 3, the pilot and the robot have wrong beliefs. If the distance d is equal to 10, after performance of the action $a_1 = obs.obstacle$, the pilot believes that there is an obstacle at the position 3, while the robot ignores that there this an obstacle at the position 3, i.e. we have: $B_p obstacle(3, do(a_1, S0))$ and $\neg B_r obstacle(3, do(a_1, S0))$. Finally, if after action a_1 the pilot performs the action $a_2 = inf.obstacle(3)$,

[4] As a matter of simplification, it is assumed here that the pilot only looks at obstacles that are foreward.

the robot and the pilot have the same beliefs about this obstacle. We have: $B_p obstacle(3, do(a_2, do(a_1, S0)))$ and $B_r obstacle(3, do(a_2, do(a_1, S0)))$.

In fact these actions can be performed only if some preconditions are satisfied. These preconditions are expressed with a particular predicate $Poss$ (see [Rei99]). The formula $Poss(a, s)$ means that in the situation s it is possible to perform the action a. For example, a precondition to perform the action adv is that the robot does not believe that there is an obstacle in a short distance sd, and there is no obstacle in front of him.

(18) $Poss(adv, s) \leftrightarrow \neg\exists x\exists y(B_r position(x, s) \wedge B_r obstacle(y, s) \wedge y - x \leq sd) \wedge \neg\exists x\exists y(position(x, s) \wedge obstacle(y, s) \wedge y = x + 1)$

3 General framework

Now we present the general framework of the extended Situation Calculus. Let L be a first order language with equality with the constant symbol $S0$, the function symbol do, and the predicate symbol $Poss$. Let L_M be an extension of language L with modal operators denoted by $B_1, \ldots, B_i, \ldots$, where modal operators can only occur in modal literals. Modal literals are of the form $B_i l$, where l is a literal of L, and l is **not formed with equality** predicate. Let's consider the theory T which contains the following axioms.

Action precondition axioms.

For each action a there is in T an axiom of the form [5]:

(G1) $Poss(a, s) \leftrightarrow \pi_a(s)$

where π_a is a formula in L_M.

Successor state axioms.

For each fluent F there is in T an axiom of the form:

(G2) $F(do(a, s)) \leftrightarrow \Gamma_F^+(a, s) \vee F(s) \wedge \neg\Gamma_F^-(a, s)$

where Γ_F^+ and Γ_F^- are formulas in L.

Successor belief state axioms.

For each modal operator B_i and each fluent F [6], there are in T two axioms of the form:

(G3) $B_i(F(do(a, s))) \leftrightarrow \Gamma_{i_1,F}^+(a, s) \vee B_i(F(s)) \wedge \neg\Gamma_{i_1,F}^-(a, s)$

(G4) $B_i(\neg F(do(a, s))) \leftrightarrow \Gamma_{i_2,F}^+(a, s) \vee B_i(\neg F(s)) \wedge \neg\Gamma_{i_2,F}^-(a, s)$

where $\Gamma_{i_1,F}^+$, $\Gamma_{i_1,F}^-$, $\Gamma_{i_2,F}^+$, and $\Gamma_{i_2,F}^-$ are formulas in L_M.

We also have in T unique name axioms for actions and for situations, and we assume that modal operators obey the (KD) logic (see [Che88]).

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[5] As a matter of simplification the arguments of function symbols are not explicited, and, for fluents, the only argument which is explicited is the situation. For instance, we could have $a(x_1)$ and $F(x_1, x_2, s)$. Also, it is assumed that all the free variables are universally quantified.

[6] To avoid to have equality in the scope of modal operators, we assume that fluent functions are expressed via fluent predicates, i.e. $y = f(x, s)$ is expressed by $F(y, x, s)$.

Moreover, it is assumed that for each fluent F we have [7]:

$(H1)\ \ \vdash T \rightarrow \forall\neg(\Gamma_F^+ \wedge \Gamma_F^-)$
$(H2)\ \ \vdash T \rightarrow \forall\neg(\Gamma_{i_1,F}^+ \wedge \Gamma_{i_1,F}^-)$
$(H3)\ \ \vdash T \rightarrow \forall\neg(\Gamma_{i_2,F}^+ \wedge \Gamma_{i_2,F}^-)$
$(H4)\ \ \vdash T \rightarrow \forall\neg(\Gamma_{i_1,F}^+ \wedge \Gamma_{i_2,F}^+)$
$(H5)\ \ \vdash T \rightarrow \forall(B_i(F(s)) \wedge \Gamma_{i_2,F}^+ \rightarrow \Gamma_{i_1,F}^-)$
$(H6)\ \ \vdash T \rightarrow \forall(B_i(\neg F(s)) \wedge \Gamma_{i_1,F}^+ \rightarrow \Gamma_{i_2,F}^-)$

The three assumptions (H4), (H5) and (H6) guarantee that if agents' beliefs are consistent in the intial state, they are consistent in all the successor states.

It can easily be shown that, if we have (H1), in the context of the theory T, successor state axioms like (G2) are equivalent to the conjunction of properties (A1), (A2), (A3), and (A4).

(A1) $\Gamma_F^+(a,s) \rightarrow F(do(a,s))$
(A2) $\Gamma_F^-(a,s) \rightarrow \neg F(do(a,s))$
(A3) $\neg\Gamma_F^-(a,s) \rightarrow [F(s) \rightarrow F(do(a,s))]$
(A4) $\neg\Gamma_F^+(a,s) \rightarrow [\neg F(s) \rightarrow \neg F(do(a,s))]$

In a similar way we have shown that, if we have (H2) and (H3), in the context of the theory T, successor belief state axioms of the form (G3) (resp. (G4)) are equivalent to the conjunction of properties (B1), (B2), (B3) and (B4) (resp. (C1), (C2), (C3) and (C4)).

(B1) $\Gamma_{i_1,F}^+(a,s) \rightarrow B_i(F(do(a,s)))$
(B2) $\Gamma_{i_1,F}^-(a,s) \rightarrow \neg B_i(F(do(a,s)))$
(B3) $\neg\Gamma_{i_1,F}^-(a,s) \rightarrow [B_i(F(s)) \rightarrow B_i(F(do(a,s)))]$
(B4) $\neg\Gamma_{i_1,F}^+(a,s) \rightarrow [\neg B_i(F(s)) \rightarrow \neg B_i(F(do(a,s)))]$
(C1) $\Gamma_{i_2,F}^+(a,s) \rightarrow B_i(\neg F(do(a,s)))$
(C2) $\Gamma_{i_2,F}^-(a,s) \rightarrow \neg B_i(\neg F(do(a,s)))$
(C3) $\neg\Gamma_{i_2,F}^-(a,s) \rightarrow [B_i(\neg F(s)) \rightarrow B_i(\neg F(do(a,s)))]$
(C4) $\neg\Gamma_{i_2,F}^+(a,s) \rightarrow [\neg B_i(\neg F(s)) \rightarrow \neg B_i(\neg F(do(a,s)))]$

Properties (B3) and (C3) show that positive beliefs remain unchanged after performance of an action as long as $\neg\Gamma_{i_1,F}^-(a,s)$ and $\neg\Gamma_{i_2,F}^-(a,s)$ holds. Properties (B4) and (C4) show that negative beliefs remain unchanged after performance of an action as long as $\neg\Gamma_{i_1,F}^+(a,s)$ and $\neg\Gamma_{i_2,F}^+(a,s)$ holds.

Definition 1. Regression operator. We define a regression operator R_T from formulas in L_M to formulas in L_M.

1. When W is a non fluent atom, including equality atoms, or when W is a fluent atom whose situation argument is the constant $S0$, $R_T[W] = W$.

2. When W is an atom formed with fluent F of the form $F(\mathbf{t}, do(\alpha,\sigma))$ whose successor state axiom in T is [8] $\forall a \forall s \forall \mathbf{x}[F(\mathbf{x}, do(a,s)) \leftrightarrow \Phi_F]$ then:

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[7] Here, we use the symbol $\forall$ to denote the universal closure of all the free variables in the scope of $\forall$.

[8] We use the notation $\mathbf{x}$ for the tuple of variables $x_1,\ldots,x_n$, and $\forall\mathbf{x}$ for $\forall x_1 \ldots \forall x_n$; $\Phi_F.\{\mathbf{x}/\mathbf{t}, a/\alpha, s/\sigma\}$ denotes the result of the application of the substitution $\{\mathbf{x}/\mathbf{t}, a/\alpha, s/\sigma\}$ to formula Φ_F.

$R_T[F(\mathbf{t}, do(\alpha,\sigma))] = R_T[\Phi_F.\{\mathbf{x}/\mathbf{t}, a/\alpha, s/\sigma\}]$

3. When W is an atom of the form $Poss(\alpha(\mathbf{t}),\sigma)$, whose action precondition axiom is $\forall s \forall \mathbf{x} Poss(\alpha(\mathbf{x}),\sigma) \leftrightarrow \Pi_\alpha(\mathbf{x}, s)$ then:

$R_T[Poss(\alpha(\mathbf{t}), s)] = R_T[\Pi_\alpha(\mathbf{x}, s).\{\mathbf{x}/\mathbf{t}, s/\sigma\}]$

4. When W is a modal literal of the form $B_i(F(\mathbf{t}, do(\alpha,\sigma)))$ or $B_i(\neg F(\mathbf{t}, do(\alpha,\sigma)))$ whose successor belief state axioms in T are $\forall a \forall s \forall \mathbf{x}[B_i(F(\mathbf{x}, do(a,s))) \leftrightarrow \Phi_{i_1,F}]$ and $\forall a \forall s \forall \mathbf{x}[B_i(\neg F(\mathbf{x}, do(a,s))) \leftrightarrow \Phi_{i_2,F}]$ then:

$R_T[B_i(F(\mathbf{t}, do(\alpha,\sigma)))] = R_T[\Phi_{i_1,F}.\{\mathbf{x}/\mathbf{t}, a/\alpha, s/\sigma\}]$ and

$R_T[B_i(\neg F(\mathbf{t}, do(\alpha,\sigma)))] = R_T[\Phi_{i_2,F}.\{\mathbf{x}/\mathbf{t}, a/\alpha, s/\sigma\}]$

5. When W is a formula in L_M [9], $R_T[\neg W] = \neg R_T[W]$ and $R_T[\exists x W] = \exists x R_T[W]$.

6. When W_1 and W_2 are formulas in L_M, $R_T[W_1 \vee W_2] = R_T[W_1] \vee R_T[W_2]$.

Theorem 2. *Let T_0 be a set of closed sentences in L_M, without Poss predicate, and whose situation argument in fluents is $S0$. Let T_{ss} be the set of precondition axioms and of successor axioms for the fluents of a given application. Let T_u be the set of unique name axioms. We use notations $T = T_u \cup T_{ss} \cup T_0$ and $T' = T_u \cup T_0$. Let $R_T^*(\phi)$ be the result of repeated applications of R_T until the result is unchanged. Let s_{gr} be a ground situation term.*

We have $\vdash T \rightarrow W(s_{gr})$ iff $\vdash T' \rightarrow R_T^*[W(s_{gr})]$

For the proof we can use the same technique as Scherl and Levesque in [SL93], but the proof is much more simple because we do not have explicit accessibility relation to represent modal operators (see next section).

This theorem shows that to prove W in situation s_{gr} comes to prove $R_T^*[W]$ in situation $S0$, droping axioms of the kind (G1), (G2), (G3) and (G4).

Theorem 2 can be used for different purposes. The most important of them, as mentioned by Reiter in [Rei99], is to check whether a given sequence of actions is executable, in the sense that after performing any of these actions, the preconditions to perform the next action are satisfied. Another one, is to check whether some property holds after performance of a given sequence of actions. These two features are essential for plan generation.

We also have the following theorem.

Theorem 3. *Let A be a formula of the form F, B_iF or $B_i\neg F$, where F is an atom formed with a fluent predicate. Let T be a theory such that for every successor axiom of the form: $A(\mathbf{x}, do(a,s)) \leftrightarrow \Gamma_A^+(\mathbf{x}, a, s) \vee A(\mathbf{x}, s) \wedge \neg \Gamma_A^-(\mathbf{x}, a, s)$, there is no other variable that occurs in Γ_A^+ or Γ_A^- than the variables in $\mathbf{x}$, or a or s. Let $\phi(s)$ be a formula in L_M such that the only variable that occurs in ϕ is s.*

If for every ground formula $A(S0)$ we have either $\vdash T \rightarrow A(S0)$ or $\vdash T \rightarrow \neg A(S0)$, then, for every ground situation term s_{gr}, which is a successor situation of $S0$, we have either $\vdash T \rightarrow \phi(s_{gr})$ or $\vdash T \rightarrow \neg\phi(s_{gr})$.

[9] The definition of R_T for universal quantifier $\forall$, conjunction $\wedge$, implication $\rightarrow$ and equivalence $\leftrightarrow$, is directly obtained from the usual definitions of these quantifier and logical connectives in function of $\exists$, $\neg$ and $\vee$.

The proof is by induction on the complexity of the formula ϕ in $S0$, and by induction on the depth of the term s_{gr}. Theorem 3 intuitively says that if we have a complete description of what the agents believe in $S0$, then we have a complete description of their beliefs in every successor situation.

4 Related works

In [SL93] Scherl and Levesque have defined an extension of Situation Calculus to Epistemic Logic for a unique modal operator, but without any restriction about formulas that are in the scope of the modal operator.

In their approach, the first idea is to define the modal operator $Knows$ in terms of an accessibility relation K which is explicitly represented in the axiomatics. Formally, they have: $Knows(\phi, s) \stackrel{\text{def}}{=} \forall s'(K(s', s) \rightarrow \phi(s'))$. The second idea is to define knowledge change by defining accessibility relation change. Moreover, two kinds of actions are distinguished: knowledge-producing actions, denoted by $\alpha_1, \ldots, \alpha_n$, and non-knowledge-producing actions. Each action α_i informs the agent in the situation $do(\alpha_i, s)$ about the fact that some formula p_i is true or false in the situation s. It is assumed that the action α_i does not change the state of the world. From a technical point of view, after the performance of action α_i, relation K selects, in the situation $do(\alpha_i, s)$, those situations where p_i has the same truth value as it has in the situation s. For instance, if p_i is true in s, then situations s', which are accessible from s and where p_i is false, are no more accessible from $do(\alpha_i, s)$. Notice that if p_i is false in all the situations which are accessible from s, there is no situation accessible from $do(\alpha_i, s)$. That means that the agent believes any formula.

This problem disappear in the logical framework presented by Shapiro et al. in [SPL00], where a plausibility degree $pl(s)$ is assigned to all the situations. From the accessibility relation $B(s', s)$, an accessibility relation $B_{max}(s', s)$ between s and the most plausible situations can be defined by $B_{max}(s', s) \stackrel{\text{def}}{=} B(s', s) \wedge \forall s''(B(s'', s) \rightarrow pl(s') \leq pl(s''))$. Then, the fact that an agent believes ϕ in s is defined as $Bel(\phi, s) \stackrel{\text{def}}{=} \forall s'(B_{max}(s', s) \rightarrow \phi(s'))$. Here, an agent can consistently believe ϕ in $do(a, s)$, while he believed $\neg\phi$ in s, provided there exists at least one most plausible situation related to $do(a, s)$ where ϕ holds.

For a non-knowledge-producing action a, it is assumed that knowledge changes in the same way as the world does. That is, if a situation s' is accessible from s, the situation $do(a, s')$ is accessible from $do(a, s)$ as well. In formal terms, the evolution of relation K is defined by the following axiom [10]:

$Poss(a, s) \rightarrow$
$[K(s'', do(a, s)) \leftrightarrow \exists s' K(s', s) \wedge s'' = do(a, s') \wedge Poss(a, s') \wedge$
$((\neg(a = \alpha_1) \wedge \ldots \wedge \neg(a = \alpha_n)) \vee$
$a = \alpha_1 \wedge (p_1(s) \leftrightarrow p_1(s')) \vee$
$\ldots$
$a = \alpha_n \wedge (p_n(s) \leftrightarrow p_n(s')))]$

[10] In fact, condition $Poss(a, s')$ is not present in [SL93], it was added in [LL98].

This successor axiom does not explicitly define which formulas are true or false in $do(a, s')$. From the examples presented in their paper we understand that the truth value of formulas in situations like s'' is defined by the successor state axioms of the type (G2). That implicitly means that: i) whenever some action has been performed the agent knows that this action has been performed, ii) the agents knows the effects of all the actions, i.e he knows all the successor state axioms, and iii) when the agent get information through a knowledge-producing action, this information is always true information, in the sense that this information is true in the situation s where he is.

How this formalisation could be extended to the context of multi-agents? The fact i) cannot be accepted in general. We can accept that an agent knows that an action has been performed when it has been performed by himself, but not necessarily when it has been performed by another agent. This problem could be solved by defining as many accessibility relations K_i as there are distinct agents, and by distinguishing for each agent those actions $\beta_1, \ldots, \beta_m$ which are performed by the agent i. For an action a which is neither of the sort β_j nor α_k, the fact that knowledge does not change can be represented by the fact that accessible situations from $do(a, s)$ are the same as accessible situations from s. That could lead to successor axioms for each relation K_i of the form:

$$
\begin{array}{l}
Poss(a, s) \rightarrow \\
{[K_i(s'', do(a, s)) \leftrightarrow} \\
(K_i(s'', s) \wedge \neg(a = \alpha_1) \wedge \ldots \wedge \neg(a = \alpha_n) \wedge \neg(a = \beta_1) \wedge \ldots \wedge \neg(a = \beta_m)) \vee \\
(\exists s' K_i(s', s) \wedge s'' = do(a, s') \wedge Poss(a, s') \wedge \\
(a = \beta_1 \vee \ldots \vee a = \beta_m \vee \\
a = \alpha_1 \wedge (p_1(s) \leftrightarrow p_1(s')) \vee \\
\ldots \\
a = \alpha_n \wedge (p_n(s) \leftrightarrow p_n(s')))]
\end{array}
$$

However, even with this extension there are still the problems related to ii) and iii). For ii), the problems is that in real situations agents may have wrong beliefs about the the evolution of the world. For instance, an agent may believe that droping a fragile object make it broken, while another agent may believe that the object is not necessarily broken, depending on his weight or on other particular circumstances. This raises the question of how to represent in this framework different evolutions of the world in the context of different agents beliefs? May be a possible answer is to have different successor state axioms, for the same fluent, to represent the "true" evolution of the world, and to represent the evolution of the world in the context of each agent's beliefs. That is, more or less, the idea we have proposed in this paper with the axioms of the type (G3) and (G4).

For iii), the problem is that there are applications where agents may receive information from different sensors, or from other agents, some of them are not necessarily reliable and may communicate wrong information. Here again, we believe that axioms like (G3) and (G4) are a possible solution, because they allow us to represent communication actions whose consequences are to generate wrong agents beliefs.

5 Conclusion

In conclusion, we have presented a general framework to solve the frame problem in the context of a limited extension of Situation Calculus to Epistemic logic. Even if for this solution strong restrictions are imposed on the language L_M, we can express non trivial properties like: $\forall s \forall x (B_r position(x,s) \rightarrow position(x,s))$, which means that in every situation the robot has true beliefs about his position, or $\forall s \forall x (B_r obstacle(x,s) \rightarrow B_p obstacle(x,s))$, which means that the robot's beliefs about obstacles are a subset of the pilot's beliefs about obstacles. Also, since in the (KD) logics we have $B_i(l \wedge l') \leftrightarrow B_i l \wedge B_i l'$, it would be a trivial extension to L_M to allow conjunction of literals in the scope of modal operators. Finally, the regression operator R_T allows us to check whether these kinds of properties can be derived from T_0. The implementation of a modal theorem prover for the restricted language L_M should not be a big issue. We are currently working on this aspect.

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