THEORY *NaturalOp*

IMPORT THEORY *Natural*

OPERATORS

add *associative commutative* ***expression*** $(n1 : iNAT, n2 : iNAT)$

direct definition

$mk\_iNAT(mk\_int(n1) + mk\_int(n2))$

miniNAT ***predicate*** $(set : \mathbb{P}(iNAT))$

well−definedness $set \neq \emptyset$

direct definition

$\exists i \cdot i \in set \wedge (\forall j \cdot j \in set \Rightarrow mk\_int(i) \leq mk\_int(j))$

THEOREMS

succ add left :

$\forall n1, n2 \cdot n1\, add\, iSucc(n2) = iSucc(n1\, add\, n2)$

succ add right :

$\forall n1, n2 \cdot iSucc(n1)\, add\, n2 = iSucc(n1\, add\, n2)$

thm3 :

$\forall set \cdot set \neq \emptyset \Rightarrow miniNAT(set)$

PROOF RULES

rulesBlock1 :

Metavariables

$n1 : iNAT$

$n2 : iNAT$

Rewrite Rules

rew1 : $n1\, add\, iSucc(n2)$

rhs1 : $\top \Rightarrow iSucc(n1\, add\, n2)$

rew2 : $iSucc(n1)\, add\, n2$

rhs1 : $\top \Rightarrow iSucc(n1\, add\, n2)$

Inference Rules

inf1 : $n1 \in iNAT, n2 \in iNAT \vdash mk\_int(n1) + mk\_int(n2) \in \mathbb{N}$

inf2 : $n1 \in iNAT, n2 \in iNAT \vdash mk\_int(n1\, add\, n2) \geq mk\_int(n1)$

inf3 : $n1 \in iNAT, n2 \in iNAT \vdash mk\_int(n1\, add\, n2) \geq mk\_int(n2)$

END