

THEORY *Natural***DATA TYPES***iNAT***CONSTRUCTORS***iZero* ()*iSucc* (*iPrec* : *iNAT*)**OPERATORS****mk_int** *expression* (*n* : *iNAT*)

recursive definition

case *n*:*iZero* \Rightarrow

0

iSucc(*m*) \Rightarrow 1 + *mk_int*(*m*)**AXIOMATIC DEFINITIONS** *xdb1*:**OPERATORS****mk_iNAT** *expression* (*x* : \mathbb{Z}) : *iNAT*well-definedness $x \in \mathbb{N}$ **AXIOMS***axm1* : *mk_iNAT*(0) = *iZero**axm2* : $\forall x \cdot x \in \mathbb{N} \Rightarrow \text{mk_iNAT}(1 + x) = \text{iSucc}(\text{mk_iNAT}(x))$ *axm3* : $\forall x \cdot x \in \mathbb{N} \Rightarrow \text{mk_int}(\text{mk_iNAT}(x)) = x$ **THEOREMS***thm1* : $\forall n \cdot n \in \text{iNAT} \Rightarrow \text{mk_int}(n) \in \mathbb{N}$ *thm2* : $\forall n \cdot n \in \text{iNAT} \Rightarrow \text{mk_iNAT}(\text{mk_int}(n)) = n$ *thm3* : $\forall n \cdot n \in \mathbb{N} \Rightarrow (\exists ni \cdot ni \in \text{iNAT} \wedge \text{mk_iNAT}(ni) = n)$ *thm4* : $\forall x \cdot x \in \mathbb{N} \Rightarrow (\exists ni \cdot \text{mk_iNAT}(ni) = \text{iSucc}(ni))$ **END**