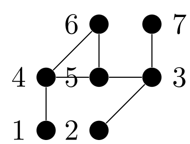
TD 1: Elementary concepts

Exercice 1 In town T., a bank robbery has been committed. According to recordings of the bank's video, 3 persons have participated in the robbery, and it is evident that they must have called each other shortly before the robbery to coordinate their action. The following listing show the phone connections established in the minutes before the robbery.

(1, 2), (2, 3), (2, 4), (7, 8), (4, 6), (6, 7), (2, 5), (5, 7), (5, 8) (4, 5)

Which groups of persons are suspect?

Exercice 2 A factory manipulates aggressive chemical substances that are mutually incompatible and may therefore never be stored in the same or adjacent rooms. Here is a plan of the factory's rooms:



How many substances can be stored in the factory at the same time?

Exercice 3 Organizing first aid in rural areas is a difficult problem: in case of an emergency, an ambulance has to reach its destination in at most 5 minutes, but to cut down costs, one wants to deploy as few ambulances as possible.

The following graph shows a map of villages (those which are ≤ 5 minutes apart are connected).

(take the graph of Exercise 2).

How many ambulances have to be deployed at least, and where?

Exercice 4 The US secret service intends to extend its Echelon system so as to get hold of every piece of information travelling on the communication lines of town T.:

(take the graph of Exercise 2).

On which routers (nodes) does it have to install its spy software?

Exercice 5 Extend the algorithm underlying the computation of Stirling numbers so as to effectively compute all k-element partitions of an n-element set (and not only determine their number).

Exercice 6 Explore the relation between independent sets and graph coloring.

Exercice 7 Consider the following imperative program[1]

[1] From: S. Muchnick, *Advanced Compiler Design and Implementation*, Morgan Kaufmann, 1997

```
s1 := 2;
s2 := 4;
s3 := s1 + s2;
s4 := s1 + 1;
s5 := s1 * s2;
s6 := s4 * 2;
```

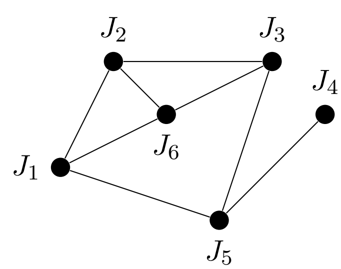
Here, the si are variables that have to be allocated to three registers r_1, r_2, r_3 with the side condition that s4 and r_1 are incompatible, i.e. should not be allocated to (for example for reasons of typing such as: s4 is an integer value, but r_1 a floating-point register).

1. Draw an *interference graph* containing the si and r_i. This graph contains as nodes all the si and r_i, and an arc between two nodes should be mapped to two different registers. This is the case for incompatible locations, and for interrupted define-use-chains: the variable would be redefined before a later use.

2. Color the graph (in our case with 3 colors, corresponding to the registers r_1, r_2, r_3).

Exercice 8 A given number of jobs $J_1, \ldots J_k$ have to be distributed on a (still unknown) number of machines $M_1, \ldots, M_n$. We suppose that the working time for each job is the same constant T. Two jobs J_a, J_b requiring the same machine are *mutually exclusive*: $J_a \not\leftrightarrow J_b$.

Consider specifically the problem of jobs $1 \ldots 6$, where two jobs are mutually exclusive if there is an arc between them:



Determine a minimal number of machines to do execute the jobs.