Evaluation of Arguments from Support Relations: Axioms and Semantics

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Abstract
This paper focuses on argumentation graphs whose nodes are arguments and edges represent supports, thus positive relations, between arguments. Furthermore, each argument has a weight reflecting its basic or intrinsic strength. For the sake of generality, the internal structure of arguments and the origin of arguments and their weights are unspecified. The paper tackles for the first time the question of evaluating the overall strengths of arguments in such graphs, thus of defining semantics for support graphs. It introduces a set of axioms that any semantics should satisfy. Then, it defines three semantics and evaluates them against the axioms.

1 Introduction
Argumentation is a social activity whose aim is to increase (or decrease) the acceptability of a given standpoint for an audience by putting forward arguments. The standpoint may be a claim which can be true or false, an action to be performed, a goal to be reached, etc. Argumentation has gained great interest in Artificial Intelligence. It is used for decision making (e.g., [Amgoud and Prade, 2009]), defeasible reasoning (e.g., [Bondarenko et al., 1997]), and negotiation (e.g., [Reed, 1998]). Interested reader can find more on applications of argumentation in [Rahwan and Simari(eds.), 2009].

Whatever the application, an argumentation-based formalism, called argumentation framework, is generally defined as a set of arguments, attacks amongst the arguments, and a semantics for evaluating the arguments. A semantics assesses to what extent an argument is acceptable. Examples of semantics are those proposed by Dung [1995] and ranking semantics (e.g., [Amgoud and Ben-Naim, 2013; Besnard and Hunter, 2001; da Costa Pereira et al., 2011; Matt and Toni, 2008]).

An attack has a negative effect since its aim is weakening its target. Cayrol and Lagasque [2005] have pointed out another meaningful relation between arguments, the so-called support. Unlike attack, this relation has positive effect and aims at strengthening its target. Several works have thus been done on the evaluation of arguments in argumentation frameworks where supports and attacks coexist [Oren and Norman, 2008; Boella et al., 2010; Brewka and Woltran, 2010; Nouioua, 2013; Polberg and Oren, 2014]. They have extended Dung’s semantics [1995] for accounting for supports. Surprisingly enough, when the attack relation is empty, these works consider all the arguments as equally acceptable. Thus, supported arguments are as acceptable as non-supported ones. This means that supports are not fully harnessed. Furthermore, these works assumed that arguments have the same basic strength, an assumption hardly ever satisfied in practice. Each argument has a basic strength representing the weight of its source [Parsons et al., 2011], or the importance degree of the value it promotes [Bench-Capon, 2003], or the importance degrees of the goals it supports [Amgoud and Prade, 2009], or the certainty degrees of its premises [Benferhat et al., 1993], . . .

In this paper, we investigate for the first time argumentation frameworks where arguments interact only in a positive way (i.e., by supporting each other). Furthermore, arguments may have different basic strengths. Such frameworks have interesting practical applications like recommendation letters. A recommendation letter contains a general recommendation justified by a list of arguments, some of them support others. The evaluation of each argument, by combining its basic strength and the overall strengths of its supporters, gives insights on the weight of the general recommendation. Another application is the evaluation of newspapers’ articles, namely argumentative essays. From a theoretical perspective, defining semantics for evaluating arguments on the basis of supports allows a better understanding of the role and impact of supports, and how they should be considered.

The contribution of the paper is threefold. First, we provide a set of axioms that a semantics should satisfy. Most of the axioms are mandatory, except three which represent pairwise incompatible choices. Second, we investigate some properties of semantics that satisfy the axioms. Third, we propose three semantics, each of which satisfies all the mandatory axioms and one of the three optional axioms.

The paper is structured as follows: Section 2 introduces the argumentation graphs we are interested in. Section 3 presents the axioms. Section 4 is devoted to properties of semantics. Section 5 defines three semantics. The last section concludes.

2 Basic Concepts
A support argumentation framework, called also support argumentation graph throughout the paper, is a set of arguments
and a binary relation representing supports amongst arguments. Arguments are abstract entities whose internal structure is not specified. Each argument has an intrinsic strength which is expressed by a numerical value in the interval $[0,1]$. The value 0 means the argument is worthless while 1 means the argument is very strong. Before introducing the graphs we are dealing with, let us first define the notion of weighting.

**Definition 1 (Weighting)** A weighting on a set $L$ is a function from $L$ to $[0,1]$.

Let us now introduce support argumentation graphs.

**Definition 2 (Support Argumentation Graph)** $A$ support argumentation graph (SAF) is an ordered tuple $A = (A,w,S)$, where $A$ is a non empty finite set of arguments, $w$ is a weighting on $A$, and $S \subseteq A \times A$ is a support relation. For $a, b \in A$, the notation $a \mathbin{S} b$ means $a$ supports $b$.

We define a semantics as a function assigning for every argument in a SAF a value between 0 and 1. This value, called strength degree, represents the overall strength of the argument, i.e., the strength issued from the aggregation of the basic strength of the argument and the overall strengths of its supporters. The higher the degree, the stronger the argument.

**Definition 3 (Semantics)** A semantics is a function $F$ transforming any SAF $A = (A,w,S)$ into a weighting on $A$. For $a \in A$, $\text{Deg}_A^F(a)$ denotes the image of argument $a$ by $F(A)$, and is called strength degree of $a$.

Below is the list of all notations used in the paper.

**Notations:** Let $A = (A,w,S)$ be a SAF and $a \in A$. We denote by $\text{Supp}_a(A)$ the set of all supports of $a$ in $A$ (i.e. $\text{Supp}_a(A) = \{ b \in A \mid b \mathbin{S} a \}$). For any two SAFs $A = (A,w,S)$ and $A' = (A',w',S')$, $A \oplus A'$ is the SAF $\langle A \cup A', w'' = w(x) + w'(x) \rangle$ where for any $x \in A$ (resp. $x \in A'$), $w''(x) = w(x)$ (resp. $w''(x) = w'(x)$).

### 3 Axiomatic Foundations of Semantics

We propose a set of 17 axioms that shed light on foundational concepts and principles behind a semantics. Some of the axioms are dual to those proposed in [Amgoud and Ben-Naim, 2016] in case of attack graphs. Before presenting the first axiom, let us recall the definition of isomorphism.

**Definition 4 (Isomorphism)** Let $A = (A,w,S)$ and $A' = (A',w',S')$ be two SAFs. An isomorphism from $A$ to $A'$ is a bijective function $f$ from $A$ to $A'$ such that:

- $\forall a \in A$, $w(a) = w'(f(a))$, and
- $\forall a, b \in A$, $a \mathbin{S} b$ iff $f(a) \mathbin{S}' f(b)$.

The first basic axiom ensures that the strength degree of an argument does not depend on the argument’s identity.

**Axiom 1 (Anonymity)** A semantics $F$ satisfies anonymity iff, for any two SAFs $A = (A,w,S)$ and $A' = (A',w',S')$, for any isomorphism $f$ from $A$ to $A'$, the following property holds: $\forall a \in A$, $\text{Deg}_A^F(a) = \text{Deg}_{A'}^F(f(a))$.

The second basic axiom, called independence, states the following: the strength degree of an argument $a$ should be independent of any argument that is not connected to $a$ (i.e., there is no path from that argument to $a$).

**Axiom 2 (Independence)** A semantics $F$ satisfies independence iff, for any two SAFs $A = (A,w,S)$ and $A' = (A',w',S')$ such that $A \cap A' = \emptyset$, the following property holds: $\forall a \in A$, $\text{Deg}_A^F(a) = \text{Deg}_{A \oplus A'}^F(a)$.

Let us consider the following running example.

**Example 1** Let $A$ be the support argumentation graph depicted below.

```
1. a
2. d
3. b
4. c
5. e
6. d'
7. e'
```

Assume that $w(a) = w(b) = w(d) = w(d') = w(e) = w(e') = \frac{1}{7}$, $w(x) = 0.2$ and $w(c) = \frac{3}{14}$. Independence ensures that the degree of $a$ is independent of $x$’s since the two arguments are not linked.

The next axiom, called non-dilution, states that supporting arguments has no impact on its own strength degree.

**Axiom 3 (Non-Dilution)** A semantics $F$ satisfies non dilution iff, for any two SAFs $A = (A,w,S)$ and $A' = (A',w',S')$ such that $A = A'$, $w = w'$ and $S' = S \cup \{(a,b)\}$ with $\text{Supp}^+_{A}(b) = \emptyset$, the following property holds: $\forall a \in A \setminus \{b\}$, $\text{Deg}_A^F(x) = \text{Deg}_{A'}^F(x)$.

**Example 1 (Cont)** Non-dilution ensures that the degree of $a$ should not take into account the fact that $a$ supports $d$.

It is worth mentioning that non dilution is not mandatory in reputation systems. PageRank, the reputation system used by Google search engine for ranking web pages, violates the axiom. It considers that the more an agent (a web page) supports other agents, the less it is credible.

The following axiom, called dummy, states that arguments that get value 0 have no impact on the arguments they support.

**Axiom 4 (Dummy)** A semantics $F$ satisfies dummy iff, for any SAF $A = (A,w,S)$, for all $a,b \in A$, if

- $w(a) = 0$,
- $\text{Supp}_A(a) = \text{Supp}_A(b) \cup \{x\}$ such that $\text{Deg}_A^F(x) = 0$, then $\text{Deg}_A^F(a) = \text{Deg}_A^F(b)$.

The next axiom, called monotony, ensures that an argument becomes stronger when its set of supporters gets bigger.

**Axiom 5 (Monotony)** A semantics $F$ satisfies monotony iff, for any SAF $A = (A,w,S)$, for all $a,b \in A$, if

- $w(a) = w(b)$,
- $\text{Supp}_A(b) \subseteq \text{Supp}_A(a)$,

then $\text{Deg}_A^F(a) \geq \text{Deg}_A^F(b)$.

**Example 1 (Cont)** Monotony ensures $\text{Deg}_A^F(d') \geq \text{Deg}_A^F(d) \geq \text{Deg}_A^F(b)$, and $\text{Deg}_A^F(e') \geq \text{Deg}_A^F(e)$.
Axiom 6 (Equivalence) A semantics $F$ satisfies equivalence iff, for any SAF $A = \langle A, w, S \rangle$, for all $a, b \in A$, if
- $w(a) = w(b)$,
- $\exists f, a$ bijective function, such that $\text{Supp}_A(a) \to \text{Supp}_A(b)$ s.t. $\forall x \in \text{Supp}_A(a), \text{Deg}_A^F(x) = \text{Deg}_A^F(f(x))$,
then $\text{Deg}_A^F(a) = \text{Deg}_A^F(b)$.

The axiom called coherence, states that the impact of support is proportional to the basic strength of its target.

Axiom 7 (Coherence) A semantics $F$ satisfies coherence iff, for any SAF $A = \langle A, w, S \rangle$, for all $a, b \in A$, if
- $w(a) > w(b)$,
- $\text{Supp}_A(a) = \text{Supp}_A(b)$,
then $\text{Deg}_A^F(a) > \text{Deg}_A^F(b)$.

The minimality axiom ensures that if an argument is not supported, its overall strength is equal to its basic strength.

Axiom 8 (Minimality) A semantics $F$ satisfies minimality iff, for any SAF $A = \langle A, w, S \rangle$, for any argument $a \in A$, if $\text{Supp}_A(a) = \emptyset$, then $\text{Deg}_A^F(a) = w(a)$.

Example 1 (Cont) From minimality, $\text{Deg}_A^F(a) = \frac{1}{3}$ and $\text{Deg}_A^F(x) = 0.2$.

The following axiom, called strengthening, defines formally the role of supports. It states that a support strengthens its target by increasing its overall strength (possibly only by an infinitesimal amount). This is particularly true when the support emanates from (even slightly) acceptable arguments (i.e., an argument s.t. $\text{Deg}_A^F(.) > 0$). If the degree of the argument is already 1, the supports are useless.

Axiom 9 (Strengthening) A semantics $F$ satisfies strengthening iff, for any SAF $A = \langle A, w, S \rangle$, for any argument $a \in A$, if $w(a) < 1$ and $\exists b \in \text{Supp}_A(a)$ s.t. $\text{Deg}_A^F(b) > 0$, then $\text{Deg}_A^F(a) > w(a)$.

Example 1 (Cont) Strengthening ensures that $\text{Deg}_A^F(b) > \frac{1}{3}$, $\text{Deg}_A^F(d) > \frac{1}{4}$, $\text{Deg}_A^F(c) > \frac{2}{3}$, $\text{Deg}_A^F(d') > \frac{1}{3}$, $\text{Deg}_A^F(e) > \frac{1}{3}$, and $\text{Deg}_A^F(e') > \frac{1}{3}$.

The next axiom, called strengthening soundness, states that the only way of increasing the overall strength of an argument is by supporting the argument with an acceptable one.

Axiom 10 (Strengthening Soundness) A semantics $F$ satisfies strengthening soundness iff, for any SAF $A = \langle A, w, S \rangle$, for any argument $a \in A$, if $\text{Deg}_A^F(a) > w(a)$ then $\exists b \in \text{Supp}_A(a)$ s.t. $\text{Deg}_A^F(b) > 0$.

The three previous axioms are the role of supports, which is strengthening arguments. The following axioms introduce two key factors that may impact the overall strength of an argument: the number of supporters and their quality. The more numerous the acceptable supporters of an argument, the stronger the argument.

Axiom 11 (Counting) A semantics $F$ satisfies counting iff, for any SAF $A = \langle A, w, S \rangle$, for all $a, b \in A$ such that
- $w(a) = w(b)$, $\text{Deg}_A^F(b) < 1$,
- $\text{Supp}_A(a) = \text{Supp}_A(b) \cup \{y\}$ with $\text{Deg}_A^F(y) > 0$,
then $\text{Deg}_A^F(a) > \text{Deg}_A^F(b)$.

Example 1 (Cont) Counting ensures $\text{Deg}_A^F(d') > \text{Deg}_A^F(d) > \text{Deg}_A^F(b)$, and $\text{Deg}_A^F(e') > \text{Deg}_A^F(e)$.

The quality of acceptable supporters is another factor impacting the overall strength of an argument. The next axiom, called reinforcement, states that if the overall strength of a supporter is increased, then its target is strengthened further provided that its strength degree is not already maximal (1).

Axiom 12 (Reinforcement) A semantics $F$ satisfies reinforcement iff, for any SAF $A = \langle A, w, S \rangle$, for all $a, b \in A$ such that
- $w(a) = w(b)$,
- $\text{Supp}_A(a) \setminus \text{Supp}_A(b) = \{x\}$,
- $\text{Supp}_A(b) \setminus \text{Supp}_A(a) = \{y\}$,
- $\text{Deg}_A^F(x) > \text{Deg}_A^F(y) > 0$,
then $\text{Deg}_A^F(a) > \text{Deg}_A^F(b)$.

Our next axiom, called boundedness, states that an argument which has a maximal degree (1) keeps the same degree if one of its supporters is strengthened.

Axiom 13 (Boundedness) A semantics $F$ satisfies boundedness iff, for any SAF $A = \langle A, w, S \rangle$, for all $a, b \in A$ such that
- $w(a) = w(b)$,
- $\text{Supp}_A(a) \setminus \text{Supp}_A(b) = \{x\}$,
- $\text{Supp}_A(b) \setminus \text{Supp}_A(a) = \{y\}$,
- $\text{Deg}_A^F(x) > \text{Deg}_A^F(y)$,
if $\text{Deg}_A^F(b) = 1$, then $\text{Deg}_A^F(a) = 1$.

The previous axioms are all mandatory and should be satisfied by any semantics. The remaining axioms are optional. The first one, called imperfection, states that an argument can never get a maximal value if its basic strength is not maximal.

Axiom 14 (Imperfection) A semantics $F$ satisfies imperfection iff, for any SAF $A = \langle A, w, S \rangle$, for all $a \in A$ if $w(a) < 1$ then $\text{Deg}_A^F(a) < 1$.

The three last axioms give an overwhelming weight to either the number of supporters, or their quality, or allow some compensation. More precisely, cardinality precedence says that an argument $a$ is stronger than an argument $b$ if the acceptable supporters of $a$ are more numerous than those of $b$.

Axiom 15 (Cardinality Precedence) A semantics $F$ satisfies cardinality precedence iff, for any SAF $A = \langle A, w, S \rangle$, for all $a, b \in A$ such that
- $w(a) = w(b)$,
- $0 < \{x \in \text{Supp}_A(b) | \text{Deg}_A^F(x) > 0\} < \{y \in \text{Supp}_A(a) | \text{Deg}_A^F(y) > 0\}$,
- $\exists x \in \text{Supp}_A(b)$ s.t $\forall y \in \text{Supp}_A(a)$, $\text{Deg}_A^F(x) > \text{Deg}_A^F(y)$.
then \( \deg^F_A(a) > \deg^F_A(b) \).

The next axiom, quality precedence, prefers the quality to the quantity of supporters. It says that an argument \( a \) is stronger than an argument \( b \), if some supporter of \( a \) is stronger than any supporter of \( b \).

**Axiom 16 (Quality Precedence)** A semantics \( F \) satisfies quality precedence iff, for any SAF \( A = \langle A, w, S \rangle \), for all \( a, b \in A \), if

- \( w(a) = w(b) \), \( \deg^F_A(a) < 1 \),
- \( 0 < | \{ x \in \text{Supp}_A(b) : \deg^F_A(x) > 0 \} | < | \{ y \in \text{Supp}_A(a) : \deg^F_A(y) > 0 \} | \),
- \( \exists x \in \text{Supp}_A(b) \ s.t \forall y \in \text{Supp}_A(a), \deg^F_A(x) > \deg^F_A(y) \),

then \( \deg^F_A(a) < \deg^F_A(b) \).

The very last axiom, called compensation, states that a small number of strong supporters compensate a greater number of weak supporters.

**Axiom 17 (Compensation)** A semantics \( F \) satisfies compensation iff, for any SAF \( A = \langle A, w, S \rangle \), for all \( a, b \in A \), if

- \( w(a) = w(b) \),
- \( |\text{Supp}_A(a)| = n, |\text{Supp}_A(b)| = m \) with \( n < m \),
- \( \forall x \in \text{Supp}_A(a), \deg^F_A(x) = d, \forall y \in \text{Supp}_A(b), \deg^F_A(x) = d' \) with \( d > d' > 0 \)

then \( \deg^F_A(a) = \deg^F_A(b) \).

## 4 Properties
Some axioms are incompatible, that is they cannot be satisfied all together by a semantics.

**Proposition 1** There is no semantics which satisfies Cardinality Precedence and Quality Precedence (respectively Compensation). There is no semantics which satisfies Quality Precedence and Compensation. There is no semantics which satisfies both Quality Precedence and Counting. There is no semantics which satisfies Quality Precedence and Compensation. Theorem 5 (Top-based Function) Let \( A = \langle A, w, S \rangle \) be a support argumentation graph. We define the top-based function \( f^*_i \) from \( A \) to \([0, +\infty)\) as follows: for any argument \( a \in A \), for \( i \in \{0, 1, 2, \ldots\} \), if \( i = 0 \) then \( f^*_i(a) = w(a) \), otherwise \( f^*_i(a) = w(a) + (1 - w(a)) \max_{b \in \text{Supp}_A(a)} f^{i-1}_i(b) \). By convention, \( \max_{b \in \text{Supp}_A(a)} f^{0}_i(b) = 0 \) if \( \text{Supp}_A(a) = \emptyset \).

The value \( f^*_i(a) \) is the score of the argument \( a \) at step \( i \). This value may change at each step, however, it converges to a unique value as \( i \) becomes high.

**Theorem 2** The function \( f^*_i \) converges.

The top-based semantics is based on the previous scoring function. The strength degree of each argument is the limit reached using the scoring function \( f^*_i \).

**Definition 5 (Top-based Function)** A semantics \( F \) is a function \( F_{\text{top}} \) transforming any support argumentation graph \( A = \langle A, w, S \rangle \) into a weighting on \( A \) such that for any \( a \in A \), \( \deg^F_{A}(a) = \lim_{i \to +\infty} f^*_i(a) \).

We show next that the limit scores of arguments satisfy a nice property, namely the equation of Definition 5.

**Theorem 3** For any support argumentation graph \( A = \langle A, w, S \rangle \), for any \( a \in A \),

\[
\deg^F_{A}(a) = w(a) + (1 - w(a)) \max_{b \in \text{Supp}_A(a)} \deg^F_{A}(b)
\]

Let us now illustrate the semantics with an example.

**Example 1 (Cont)** The strength degrees of the arguments of graph \( A \) under semantics \( F \) are: \( \deg^F_{A}(a) = 1 \), \( \deg^F_{A}(b) = \frac{5}{7} \), \( \deg^F_{A}(c) = \frac{8}{9} \), \( \deg^F_{A}(d) = \frac{27}{28} \), \( \deg^F_{A}(d') = \frac{27}{28} \), \( \deg^F_{A}(e) = \frac{27}{28} \), \( \deg^F_{A}(e') = \frac{27}{28} \), and \( \deg^F_{A}(x) = 0.2 \). Notice that \( \text{Supp}_A(e) = \{ e \} \) and \( \text{Supp}_A(d) = \{ a, b \} \). The argument \( e \) has thus less supporters but its supporter \( e \) is stronger than both supporters of \( d \). Since \( Tbs \) satisfies quality precedence, then \( e \) is stronger than \( d \). Furthermore, \( d' \approx e' > e > c > d > b > a > x \), where \( c > e \) means \( e \) is stronger than \( c \) and \( d' \approx e' \) means \( d' \) is as strong as \( e' \).

The Top-based semantics satisfies quality precedence as well as all the mandatory axioms which are compatible with it.
Theorem 4 Table 1 summarizes the axioms satisfied (violated) by top-based semantics.

From Theorems 1 and 4, it follows that the strength degree of each argument \( a \) is in the interval \([w(a), 1]\).

Corollary 1 For any support argumentation graph \( A = \langle A, w, S \rangle \), for any \( a \in A \), \( \text{Deg}^\text{Rbs}_A(a) \in [w(a), 1] \).

5.2 Reward-based Semantics

The second semantics, called reward-based semantics, favours the number of supporters over their quality. Its basic idea is the following: an argument receives a reward for each of its supporters. The greater the number of supporters, the smaller the amount of the reward. The reward concerning the last supporter takes into account the quality of the supporters. Note that no particular order of arguments is needed. Furthermore, since arguments having degree 0 have no impact on the arguments they support (Dummy axiom), such supporters are not taken into account. We thus consider only founded supporters. An argument is founded if there exists at least one path leading to it such that the basic strength of the source of the path is not 0. As we will see later, a founded argument has necessarily a positive degree.

Definition 7 (Founded Argument) Let \( A = \langle A, w, S \rangle \) be a support argumentation graph and \( a \in A \). The argument \( a \) is founded iff there exists a finite sequence \( \langle a_0, a_1, \ldots, a_n \rangle \) of arguments such that \( w(a_0) > 0, a_n = a \) and for all \( i = 0, 1, \ldots, n - 1 \), \( a_i S a_{i+1} \). It is unfounded otherwise. Let \( \text{Supp}^F_A(a) \) denote the set of founded supporters of \( a \).

It is easy to show that the basic strength of an unfounded argument is 0. Furthermore, if at least one supporter of an argument is founded, then the argument itself is founded.

Proposition 3 Let \( A = \langle A, w, S \rangle \) be a support argumentation graph and \( a \in A \).

- If \( \text{Supp}^F_A(a) \neq \emptyset \), then \( a \) is founded.
- \( a \) is unfounded iff \( \text{Supp}^F_A(a) = \emptyset \) and \( w(a) = 0 \).

The reward-based semantics is based on a scoring function which assigns a numerical value to each argument. If an argument is not founded, then it receives a score 0. Otherwise, the function proceeds in multiple steps. In the initial step, it assigns to each argument its basic strength. Then in each step, it recomputes all the scores by taking into account the basic strength, the number of founded supporters and their scores in the previous step.

Definition 8 (Reward-based Function) Let \( A = \langle A, w, S \rangle \) be a support argumentation graph. We define the reward-based function \( f_r \) from \( A \) to \([0, +\infty)\) as follows: for any argument \( a \in A \), if \( a \) is unfounded, then \( f_r^0(a) = 0 \) for any \( i \in \{0, 1, 2, \ldots\} \). If \( a \) is founded, then for \( i \in \{0, 1, 2, \ldots\} \), if \( i = 0 \) then \( f_r^i(a) = w(a) \), otherwise

\[
f_r^i(a) = w(a) + (1 - w(a))\left(\frac{1}{2} \sum_{j=1}^{n-1} m + \frac{m}{2^n}\right),
\]

where \( n = |\text{Supp}^F_A(a)| \) and \( m = \frac{\sum_{b \in \text{Supp}^F_A(a)} f_r^{i-1}(b)}{n} \). By convention, \( \sum_{j=1}^{n-1} \frac{1}{2^j} + \frac{m}{2^n} = 0 \) if \( \text{Supp}^F_A(a) = \emptyset \).

The value \( f_r^i(a) \) is the score of the argument \( a \) at step \( i \). This value converges to a unique value as \( i \) becomes high.

Theorem 5 The function \( f_r^i \) converges.

The reward-based semantics assigns to each argument a score which is equal to the limit reached by the reward-based function \( f_r \).

Definition 9 (Reward-based Semantics) The reward-based semantics is a function \( Rbs \) transforming any support argumentation graph \( A = \langle A, w, S \rangle \) into a weighting on \( A \) such that for any \( a \in A \), \( \text{Deg}^\text{Rbs}_A(a) = \lim_{i \to +\infty} f_r^i(a) \).

We show next that the limit scores of arguments satisfy the equation of Definition 8.

Theorem 6 For any support argumentation graph \( A = \langle A, w, S \rangle \), for any \( a \in A \), \( \text{Deg}^\text{Rbs}_A(a) = 0 \) if \( a \) is unfounded. Otherwise,

\[
\text{Deg}^\text{Rbs}_A(a) = w(a) + (1 - w(a))\left(\sum_{j=1}^{n-1} \frac{1}{2^j} + \frac{m}{2^n}\right)
\]

where \( n = |\text{Supp}^F_A(a)| \) and \( m = \frac{\sum_{b \in \text{Supp}^F_A(a)} \text{Deg}^\text{Rbs}_A(b)}{n} \).

Let us illustrate the semantics with an example.

Example 1 (Cont) It is worth noticing that all the eight arguments are founded. Their strength degrees under semantics \( Rbs \) are: \( \text{Deg}^\text{Rbs}_A(a) = 144, \text{Deg}^\text{Rbs}_A(b) = 144, \text{Deg}^\text{Rbs}_A(c) = 27, \text{Deg}^\text{Rbs}_A(d) = 29, \text{Deg}^\text{Rbs}_A(e) = 31, \text{Deg}^\text{Rbs}_A(f) = 13, \text{Deg}^\text{Rbs}_A(x) = 0.2 \). According to \( Rbs \), \( d' > c > e' > d > c > b > a > x \).

Theorem 7 Table 1 summarizes the axioms satisfied (violated) by reward-based semantics.

From Theorems 1 and 4, it follows that the strength degree of each argument \( a \) is in the interval \([w(a), 1]\).

Corollary 2 For any support argumentation graph \( A = \langle A, w, S \rangle \), for any \( a \in A \), \( \text{Deg}^\text{Rbs}_A(a) \in [w(a), 1] \).

The following proposition shows additional basic properties of reward-based semantics.

Proposition 4 Let \( A = \langle A, w, S \rangle \) be a support argumentation graph and \( a \in A \).

- If \( a \) is founded, then \( \text{Deg}^\text{Rbs}_A(a) > 0 \).
- If \( \text{Supp}^F_A(a) = \emptyset \), then \( \text{Deg}^\text{Rbs}_A(a) = w(a) \).

5.3 Aggregation-based Semantic

Our last semantics satisfies compensation axiom, i.e., allows a small number of strong supporters to compensate a large number of weaker supporters. This semantics makes use of a scoring function which assigns a numerical value to each argument. Like the two previous functions, the new function proceeds in several steps. It starts by assigning to each argument its basic strength. Then at each step, it recomputes the score of the argument by adding to the basic strength a certain percentage of the scores of its supporters at the previous step.
The strength degrees of the arguments Example 1 (Cont) A of graph Gbs aggregation-based semantics. The symbol • (resp. ◦) means the axiom is satisfied (resp. violated).

<table>
<thead>
<tr>
<th>Axioms - Semantics</th>
<th>Tbs</th>
<th>Rbs</th>
<th>Gbs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Anonymity</td>
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<tr>
<td>Independence</td>
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<td>Non-Dilution</td>
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<td>Monotony</td>
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<td>Equivalence</td>
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<td>Counting</td>
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<td>○</td>
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<td>○</td>
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<tr>
<td>Quality Precedence</td>
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<td>○</td>
<td>○</td>
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<tr>
<td>Compensation</td>
<td>○</td>
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</tr>
</tbody>
</table>

Table 1: Axioms satisfaction by the three semantics.

**Definition 10** Let $A = (A, w, S)$ be a support argumentation graph. We define the aggregation function $\mathcal{f}_g$ from $A$ to $[0, +\infty)$ as follows: for any argument $a \in A$, for $i \in \{0, 1, 2, \ldots\}$, if $i = 0$, then $\mathcal{f}_g^i(a) = w(a)$ otherwise $\mathcal{f}_g^i(a) = w(a) + (1 - w(a)) \frac{\sum_{b \in \text{Supp}_A(a)} \mathcal{f}_g^{i-1}(b)}{1 + \sum_{b \in \text{Supp}_A(a)} \mathcal{f}_g^{i-1}(b)}$.

By convention, $\sum_{b \in \text{Supp}_A(a)} \mathcal{f}_g^{i-1}(b) = 0$ if $\text{Supp}_A(a) = \emptyset$.

The value $\mathcal{f}_g^i(a)$ is the score of the argument $a$ at step $i$. This value may change at each step, however, it converges to a unique value as $i$ becomes high.

**Theorem 8** The function $\mathcal{f}_g^i$ converges.

The aggregation-based semantics is based on the previous scoring function. The strength degree of each argument is the limit reached using the scoring function.

**Definition 11** (Aggregation-based Semantics) The aggregation-based semantics is a function $\text{Gbs}$ transforming any support argumentation graph $A = (A, w, S)$ into a weighting on $A$ such that for any $a \in A$, $\text{Deg}_{\text{Gbs}}(a) = \lim_{i \to \infty} \mathcal{f}_g^i(a)$.

We show next that the limit scores of arguments satisfy a nice property, namely the definition of Definition 10.

**Theorem 9** For any support argumentation graph $A = (A, w, S)$, for any $a \in A$, $\text{Deg}_{\text{Gbs}}(a) = w(a) + (1 - w(a)) \frac{\sum_{b \in \text{Supp}_A(a)} \text{Deg}_{\text{Gbs}}(b)}{1 + \sum_{b \in \text{Supp}_A(a)} \text{Deg}_{\text{Gbs}}(b)}$.

Consider again our running example.

**Example 1 (Cont)** The strength degrees of the arguments of graph $A$ under semantics $\text{Gbs}$ are: $\text{Deg}_{\text{Gbs}}(a) = \frac{1}{3}$, $\text{Deg}_{\text{Gbs}}(b) = \frac{1}{2}$, $\text{Deg}_{\text{Gbs}}(c) = \frac{5}{6}$, $\text{Deg}_{\text{Gbs}}(d) = \frac{21}{33}$, $\text{Deg}_{\text{Gbs}}(d') = 0.74$, $\text{Deg}_{\text{Gbs}}(e) = \frac{21}{33}$, $\text{Deg}_{\text{Gbs}}(e') = 0.72$, and $\text{Deg}_{\text{Gbs}}(x) = 0.2$. Recall that $\text{Supp}_A(e) = \{e\}$ and $\text{Supp}_A(d) = \{a, b\}$. The argument $e$ has less supporters but its supporter $c$ is stronger than both supporters of $d$. Since $\text{Gbs}$ satisfies compensation, then $d$ is as strong as $e$. $\text{Tbs}$ promotes the quality, thus it declares $e$ stronger than $d$. Finally, $\text{Rbs}$ promotes the cardinality, thus declares $d$ stronger than $e$.

Aggregation-based semantics satisfies compensation as well as all the mandatory axioms.

**Theorem 10** Table 1 summarizes the axioms satisfied (violated) by aggregation-based semantics.

From Theorems 1 and 4, it follows that the strength degree of each argument $a$ is in interval $[w(a), 1]$.

**Corollary 3** For any support argumentation graph $A = (A, w, S)$, for any $a \in A$, $\text{Deg}_{\text{Gbs}}(a) \in [w(a), 1]$.

It is worth noticing from Table 1, that $\text{Tbs}$ is the only semantics that violates Imperfection. Indeed, according to this semantics, an argument whose intrinsic strength is 0 may become very strong (i.e., gets degree 1) due to its supporters. This scenario is not possible with the two other semantics.

**6 Conclusion**

This paper presented the first study on support argumentation graphs. It tackled the problem of assessing the overall strengths of arguments in such graphs. It proposed a set of axioms guiding the well definition of semantics. Then, it proposed three semantics that satisfy most of the axioms.

Our future work consists of extending these semantics for evaluating arguments in bipolar graphs (graphs containing attacks and supports). Recall that the existing semantics in the argumentation literature do not treat properly supports. For instance, if the attack relation is empty, then they declare all the arguments as acceptable, neglecting thus the supports.

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References


