Worst case delay analysis for a wireless point-to-point transmission

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***Abstract*—Wireless technologies are currently being intensively investigated for real-time applications because of their appealing ease of deployment and scalability. Dimensioning a wireless network for safety-critical applications is still an open problem mainly because of the intrinsic non-deterministic nature of the wireless medium. This paper discusses the derivation of a worst case delay (WCD) measure for a point-to-point wireless transmission. A WCD performance measure is central to the performance evaluation of wireless networks subject to hard real-time constraints. To capture the non-deterministic nature of the wireless channel, our measure relies on a probabilistic link model where transmissions are guaranteed using an acknowledgement mechanism. The delay is expressed by the number of emissions necessary for a packet to arrive at its destination. The WCD is expressed as the P_d-percentile of this number of emissions. The proposed WCD metric is computed for an interference-free scenario considering AWGN and Rayleigh fading channels. Interference-limited scenarios are discussed as well to highlight the perspectives of this work.**

***Keywords*-Wireless transmission, unreliable link model, performance evaluation, worst case delay analysis**

I. INTRODUCTION

The deployment of wireless technologies for real-time applications is rapidly gaining momentum because of their appealing ease of deployment and scalability. First analysis of legacy wireless protocols [1] (e.g. IEEE802.11, Bluetooth or IEEE802.15.4) in the factory automation context called for the design of novel solutions meeting the needs of real-time systems. New protocols have been specified for industrial process control such as WirelessHART [2] [3] or ISA100.11a [4]. Both solutions provide a pure time division multiple access to its real time users to prevent unbounded channel access delays. Channel hopping techniques with blacklisting is implemented at the physical layer to be more robust to interference. In the context of nuclear plant or warship monitoring, dedicated wireless sensor network protocols such as OCARI and MACARI [5] have been developped.

Temporal behavior of such protocols have to be thoroughly assessed for such critical applications. As such, a comprehensive performance evaluation of transmission delay is needed. Together with controlling the variance of transmission delays, it is of foremost importance to derive a safe bound on the worst case transmission delay. This safe bound can be accounted for to check that transmission delays meet their temporal requirements in the protocol integration process. Worst case delay analysis in wired networks has been performed using two types of derivations: deterministic (network calculus [6], trajectory approach [7]) and probabilistic (stochastic network calculus [8]).

In this paper we propose a probabilistic derivation of the worst case delay (WCD) bound for a point-to-point wireless communication. This choice is clearly motivated by the non-deterministic nature of the wireless channel whose most valid models are stochastic. Thus, our WCD bound relies on a probabilistic link model where transmissions are guaranteed using an acknowledgement mechanism. The overall transmission delay is measured as a function of the number of emissions necessary for a packet to arrive and be decoded at its destination. The WCD delay is defined as the P_d-percentile of the overall transmission delay. As such, there is a probability of $P_{th} = (1 - P_d)/100$ for the delay to be larger than the WCD, providing a confidence level on the calculated WCD bound. The proposed WCD metric is completely derived and calculated for an interference-free scenario considering AWGN and Rayleigh fading channels. Interference-limited scenarios are discussed at the end of the paper to show the perspectives of this work.

This paper is organized as follows. Section II presents our WCD analysis for a point-to-point interference-free transmission. Next, Section III discusses the main issues related to the WCD analysis in interference-limited scenarios. Section IV concludes the paper.

II. WORST CASE DELAY FOR INTERFERENCE-FREE TRANSMISSIONS

This section details firstly the unreliable wireless link model, then it briefly presents the average transmission delay computation before introducing the derivation of the stochastic WCD bound.

A. Unreliable wireless link model

The unreliable link model captures the wireless link availability between two nodes i and j. It is defined as the *probability* p_ℓ of a successful transmission over the link $\ell = (i, j)$. Characterization of the link probability is

Table I
TRANSMISSION PARAMETERS [12]

Symbol	Description	Value
N_b	Number of bits per packet	2560
R	Transmission bit rate	$1\ Mbps$
N_0	Noise level	$-154dBm/Hz$
f_c	Carrier frequency	$2.4GHz$
G_T	Transmitter antenna gain	1
G_R	Receiver antenna gain	1
α	Path-loss exponent	3
L	Circuitry losses	1

impacted by enhancements and impairments at the physical layer: transmission power, modulation type, channel fading, etc. Such a realistic link model captures the non-deterministic nature of a wireless transmission and has been used in recent performance studies [9] [10] [11] focusing on various metrics such as energy consumption, average delay or reliability. It is derived for the transmission of a packet of N_b bits. Formally,

$$p_\ell(\gamma_\ell) = (1 - BER(\gamma_\ell))^{N_b} \tag{1}$$

where $BER(\gamma_\ell)$ is the bit error rate (BER) corresponding to the signal to noise ratio (SNR) γ_ℓ on link ℓ. Note that consequently $p_\ell(\gamma_\ell) = 1 - PER(\gamma_\ell)$, with $PER(\gamma_\ell)$ the corresponding packet error rate. The BER depends on the transmission chain technology (modulation, coding, etc.) and channel type (AWGN, Rayleigh, Rician). It is defined as the average probability to decode one bit. Thus it is a function of the SNR γ_ℓ experienced by the destination calculated by [12]:

$$\gamma_\ell = \frac{K_1 \cdot P^t \cdot d_\ell^{-\alpha}}{N_0 \cdot B}, \tag{2}$$

with

$$K_1 = \frac{G_T \cdot G_R \cdot \lambda^2}{(4\pi)^2 \cdot L}, \tag{3}$$

where d_ℓ is the transmission distance between nodes i and j, $\alpha \geq 2$ is the path loss exponent, P^t is the transmission power, N_0 the noise power density in mW/Hz, G_T and G_R are the antenna gains for the emitter and receiver respectively, B is the bandwidth of the channel and is set to the emission rate ($B = R$), λ is the wavelength and $L \geq 1$ summarizes losses through the transmitter and receiver circuitry. For a given transmission technology, K_1 is constant and $p_\ell(\gamma_\ell)$ is a function of d_ℓ and P^t: $p_\ell(d_\ell, P^t)$. Values considered herein are listed in Table I.

In the following, a transmission scenario using Binary Phase Shift Keying (BPSK) modulation and coherent detection is assumed. Closed form expressions of $BER(\gamma_\ell)$ for AWGN (Additive White Gaussian Noise) and Rayleigh flat fading channels as follows:

AWGN CHANNEL: Derivation of $BER(\gamma_\ell)$ for BPSK and coherent detection follows the derivation in [13]:

$$BER(\gamma_\ell) = \alpha_m \mathrm{Q}(\sqrt{\beta_m \gamma_\ell}) \tag{4}$$

with the Q function, $\mathrm{Q}(x) = \int_x^\infty \frac{1}{\sqrt{\pi}} e^{-u^2/2} du$ and α_m, β_m the modulation type and order, respectively. For BPSK, $\alpha_m = 1$ and $\beta_m = 2$.

RAYLEIGH FLAT FADING: The general expression for the BER in Rayleigh flat fading channel for $\gamma_\ell \geq 5$ [13] is assumed:

$$BER_f(\gamma_\ell) \approx \frac{\alpha_m}{2\beta_m \gamma_\ell} \tag{5}$$

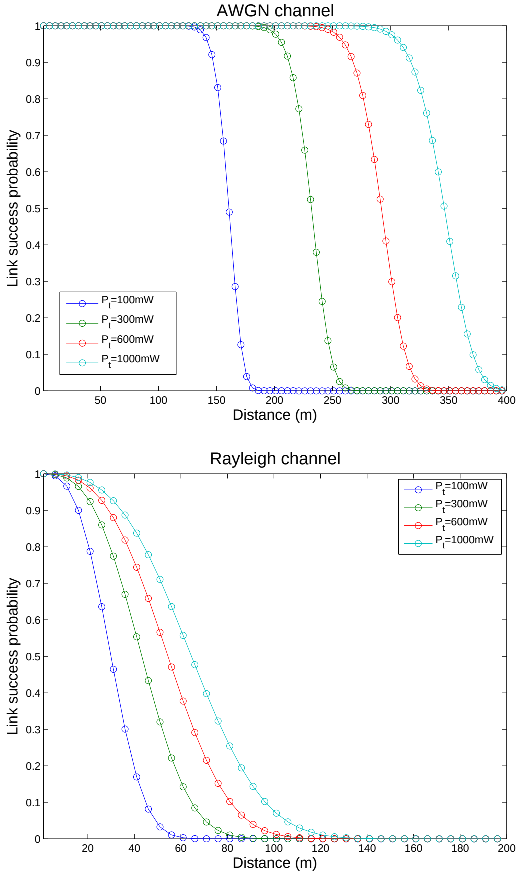

Figure 1. Link probability as a function of distance at different transmission power values, for AWGN and Rayleigh flat fading channels.

Link probability values for both channel types and different values of P^t are represented in Figure 1. The curves can be divided into three parts: reliable transmission, unreliable transmission and impossible transmission. For instance, in an AWGN channel at a transmission power of 100mW, transmission is always successful until about 150 meters. Transmission is impossible beyond 180 meters and in between, the transmission is unreliable. In the Rayleigh flat fading environment, the perfectly reliable transmission is nearly inexistent and most of the links are unreliable links. Rayleigh fading characterizes harsher propagation environments where nodes are usually not in line of sight and transmission is deeply affected by multi-path such as it is the case in heavily built up city centers. There is no main line of sight transmission component. In this case, the envelope of the received SNR is Rayleigh distributed. Other channel models can be considered, depending on the environment the wireless network is deployed in. For instance, Rician fading is appropriate when communication with a direct line of sight is possible in a harsh propagation environment with lots of scatterers.

The probabilistic model contrasts to previous models such as the switched link model where a transmission between nodes i and j is successful if and only if the SNR is above a minimal threshold value. With the switched link model, there are either completely reliable links or no communication is possible. Unreliable links have been leveraged to properly evaluate connectivity [14], derive multi-objective performance trade-offs [10] [11] and design optimal routing and resource allocation strategies [11]. We show in this paper that the unreliable link model is particularly suited to become the building block of a worst case delay analysis of wireless networks.

B. *Average delay metric*

To combat packet losses on an unreliable radio link, we assume here a general acknowledgement procedure where the complete packet is retransmitted if no acknowledgement is received before T_{NACK} milliseconds have elapsed. A maximum number of retransmissions N_R^{max} can be set. If transmission is successful, the acknowledgement packet is received within T_{ACK} milliseconds. For simplicity, we assume $T_{ACK} = T_{NACK}$ but different, realistic values of both durations can be accounted for if needed.

The delay for a packet to be emitted once by i and acknowledged by j over ℓ, d_1, is the sum of three delay components. The first component is the queuing delay during which a packet waits at i for being transmitted. The focus of this paper is on the delay introduced by the transmission and thus, queuing delay is out of the scope of this analysis. The second component is the transmission delay equal to N_b/R and the third component is T_{ACK}. Propagation delay is neglected because transmission distances in current technologies emitting in the 2.4GHz band are usually short (≤ 100m).

T_{ACK} and N_b/R being constant, d_1 is set to be 1 unit. Due to link unreliability, packets suffer from the delay introduced by their possible retransmissions. As such, we introduce the random variable N_R which represents the number of retransmissions needed before receiving a positive acknowledgement. For a given value of N_R, the complete transmission delay D_ℓ over ℓ is thus given by:

$$D_\ell = (N_R + 1) \cdot d_1 \tag{6}$$

since N_R unsuccessful and one successful transmissions are needed.

From (6), D_ℓ is a random variable giving the time before a positive acknowledgement is received in j. Having d_1 constant, the expectation of random variable D_ℓ is derived from the average number of retransmissions $\overline{N_R}$ [15] using $\overline{D_\ell} = (\overline{N_R} + 1) \cdot d_1$. Assuming a maximum number of retransmissions N_R^{max}, $\overline{N_R}$ follows:

$$\overline{N_R} = \sum_{r=0}^{N_R^{max}} r \cdot P[N_R = r] \tag{7}$$

with $P[N_R = r] = p_\ell \cdot (1 - p_\ell)^r$ the probability for a packet to necessitate r retransmissions. For a perfect transmission, $N_R^{max} = \infty$ and $\overline{N_R} = 1/p_\ell(d_\ell, P^t)$.

C. *Worst case delay metric*

The distribution of N_R knowing the link probability p_ℓ is given by $P[N_R = x] = p_\ell \cdot (1 - p_\ell)^x$. The distribution of the delay D_ℓ is derived according to (6):

$$P[D_\ell = x] = P[N_R = \frac{x}{d_1} - 1] \tag{8}$$

Definition The *worst case delay* is defined in this paper by the value D_ℓ^w of D_ℓ below which P_d percent of the observations fall, with $P_d = (1 - P_{th}) * 100$. Formally:

$$\max_{D_\ell^w \in D_\ell} D_\ell^w \text{ s.t. } P[D_\ell \geq D_\ell^w] \leq P_{th} \tag{9}$$

The worst case delay D_ℓ^w is the P_d-percentile of the transmission delay D_ℓ on link ℓ. D_ℓ^w is a function of the random variable N_R. It is thus a probabilistic bound that can be exceeded with probability P_{th}. Closed form expression of D_ℓ^w is:

$$D_\ell^w = \left\lceil \frac{d_1 \cdot \ln(P_{th})}{\ln(1 - p_\ell)} \right\rceil \tag{10}$$

Proof: We have $P[D_\ell \leq D_\ell^w] = p_\ell \cdot \sum_{x=0}^{\frac{D_\ell^w}{d_1}-1} (1 - p_\ell)^x$ from (8). This is a geometric serie of rate $(1 - p_\ell)$ and thus

$$P[D_\ell \leq D_\ell^w] = p_\ell \frac{(1 - p_\ell)^{D_\ell^w/d_1} - 1}{-p_\ell} = 1 - (1 - p_\ell)^{D_\ell^w/d_1}$$

From (9), $1 - (1 - p_\ell)^{D_\ell^w/d_1} \leq 1 - P_{th}$, leading to

$$D_\ell^w \leq \frac{d_1 \cdot \ln(P_{th})}{\ln(1 - p_l)}.$$

Since we are looking for the largest integer value of D_ℓ satisfying (9), we have:

$$D_\ell^w = \left\lceil \frac{d_1 \cdot \ln(P_{th})}{\ln(1 - p_l)} \right\rceil$$

■

Average delay $\overline{D_\ell}$ and worst case bounds D_ℓ^w expressed for different P_{th} values are represented in Figures 2 and 3, for both AWGN and Rayleigh fading channels. Figure 2

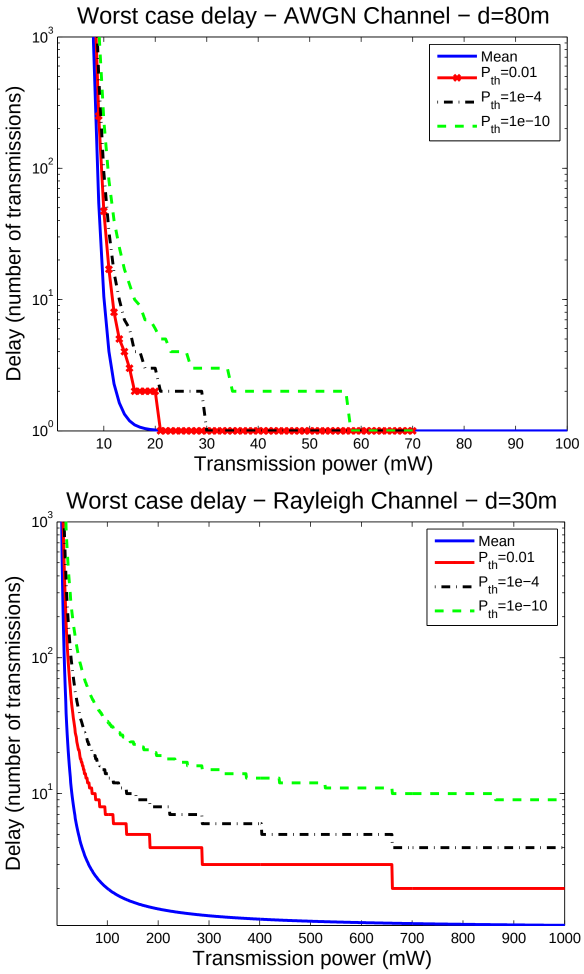

Figure 2. Mean and worst-case delay as a function of the transmission power for different percentile values, for AWGN and Rayleigh flat fading channels.

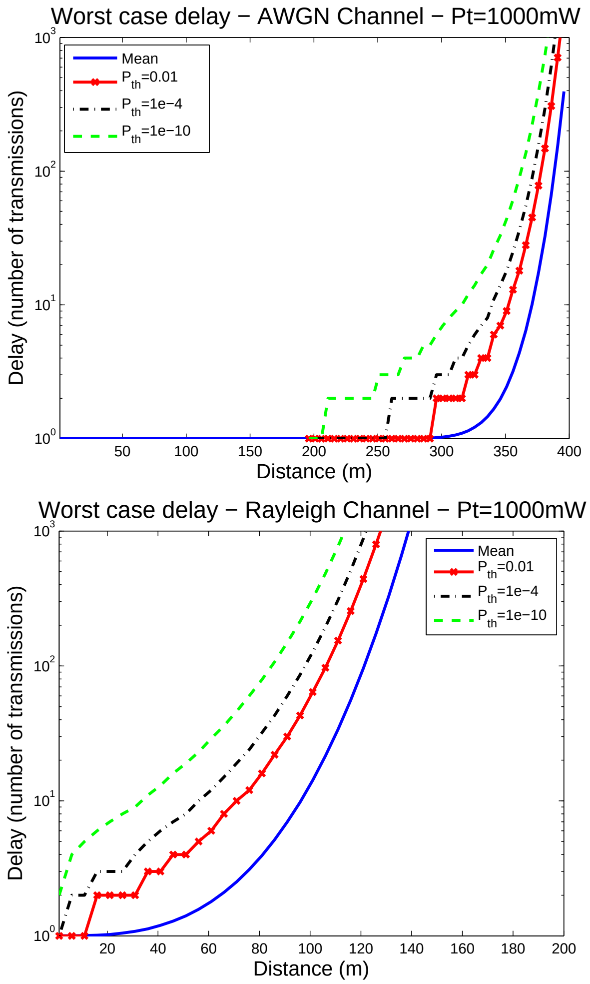

Figure 3. Mean and worst-case delay as a function of the distance for different percentile values, for AWGN and Rayleigh flat fading channels.

focuses on the impact of the transmission power for a fixed inter-node distance while Figure 3 concentrates on the impact of the inter-node distance for a fixed transmission power.

In Figure 2, delay decreases with the increase in power. Indeed, as power is increased for a fixed inter-node distance, the link becomes more and more reliable, reducing the number of retransmissions needed to transmit a packet. For the AWGN channel, no communication is possible for a power below 20 mW: average and WC delay are infinite. Practically, infinite delays are not tolerable in a transmission and a maximum number of retransmissions is introduced N_R^{max} (which is not represented in this figure). WCD bounds are presented for P_{th} values as small as 1.10^{-10}, providing a really tight probabilistic bound on the worst case delay in this context.

Impact of inter-node distance at fixed power is represented in Figure 3. As expected, delay (and thus link reliability) increases with distance. Similar conclusions to Figure 2 can be drawn here: a tight bound is obtained, at the cost of little computation since a closed form expression exists in (10).

III. ACCOUNTING FOR INTERFERENCE IN WCD ANALYSIS

This section introduces the main issues in accounting for interference created by multiple concurrent transmissions in our WCD analysis. Firstly, we concentrate on the interference-limited unreliable link model. Next, we discuss the main steps and problems to integrate elaborated channel access protocols if interference-free medium access is not achievable.

A. *Interference-limited link model*

In this section, we still discuss a point-to-point wireless communication on link ℓ between two nodes i and j. We assume here that this communication is interference-limited due to the other active links in the network as represented in

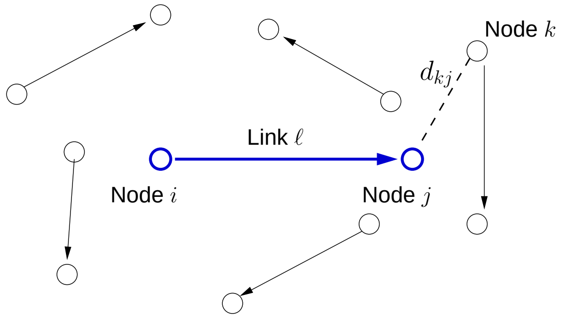

Figure 4. Interference-limited link model.

Figure 4. The complete network is static. More specifically, this scenario illustrates the study case where i is transmitting data to an access point j. Other nodes may interfere this communication because they have ad hoc communications with other nodes and can not detect the ongoing transmission between i and j for some reason (hidden terminal problem for instance).

Interference originates from concurrent transmissions in the wireless channel link ℓ. Medium access control prevents nodes from the same network to interfere with each other. Interference can be completely mitigated using Time Division Multiple Access (TDMA) or Frequency Division Multiple Access (FDMA). In this case, each user is assigned its own resource (time slot or frequency) and no other node is allowed to transmit in this resource. Worst case delay analysis resumes in this case to the previously defined point-to-point interference-free model of Section II.

TDMA or FDMA medium access may suffer from both under-utilization of the network bandwidth and additional overhead for resource allocation. This is mostly the case when the network is lightly loaded. In this case, Carrier Sense Medium Access (CSMA) is an alternative that reduces resource allocation overhead and provides a faster access to the wireless channel. The drawback of CSMA is that interference can not be completely mitigated anymore, mostly because of the hidden terminal problem.

As for the interference-free case, the formulas for the BER hold, but this time they depend on the Signal to Noise and Interference ratio γ_ℓ^I (SINR) instead of the SNR γ_ℓ. Interference I_ℓ experienced at receiver j is added to the thermal noise in equation (2) to derive the SINR:

$$\gamma_\ell^D = \frac{K_1 \cdot P_i^t \cdot d_\ell^{-\alpha}}{N_0 \cdot B + I_\ell^D} \tag{11}$$

with I_ℓ^D defined as the sum of the power at j received from all other emitters transmitting at the same time. In this notation, D represents the set of interfering nodes. In this formulation, nodes can use different transmission power values. Thus, P_i^t represents the transmission power a node i is using. Formally, I_ℓ^D is defined as:

$$I_\ell^D = \sum_{k \in D} K_2 \cdot P_k^t \cdot d_{kj}^{-\alpha} \tag{12}$$

where $K_2 = G_T \cdot (\lambda/4\pi)^2$ and d_{kj} the distance between interferer k and destination node j. This computation of interference power captures the geometry of the network. As such, if the location of all nodes in the network is known, I_ℓ^D can be calculated using (12) and its corresponding link probability using (11) and (1). Similarly, if the node distribution follows a given law (e.g. a Poisson point process or a power law distribution for scale-free networks), interference distribution may be derived as well.

B. *Worst case delay metric and medium access control*

The set of interferers D affecting the communication on link ℓ depends on the decisions made by the medium access control (MAC) layer. For ideal TDMA (one user is assigned to one time slot at any time), the set D is empty. For a CSMA-oriented MAC protocol, we are interested in deriving the distribution of the bit error rate values over all possible interfering sets.

A set of interferers D belongs to the power set $\mathcal{P}(N)$ of N, with N the set of all nodes of the network different from i and j. For each set of interferers $D \in \mathcal{P}(N)$, a SINR value can be computed with (11) and its corresponding BER using (4) or (5). The distribution of $BER(\gamma_\ell^D)$ is given by the distribution of the set of interferers: $P[BER(\gamma_\ell^D) = x] = P[D \text{ active}]$, with D the set producing $BER(\gamma_\ell^D)$.

A node is said to be active if it can emit in the same channel than i. The activity of a node is captured by the probability it is emitting on the channel as proposed in [11]. Two types of such *emission probabilities* may be considered:

1. Independent emission probability: It is captured by τ_i, the probability node i is emitting. Using τ_i values, it is shown in [11] that it is possible to derive the probability of any set D of interferers to be active using:

$$P[D \text{ active}] = \prod_{i \in D} \tau_i \cdot \prod_{j \in N \setminus D} (1 - \tau_j)$$

The average link probability is deduced from the distribution of BER values using the law of total probabilities:

$$\bar{p}_\ell = \sum_{D \in \mathcal{P}(N)} p_{\ell,D} \cdot P[D \text{ active}] \tag{13}$$

where $p_{\ell,D} = \left[1 - BER(\gamma_\ell^D)\right]^{N_b}$ is the link probability experienced for the set D of interferers.

2. Conditional emission probability: The independent channel access model is a simplified model where transmission decisions are independent from eachother, which is usually not the case in a MAC protocol. Interaction between nodes could be for instance captured by $\tau_{i/j}$, the probability the channel is occupied by a transmission of node i knowing j is *not* transmitting. This conditional channel probability can be leveraged to derive the probability of a set of interferer K to be active.

Two types of worst case delays can be computed. The first one can be derived from (10) using the average link probability $\bar{p}_\ell$ derived in (13). A safer estimation but more pessimistic probabilistic bound can be computed from the *worst case link probability* which is experienced as the channel between i and j is the *most interfered*. Knowing the BER distribution and similarly to the definition of the worst case delay D_l^w, we can define the worst case link probability.

Definition The *worst case link probability* is defined as the value p_ℓ^w of p_ℓ above which P_d percent of the observations fall, with $P_d = (1 - P_{th}) * 100$. Formally:

$$\min_{p_\ell^w \in p_\ell} p_\ell^w \text{ s.t. } P[p_\ell \leq p_\ell^w] \leq P_{th} \tag{14}$$

The safer delay bound is then computed from (10) using the worst case link probability of (14).

Worst case delay bounds are straightforward to calculate if channel activity of each node is known (i.e. node emission probabilities). Different medium access protocols can be characterized using such node emission probabilities. Future studies will study the impact of these node emission probabilities on the worst case delay, and work on modeling medium access decisions either as independent or conditional emission probabilities, possibly accounting for incoming traffic models, memory size or node distribution.

IV. CONCLUSION

This paper discusses the derivation of the worst case delay (WCD) bound for a point-to-point wireless communication. This bound is guaranteed not to be exceeded with a probability of $(1 - P_{th})$, with P_{th} arbitrarily small. The proposed WCD metric is computed for an interference-free scenario considering AWGN and Rayleigh fading channels. Interference-limited scenarios are discussed as well to highlight the perspectives of this work. The next step is to fully characterize the WCD for the interference-limited case and concentrate on mapping MAC protocol decisions to emission probabilities. Therefore, protocol performance evaluation models derived from the one proposed by Bianchi in [16] for IEEE802.11 DCF can be leveraged.

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