Hough transform

I- Introduction

The Hough transform is used to isolate features of a particular shape within an image. Because it requires that the desired features be specified in some parametric form, the **classical Hough transform** is most commonly used for the **detection of regular curves** such as lines, circles, ellipses, etc. A **generalized Hough transform** can be employed in applications where a simple analytic description of a feature(s) is not possible.

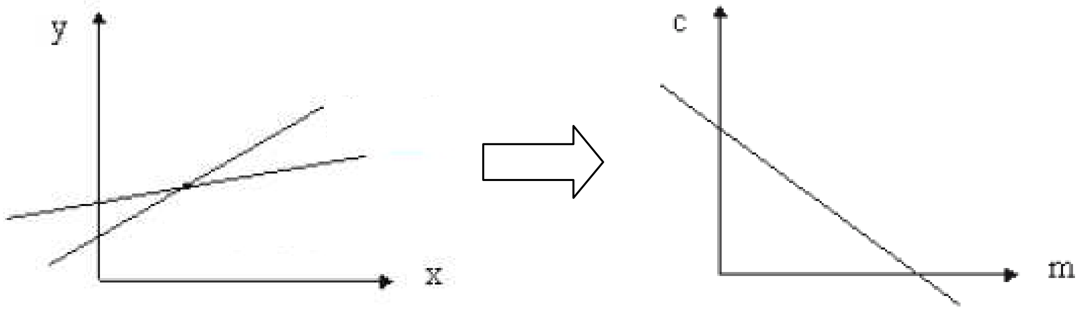
This transform was invented by Paul Hough in 1959.
It was made popular by Duda and Hart: Duda, R. O. and P. E. Hart, "Use of the Hough Transformation to Detect Lines and Curves in Pictures", 1972.

II- Method

1. Description

Earliest versions of the Hough transform used a line equation of: y = mx + c.

In the Hough transform, we will create a dual space in which (m,c) are the parameters:
c = -mx + y



For a point p, all lines passing through this point correspond to a single line in space (m,c).

For a point q, all lines passing through this point correspond to a single line in space (m, c).

These lines in space (x, y) share the line connecting points p and q. This line (pq) is the intersection of 2 lines D_p and D_q representing p and q in space (m, c).

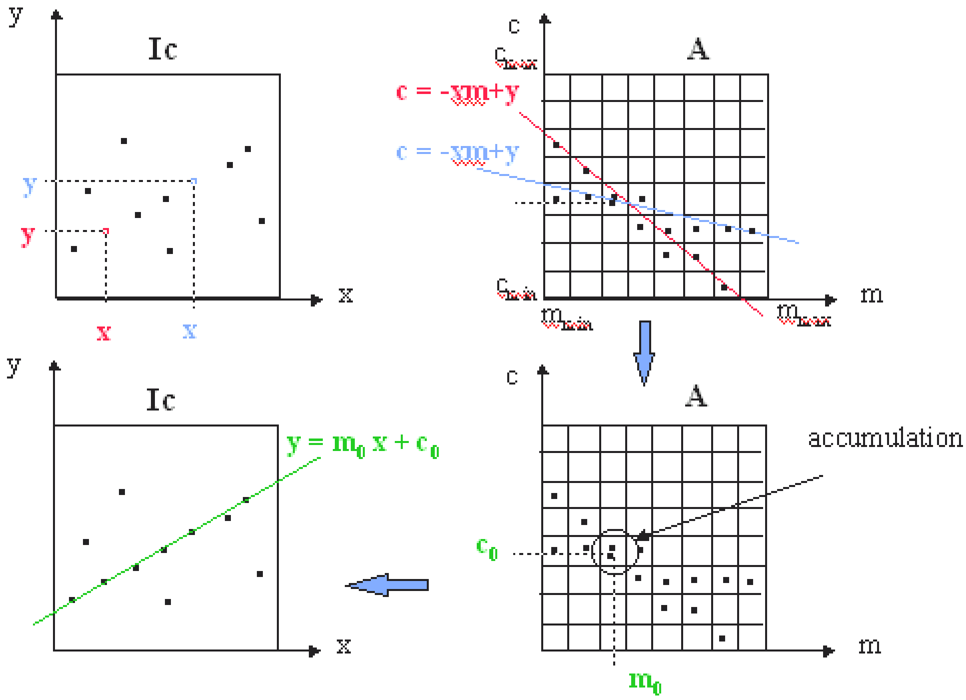
All points on the same line D in space (x,y) are represented by lines which all pass through the same point in space (m,c).

2. Algorithm

We will create an accumulator array h(m,c) for discrete values of (m,c).
This poses the problem of how to "discretize" m, c.

For each point x, y, which is a local maxima in the gradient (for which $I(x,y) \neq 0$), we will evaluate the set of possible values of m and c (a step). For each possible value we will increment h(m,c).

Allocate a table h(m,c) initially set to 0
for each x, y
    if I(x, y) ≠ 0 then
    for all m
        c = –mx+y
        h(m,c) = h(m,c) + 1
    end
end



Exercise 1: Basics of the Hough Transform

Question 1: Consider an image of 5 x 5 pixels identified by their coordinates between 0 and 4. P_1 is the pixel coordinate (2,3). We use the parameters m and c to represent the equation of a line in the form y = mx + c.
Give an equation of the line representative point P_1.

Question 2: Let P_2, the coordinates (1,1). Determine the equation of the line through P1 (cf. Question 1) and P2.

Exercise 2: Implementation
The Hough space is represented by an 11x11 matrix where the slope coefficient m and translation coefficient c can take integer values between -5 and 5. Give the configuration of the Hough space when the shape points of the image are the following:

a) $P_1(1,0)$, $P_2(2,2)$ and $P_3(3,4)$,
b) $P_1(1,0)$, $P_2(2,2)$ and $P_3(0,0)$,
c) $P_1(1,0)$, $P_2(1,2)$ and $P_3(1,4)$.

3. Discretization

A second, more useful parameterization is to represent line by its shortest distance from the origin ρ and its orientation θ. We build an accumulator array: h(ρ ,θ).

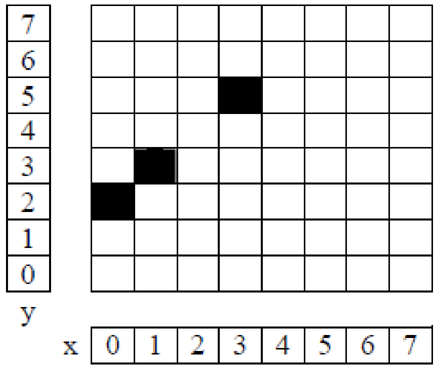
This is representation in polar coordinates: $\rho = x \cos(\theta) + y \sin(\theta)$

Allocate a table h(ρ ,θ) initially set to 0.
for each x, y of the image
    for each θ
        $\rho = x \cos(\theta) + y \sin(\theta)$
        $h(\rho,\theta) = h(\rho,\theta) + I(x,y)$
    end
end

The resulting table accumulates contrast.
Peaks in h(ρ ,θ) correspond to line segments in the image.

Exercise 3: Let the binary image I as follows. Complete the following table T to express the Hough transform in polar coordinates corresponding.

I



T

θ \ ρ	0	1	$\sqrt{2}$	2	$2\sqrt{2}$	3	$4\sqrt{2}$	5
0								
$\pi/4$								
$\pi/2$								
$3\pi/4$								

Identify the item receiving the most votes and deduce the equation of the line in polar coordinates, which connects the 3 points of I.

4. Extension to circular form

We can represent a circle with the equation: $(x - a)^2 + (y - b)^2 = r^2$.

We can use this to create a Hough space h(a, b, r) for limited ranges of r.
The ranges of a and b are the possible positions of circles.

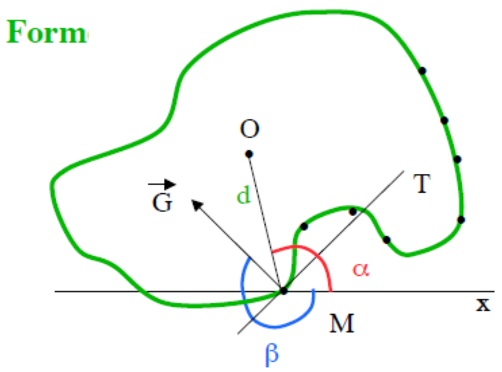
Allocate a table h(a, b, r) initially set to 0.
for each x, y of the image
    for r from r_{min} to r_{max}
        for a from 0 to a_{max}
            $b = y \pm sqrt(r^2 - (x - a)^2)$
            $h(a,b,r) = h(a,b,r) + I(x,y)$.
        end
    end
end

Exercise 4: We have a circular shape of radius 2, using the equation: $(x - a)^2 + (y - b)^2 = 2^2$
Provide for three points, the configuration Hough space associated:
$P_1(0,2)$, $P_2(2,0)$ and $P_3(4,2)$.

5											
4											
3											
2											
1											
0											
-1											
-2											
-3											
-4											
-5											
a/b	-5	-4	-3	-2	-1	0	1	2	3	4	5

III- Generalization of the Hough Transform

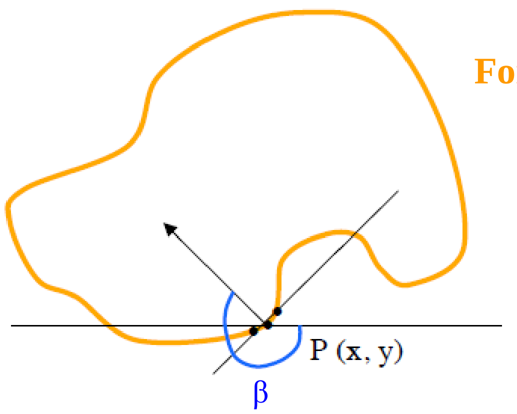
1. Modeling



- O: gravitational center
- MT: tangent to the shape in M
- $\vec{G}$: perpendicular MT (gradient orientation)
- $d = \overline{MO}$

Model = list of triplets {β, α, d}

2. Recognition



We search the triplets {β, α, d}. These are the points of the contour model with a perpendicular parallel to that of P. We can therefore say that the portion of contour centered on P vote for a gravitational center whose coordinates are given by α and d:

- $x_0 = x + d\cos(\alpha)$
- $y_0 = y + d\sin(\alpha)$

If many points vote for the same item, the form will be recognized.

3. Algorithm

Modeling

Grouping the triplets of the model which have the same β and the value is stored in a table T whose rows are indexed by different values of β:

T

β_1	(α_{11},d_{11}) (α_{12},d_{12}) ... (α_{1j},d_{1j})
β_2	(α_{21},d_{21}) (α_{22},d_{22}) ... (α_{1k},d_{1k})
β_n	(α_{n1},d_{n1}) (α_{n2},d_{n2}) ... (α_{nr},d_{nr})

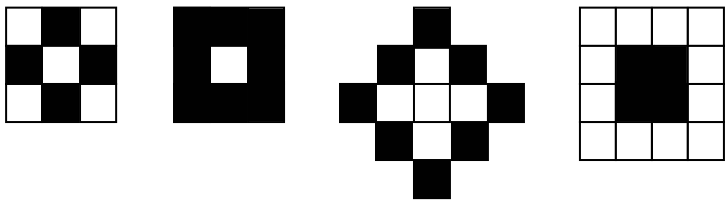
Recognition

Initialize an array $A[x_{min} .. x_{max} ; y_{min} .. y_{max}]$
for each shape point $I(x, y)$
    Compute β (which provides an entry in table T)
    for each couple of the line β of T
        Compute $x_0 = x + d\cos(\alpha)$ and $y_0 = y + d\sin(\alpha)$
        Increment $A[x_0, y_0]$
    end
end
Determine the couple(x, y) that maximizes $A[x, y]$

Exercise 5: The following table contains only partial data to model a form (the angles are expressed in degrees):

β	**list of couples (α,d)**
0	(0,1)
90	(90,1)
180	(180,1)
270	(270,1)

Given this information, what is (are) the shape(s) that correspond(s) to the complete model? We consider that the outline shape is represented by black pixels.



Exercise 6: A picture of a printed circuit board shows in particular components squared. Assume that ideally these components appear as squares of 3x3 pixels. To detect their shape and position, we chose to use the Hough Transform.

Give an expression of the model. What are the values in the Hough space when the image contour is that corresponding to the matrix below?

0	0	0	0	0
0	1	1	1	0
0	1	0	1	0
0	1	1	1	0
0	0	0	0	0