International Journal of Approximate Reasoning 99 (2018) 39–55

Contents lists available at ScienceDirect

International Journal of Approximate Reasoning

www.elsevier.com/locate/ijar

Evaluation of arguments in weighted bipolar graphs ☆,☆☆

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ARTICLE INFO

Article history:
Received 1 December 2017
Received in revised form 27 April 2018
Accepted 23 May 2018
Available online 25 May 2018

Keywords:
Argumentation
Weighted bipolar graphs
Semantics

ABSTRACT

The paper tackled the issue of arguments evaluation in *weighted bipolar argumentation graphs* (i.e., graphs whose arguments have basic strengths, and may be both supported and attacked). We introduce principles that an evaluation method (or semantics) could satisfy. Such principles are very useful for understanding the foundations of semantics, judging them, and comparing semantics. We then analyze existing semantics on the basis of our principles, and finally propose a new semantics for the class of acyclic graphs. We show that it satisfies all the principles.

1. Introduction

Argumentation is a form of common-sense reasoning consisting of the justification of claims by arguments. An argument is made of a set of *premises* (called reason), a *conclusion* (the justified claim), and the two are related with a link. An argument has also generally a basic strength which may represent different issues like the certainty degree of its premises [2], the strength of its link [3], the importance of values supported by the argument [4], or the trustworthiness of the source providing the argument [5].

Despite its explanatory power, an argument does not guarantee the validity of its conclusion. Indeed, its premises may be wrong, its link may be flawed, and in some cases the premises may be irrelevant to the conclusion. These flaws of an argument may themselves be supported by arguments, which are seen as attackers of the original one. An argument may also be supported by other arguments, which endorse either its premises, its conclusion, or its link. This leads to *weighted bipolar argumentation graphs*, i.e., graphs whose nodes represent arguments with numerical basic strengths, and edges represent attack and support relationships between pairs of arguments.

An evaluation of the overall strength of each argument is crucial for deciding whether or not one may rely on the argument's conclusion. Phan Ming Dung was the first to investigate in [6] this evaluation issue. He focused on a simple input: a set of arguments, having all the same basic strength, and an attack relation between pairs of arguments. Leaving the origin and the nature of arguments/attacks unspecified, Dung proposed several semantics specifying which sets of arguments (called extensions) are acceptable. Such graphs may have zero, one, or several extensions. A *single qualitative status* is then assigned to each argument as follows: an argument is *accepted* if it belongs to all extensions, and *rejected* otherwise. This status represents the *overall strength* of the argument.

☆ This paper extends the content of the conference paper [1].
☆☆ This paper is part of the Virtual special issue on the 14th European Conference on Symbolic and Quantitative Approaches to Reasoning with Uncertainty (ECSQARU 2017), Edited by Alessandro Antonucci, Laurence Cholvy and Odile Papini.

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https://doi.org/10.1016/j.ijar.2018.05.004
0888-613X/

This seminal paper has led to substantial work either on proposing new alternative semantics dealing with the same input (e.g., [7,8]), or on extending Dung's semantics for dealing with richer input, i.e., previous flat graphs with one of the following features: preferences between arguments (or basic strengths of arguments) [3,4,9], weights on attacks [10–12], or support relation between arguments [13–18]. To the best of our knowledge there is no extension semantics dealing with weighted bipolar argumentation graphs.

More recently, another family of semantics, called weighted semantics, is gaining interest (e.g., [1,5,19–21]). These semantics focus on the evaluation of individual arguments rather than sets of arguments. Furthermore, unlike extension-based semantics which assign a qualitative overall strength (accepted, rejected) to each argument, they assign a numerical value to each argument. Finally, instead of a coarse classification of arguments as accepted/rejected, weighted semantics allow fine-grained classifications. Most existing semantics deal only with *unipolar* graphs (i.e., graphs that consider either attack relation or support one but not both). Two notable exceptions are QuAD semantics [22] and DF-QuAD [23]. In [24] the authors discussed advantages of weighted semantics in case of bipolar argumentation graphs, but they did not propose concrete semantics.

While there is a consensus in the argumentation community on the role of attackers and how they should be taken into account in the evaluation of individual arguments, the situation is less clear for supporters. Indeed, different interpretations are given to support relation (deductive [17], evidence [15], necessary [18]), leading to semantics which may return completely different evaluations of arguments of the same graph. This complicates the comparison of existing semantics for weighted bipolar graphs. Another source of difficulty is the absence of formal principles that guide the well-definition and formal comparisons of semantics.

This paper focuses on the evaluation of arguments in weighted bipolar argumentation graphs. It extends our previous works on axiomatic foundations of semantics for unipolar graphs (support graphs [25] and attack graphs [26]). It defines principles that a semantics would satisfy in a bipolar setting. Such principles are very useful for judging and understanding the underpinnings of semantics, and also for comparing semantics of the same family, and those of different families. Some of the proposed principles are simple combinations of those proposed in [25,26]. Others are new and show how support and attack might be aggregated. The second contribution of the paper consists of analyzing existing semantics against the principles. The main conclusion is that extension semantics do not harness the potential of support relation. Indeed, when the attack relation is empty, the existing semantics declare all (supported, non-supported) arguments of a graph as equally accepted. Weighted semantics take into account supporters in this particular case, however they violate some key principles. The third contribution of the paper is the definition of a novel weighted semantics for the sub-class of acyclic bipolar graphs. We show that it satisfies all the proposed principles. Furthermore, it avoids the *big jump* problem that may impede the relevance of existing weighted semantics for practical applications, like dialogue.

The paper is structured as follows: Section 2 introduces basic notions, Section 3 presents our list of principles, Section 4 analyses existing semantics, and Section 5 introduces our new semantics and discusses its properties.

2. Main concepts

This section introduces the main concepts of the paper. Let us begin with the useful notion of weightings.

Definition 1 *(Weighting).* A *weighting* on a set X is a function from X to $[0, 1]$.

Next, we introduce the argumentation graphs (called frameworks in the literature) we are interested in, namely weighted bipolar argumentation graphs (wBAGs).

Definition 2 *(wBAG).* A *weighted bipolar argumentation graph* (wBAG) is a quadruple $\mathbf{A} = \langle \mathcal{A}, w, \mathcal{R}, \mathcal{S} \rangle$, where $\mathcal{A}$ is a finite set of arguments, w a weighting on $\mathcal{A}$, $\mathcal{R} \subseteq \mathcal{A} \times \mathcal{A}$, and $\mathcal{S} \subseteq \mathcal{A} \times \mathcal{A}$. Let $\mathtt{wBAG}$ denote the set of all possible wBAGs.

Given two arguments a and b, $a\mathcal{R}b$ (resp. $a\mathcal{S}b$) means a *attacks* (resp. *supports*) b, and $w(a)$ is the *basic strength* of a. The latter may represent various issues like the certainty degree of the argument's premises, trustworthiness of the argument's source, ….

We turn to the core concept of the paper. A semantics is a function transforming any weighted bipolar argumentation graph into a weighting on the set of arguments. The weight of an argument given by a semantics represents its *overall strength*. It is obtained from the aggregation of its basic strength and the overall strengths of its attackers and supporters. Arguments that get value 1 are *extremely strong* whilst those that get value 0 are *worthless*.

Definition 3 *(Semantics).* A *semantics* is a function $\mathbf{S}$ transforming any $\mathbf{A} = \langle \mathcal{A}, w, \mathcal{R}, \mathcal{S} \rangle \in \mathtt{wBAG}$ into a weighting $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}}$ on $\mathcal{A}$. Let $a \in \mathcal{A}$, $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}}(a)$ denote the *overall strength* of a.

Let us recall the notion of *isomorphism* between graphs.

Definition 4 *(Isomorphism).* Let $\mathbf{A} = \langle \mathcal{A}, w, \mathcal{R}, \mathcal{S} \rangle$, $\mathbf{A}' = \langle \mathcal{A}', w', \mathcal{R}', \mathcal{S}' \rangle \in \mathtt{wBAG}$. An *isomorphism* from $\mathbf{A}$ to $\mathbf{A}'$ is a bijective function f from $\mathcal{A}$ to $\mathcal{A}'$ such that the following hold:

- $\forall a \in \mathcal{A}$, $w(a) = w'(f(a))$,
- $\forall a, b \in \mathcal{A}$, $a\mathcal{R}b$ iff $f(a)\mathcal{R}'f(b)$,
- $\forall a, b \in \mathcal{A}$, $a\mathcal{S}b$ iff $f(a)\mathcal{S}'f(b)$.

Let us recall the notion of path between two nodes in a graph.

Definition 5 *(Path).* Let $\mathbf{A} = \langle \mathcal{A}, w, \mathcal{R}, \mathcal{S} \rangle \in \mathtt{wBAG}$, and $a, b \in \mathcal{A}$. A *path* from b to a is a finite non-empty sequence $\langle x_1, \ldots, x_n \rangle$ such that $x_1 = b$, $x_n = a$, and $\forall i < n$, $x_i \mathcal{R} x_{i+1}$ or $x_i \mathcal{S} x_{i+1}$.

Below is the list of all notations used in the paper.

Notations. Let $\mathbf{A} = \langle \mathcal{A}, w, \mathcal{R}, \mathcal{S} \rangle \in \mathtt{wBAG}$ and $a \in \mathcal{A}$. We denote by $\mathtt{Att}_{\mathbf{A}}(a)$ the set of all attackers of a in $\mathbf{A}$ (i.e., $\mathtt{Att}_{\mathbf{A}}(a) = \{b \in \mathcal{A} \mid b\mathcal{R}a\}$), and by $\mathtt{sAtt}_{\mathbf{A}}(a)$ the set of all *significant attackers* of a, i.e., attackers x of a such that $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}}(x) \neq 0$. Similarly, we denote by $\mathtt{Supp}_{\mathbf{A}}(a)$ the set of all supporters of a (i.e., $\mathtt{Supp}_{\mathbf{A}}(a) = \{b \in \mathcal{A} \mid b\mathcal{S}a\}$) and by $\mathtt{sSupp}_{\mathbf{A}}(a)$ the *significant supporters* of a, i.e., supporters x such that $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}}(x) \neq 0$.

Let now $\mathbf{A}' = \langle \mathcal{A}', w', \mathcal{R}', \mathcal{S}' \rangle \in \mathtt{wBAG}$ be such that $\mathcal{A} \cap \mathcal{A}' = \emptyset$. We denote by $\mathbf{A} \oplus \mathbf{A}'$ the element $\langle \mathcal{A}'', w'', \mathcal{R}'', \mathcal{S}'' \rangle$ of $\mathtt{wBAG}$ such that $\mathcal{A}'' = \mathcal{A} \cup \mathcal{A}'$, $\mathcal{R}'' = \mathcal{R} \cup \mathcal{R}'$, $\mathcal{S}'' = \mathcal{S} \cup \mathcal{S}'$, and $\forall x \in \mathcal{A}''$, the following holds: $w''(x) = w(x)$, if $x \in \mathcal{A}$; $w''(x) = w'(x)$, if $x \in \mathcal{A}'$.

3. Principles for semantics

In what follows, we propose principles that shed light on foundational choices made by semantics. In other words, properties that help us to better understand the underpinnings of semantics, and that facilitate their comparisons. The first nine principles are simple *combinations* of axioms proposed for graphs with only one type of interactions (support in [25], attack in [26]). The three next principles are new and show how the overall strengths of supporters and attackers of an argument might be aggregated, and the last one shows how to regulate the intensity of support in case of weighted bipolar argumentation graphs.

The first very basic principle, Anonymity, states that the strength of an argument is independent of its identity. It combines the two Anonymity axioms from [25,26].

Principle 1 *(Anonymity). A semantics* **S** *satisfies* anonymity *iff, for any* $\mathbf{A} = \langle \mathcal{A}, w, \mathcal{R}, \mathcal{S} \rangle, \mathbf{A}' = \langle \mathcal{A}', w', \mathcal{R}', \mathcal{S}' \rangle \in \mathtt{wBAG}$*, for any isomorphism* f *from* $\mathbf{A}$ *to* $\mathbf{A}'$*, the following property holds:* $\forall a \in \mathcal{A}$, $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}}(a) = \mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}'}(f(a))$.

Bi-variate independence principle states the following: the overall strength of an argument a should be independent of any argument b that is not connected to it (i.e., there is no path from b to a, ignoring the direction of the edges). This principle combines the two independence axioms from [25,26].

Principle 2 *(Bi-variate Independence). A semantics* **S** *satisfies* bi-variate independence *iff, for all* $\mathbf{A} = \langle \mathcal{A}, w, \mathcal{R}, \mathcal{S} \rangle$, $\mathbf{A}' = \langle \mathcal{A}', w', \mathcal{R}', \mathcal{S}' \rangle \in \mathtt{wBAG}$ *such that* $\mathcal{A} \cap \mathcal{A}' = \emptyset$*, the following property holds:* $\forall a \in \mathcal{A}$, $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}}(a) = \mathtt{Deg}^{\mathbf{S}}_{\mathbf{A} \oplus \mathbf{A}'}(a)$.

Bi-variate directionality principle combines Non-Dilution from [25] and Circumscription from [26]. It states that the overall strength of an argument should depend only on its incoming arrows, and thus not on the arguments it itself attacks or supports.

Principle 3 *(Bi-variate Directionality). A semantics* **S** *satisfies* bi-variate directionality *iff, for all* $\mathbf{A} = \langle \mathcal{A}, w, \mathcal{R}, \mathcal{S} \rangle$, $\mathbf{A}' = \langle \mathcal{A}', w', \mathcal{R}', \mathcal{S}' \rangle \in \mathtt{wBAG}$ *such that* $\mathcal{A} = \mathcal{A}'$, $\mathcal{R} \subseteq \mathcal{R}'$*, and* $\mathcal{S} \subseteq \mathcal{S}'$*, the following holds: for all* $a, b, x \in \mathcal{A}$*, if* $\mathcal{R}' \cup \mathcal{S}' = \mathcal{R} \cup \mathcal{S} \cup \{(a, b)\}$ *and there is no path from* b *to* x*, then* $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}}(x) = \mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}'}(x)$.

Bi-variate Equivalence principle ensures that the overall strength of an argument depends *only* on its basic strength and on the overall strengths of its direct attackers and supporters. It combines the two equivalence axioms from [25,26].

Principle 4 *(Bi-variate Equivalence). A semantics* **S** *satisfies* bi-variate equivalence *iff, for any* $\mathbf{A} = \langle \mathcal{A}, w, \mathcal{R}, \mathcal{S} \rangle \in \mathtt{wBAG}$*, for all* $a, b \in \mathcal{A}$*, if:*

- $w(a) = w(b)$,
- *there exists a bijective function* f *from* $\mathtt{Att}_{\mathbf{A}}(a)$ *to* $\mathtt{Att}_{\mathbf{A}}(b)$ *such that* $\forall x \in \mathtt{Att}_{\mathbf{A}}(a)$, $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}}(x) = \mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}}(f(x))$*, and*
- *there exists a bijective function* f' *from* $\mathtt{Supp}_{\mathbf{A}}(a)$ *to* $\mathtt{Supp}_{\mathbf{A}}(b)$ *such that* $\forall x \in \mathtt{Supp}_{\mathbf{A}}(a)$, $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}}(x) = \mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}}(f'(x))$,

then $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}}(a) = \mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}}(b)$.

Stability axiom combines Minimality [25] and Maximality [26] axioms. It states the following: if an argument is neither attacked nor supported, its overall strength should be equal to its basic strength.

Principle 5 *(Stability). A semantics* **S** *satisfies* stability *iff, for any* $\mathbf{A} = \langle \mathcal{A}, w, \mathcal{R}, \mathcal{S} \rangle \in \mathtt{wBAG}$, *for any* $a \in \mathcal{A}$, *if* $\mathtt{Att}_{\mathbf{A}}(a) = \mathtt{Supp}_{\mathbf{A}}(a) = \emptyset$, *then* $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}}(a) = w(a)$.

Neutrality axiom generalizes Dummy axiom [25] and Neutrality one from [26]. It states that worthless attackers or supporters have no effect.

Principle 6 *(Neutrality). A semantics* **S** *satisfies* neutrality *iff, for any* $\mathbf{A} = \langle \mathcal{A}, w, \mathcal{R}, \mathcal{S} \rangle \in \mathtt{wBAG}$, *for all* $a, b, x \in \mathcal{A}$, *if:*

- $w(a) = w(b)$,
- $\mathtt{Att}_{\mathbf{A}}(a) \subseteq \mathtt{Att}_{\mathbf{A}}(b)$,
- $\mathtt{Supp}_{\mathbf{A}}(a) \subseteq \mathtt{Supp}_{\mathbf{A}}(b)$,
- $\mathtt{Att}_{\mathbf{A}}(b) \cup \mathtt{Supp}_{\mathbf{A}}(b) = \mathtt{Att}_{\mathbf{A}}(a) \cup \mathtt{Supp}_{\mathbf{A}}(a) \cup \{x\}$, *and* $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}}(x) = 0$,

then $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}}(a) = \mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}}(b)$.

Bi-variate Monotony states the following: an argument is all the stronger when it is less attacked and more supported. This means that attacks cannot be beneficial to their targets and supports cannot be harmful. This axiom generalizes four axioms from the literature (Monotony and Counting [25] for supports, and the same axioms from [26] for attacks).

Principle 7 *(Bi-variate Monotony). A semantics* **S** *satisfies* bi-variate monotony *iff, for any* $\mathbf{A} = \langle \mathcal{A}, w, \mathcal{R}, \mathcal{S} \rangle \in \mathtt{wBAG}$, *for all* $a, b \in \mathcal{A}$ *such that:*

- $w(a) = w(b)$,
- $\mathtt{Att}_{\mathbf{A}}(a) \subseteq \mathtt{Att}_{\mathbf{A}}(b)$,
- $\mathtt{Supp}_{\mathbf{A}}(b) \subseteq \mathtt{Supp}_{\mathbf{A}}(a)$,

the following hold:

- $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}}(a) \geq \mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}}(b)$; ***(Monotony)***
- *if* $(\mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}}(a) > 0$ *and* $\mathtt{sAtt}_{\mathbf{A}}(a) \subset \mathtt{sAtt}_{\mathbf{A}}(b))$ *or* $(\mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}}(b) < 1$ *and* $\mathtt{sSupp}_{\mathbf{A}}(b) \subset \mathtt{sSupp}_{\mathbf{A}}(a))$, *then* $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}}(a) > \mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}}(b)$. ***(Strict Monotony)***

The next axiom concerns the quality of attackers and supporters. It states that any argument becomes stronger if the quality of its attackers is reduced and the quality of its supporters is increased. It combines the two Reinforcement axioms from [25,26].

Principle 8 *(Bi-variate Reinforcement). A semantics* **S** *satisfies* bi-variate reinforcement *iff, for any* $\mathbf{A} = \langle \mathcal{A}, w, \mathcal{R}, \mathcal{S} \rangle \in \mathtt{wBAG}$, *for all* $C, C' \subseteq \mathcal{A}$, *for all* $a, b \in \mathcal{A}$, *for all* $x, x', y, y' \in \mathcal{A} \setminus (C \cup C')$ *such that*

- $w(a) = w(b) > 0$,
- $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}}(x) \leq \mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}}(y)$,
- $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}}(x') \geq \mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}}(y')$,
- $\mathtt{Att}_{\mathbf{A}}(a) = C \cup \{x\}$,
- $\mathtt{Att}_{\mathbf{A}}(b) = C \cup \{y\}$,
- $\mathtt{Supp}_{\mathbf{A}}(a) = C' \cup \{x'\}$,
- $\mathtt{Supp}_{\mathbf{A}}(b) = C' \cup \{y'\}$,

the following hold:

- $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}}(a) \geq \mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}}(b)$; ***(Reinforcement)***
- *if* $(\mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}}(a) > 0$ *and* $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}}(x) < \mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}}(y))$ *or* $(\mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}}(b) < 1$ *and* $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}}(x') > \mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}}(y'))$, *then* $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}}(a) > \mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}}(b)$. ***(Strict Reinforcement)***

We have shown previously that an attacker may weaken (respectively a supporter may strengthen) a target. However, nothing is said about the *intensity* of an attack or support, i.e., to what extent an attack or a support may impact a targeted argument. Can an attack completely kill an argument? Can a support fully rehabilitate a weak argument? The answers to these questions depend on the nature of arguments. For instance, deductive arguments whose premises are information that may be true or false may be killed by attacks. Consider the two arguments A and B below.

(A) Tweety is a bird, therefore it flies.
(B) Tweety is a penguin, therefore the rule "birds fly" is not applicable.

Clearly, B undercuts A [27], and A may be fully rejected since the rule "birds fly" is indeed not applicable in the particular case of penguins. Consider now the two arguments C and D provided respectively by Paula and Paul:

(C) Senor Taco has the best Mexican food, therefore we go there.
(D) Food is much better at COATL restaurant.

The argument D denies the premise of C. However, both arguments are based on personal opinions of Paula and Paul and there is no reason for fully rejecting C.

The same reasoning holds for support relations. Indeed, in some cases it is reasonable to fully rehabilitate an argument with supporters. However, irrational behaviors, like fully accepting fallacious arguments that are supported are also possible and should be avoided. The argument E below remains fallacious even if it is clearly supported by the argument F.

(E) Tweety needs fuel, since it flies like planes.
(F) Indeed, Tweety flies. It is a bird.

In this paper, arguments are abstract entities and thus their internal structure, content, and nature are unspecified. Thus, it is not possible to distinguish between cases where killing is suitable for attacks and cases where it is not. Similarly, cases of full rehabilitation of support cannot be identified. Thus, in this paper we follow a cautious approach by avoiding both forms (killing, full rehabilitation). For that purpose, we combine Imperfection axiom from [25] with Resilience axiom from [26]. Imperfection states that an argument whose basic strength is less than 1 cannot be fully rehabilitated by supports. In other words, it cannot get an overall strength 1 due to supports. Resilience in [26] states that an argument whose basic strength is positive cannot be completely destroyed by attacks. Unlike the previous principles, the next one is not mandatory since its suitability depends on the nature of arguments being evaluated.

Principle 9 *(Resilience). A semantics* **S** *satisfies* resilience *iff, for any* $\mathbf{A} = \langle \mathcal{A}, w, \mathcal{R}, \mathcal{S} \rangle \in \mathtt{wBAG}$*, for all* $a \in \mathcal{A}$*, if* $0 < w(a) < 1$*, then* $0 < \mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}}(a) < 1$.

Resilience forbids an argument from getting an overall strength equal to 1 due simply to supporters. However, it allows an argument whose basic weight is, for instance, 0.1 to get an overall strength 0.9 if it is supported by one strong argument. This phenomenon, called *big jump*, may be undesirable. Consider the analogical arguments G and H below:

(G) Both restaurants X and Y are Italian, X serves good food, therefore Y serves good food as well.
(H) The two restaurants X and Y use the same products.

The link between the conclusion and the premises in G is clearly very weak. Strengthening this analogical argument amounts to finding important additional similarities between the compared objects (namely X and Y). However, pointing out one very important similarity may not be sufficient for making G very strong. The argument H supports G since it points out one additional similarity between the two restaurants. However, even if H is very strong (its premises are true, and it is not attacked), the link in G is still weak since the two restaurants may not have the same *chef de cuisine*. Thus, if the basic weight of G was initially 0.1 (due to its weak link), its overall strength cannot become for instance 0.9 simply due to H.

As for Resilience, there are cases where a weak argument may become very strong due to a single supporter. However, since arguments are abstract entities in our setting, we follow a cautious approach by forbidding big jumps between the basic weight of an argument and its overall strength. The next principle is also about the intensity of support. It aims at preventing supporters from having an exaggerated impact on their targets. More precisely, the idea is the following: if we add a new supporter (of any strength) to an argument A, then the distance between the strength of A and 1 cannot be reduced more than the half. This halfway philosophy seems to well-balance freedom of movement and prevention of exaggerated movements. It is worth mentioning that this principle concerns the impact of a single supporter, and does not prevent a weak argument from becoming very strong due to the combined effect of several supporters.

Principle 10 *(Inertia). A semantics* **S** *satisfies* inertia *iff, for any* $\mathbf{A} = \langle \mathcal{A}, w, \mathcal{R}, \mathcal{S} \rangle \in \mathtt{wBAG}$*, for all* $a, b, x \in \mathcal{A}$*, if*

- $w(a) = w(b)$,
- $\mathtt{Supp}_{\mathbf{A}}(b) = \mathtt{Supp}_{\mathbf{A}}(a) \cup \{x\}$,

- $\mathtt{Att}_{\mathbf{A}}(b) = \mathtt{Att}_{\mathbf{A}}(a)$,

then $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}}(b) \leq \mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}}(a) + [1 - \mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}}(a)]/2$.

The next three axioms answer the same question: how the overall strengths of attackers and supporters of an argument are aggregated? To answer this question, it is important to specify first which of the two types of interactions is more important. There are three options:

- Attacks are as important as supports,
- Attacks are more important than supports,
- Supports are more important than attacks.

The first option makes perfect sense in a decision making context. Indeed, in multiple criteria decision making, each argument promotes a criterion (see e.g., [28,29]). A supporter is an argument showing that a criterion is satisfied while an attacker shows a criterion that is violated. In this context, if an attacker and a supporter of the same argument have equal strength, they counter-balance each other. This principle is used in [28] for aggregating arguments of options/alternatives in decision making context. In another context like reasoning with inconsistent/defeasible information, supporters (respectively attackers) aim at confirming (respectively denying) parts of an argument. Thus, the exact part that is confirmed/denied plays a role. However, even if a supporter and an attacker target the same part, they do not necessarily counter-balance each other. Consider again the previous analogical argument G. Assume that it is supported by H and attacked by the following argument I:

(I) The two restaurants X and Y have different chef de cuisine.

Even if we assume that I is as strong as H (because for instance they both use certain information and are not attacked), the analogy used in G is weakened since there is one important feature on which the two compared restaurants X and Y differ. Please recall that an analogy is all the stronger when the number of "important" properties shared between X and Y is high and the number of different important properties is low. This example suggests that attacks take precedence over supports.

The third option (supports take precedence over attack) is not reasonable. An argument can be seen as a chain made of different components (premises, conclusion, link). Attacking one of the components is sufficient for weakening or destroying the whole chain. However, supporting one element of the chain does not necessarily make an argument strong. Thus, an attack cannot be ignored even in presence of (several) supporters.

The next principle captures the two first options. Franklin principle states that a supporter may never be more important than an attacker of equal strength while Strict Franklin states that an attacker and a supporter of equal strength counter-balance each other.

Principle 11 *(Franklin). A semantics* **S** *satisfies* Franklin *iff, for any* $\mathbf{A} = \langle \mathcal{A}, w, \mathcal{R}, \mathcal{S} \rangle \in \mathtt{wBAG}$, *for all* $a, b, x, y \in \mathcal{A}$, *if*

- $w(b) = w(a)$,
- $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}}(x) = \mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}}(y)$
- $\mathtt{Att}_{\mathbf{A}}(a) = \mathtt{Att}_{\mathbf{A}}(b) \cup \{x\}$,
- $\mathtt{Supp}_{\mathbf{A}}(a) = \mathtt{Supp}_{\mathbf{A}}(b) \cup \{y\}$,

then the following hold:

- $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}}(a) \leq \mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}}(b)$, ***(Franklin)***
- $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}}(a) = \mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}}(b)$. ***(Strict Franklin)***

We show that attacks and supports of equal strengths eliminate each other when a semantics satisfies Strict Franklin.

Proposition 1. *Let* **S** *be a semantics that satisfies Bi-variate Independence, Bi-variate Directionality, Stability and Strict Franklin. For any* $\mathbf{A} = \langle \mathcal{A}, w, \mathcal{R}, \mathcal{S} \rangle \in \mathtt{wBAG}$, *for any* $a \in \mathcal{A}$, *if there exists a bijective function* f *from* $\mathtt{Att}_{\mathbf{A}}(a)$ *to* $\mathtt{Supp}_{\mathbf{A}}(a)$ *such that* $\forall x \in \mathtt{Att}(a)$, $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}}(x) = \mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}}(f(x))$, *then* $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}}(a) = w(a)$.

Weakening states that if attackers overcome supporters, the argument should lose weight. The idea is that supports are not sufficient for counter-balancing attacks. Please note that this does not mean that supports will not have an impact on the overall strength of an argument. They may mitigate the global loss due to attacks.

Principle 12 *(Weakening). A semantics* **S** *satisfies* weakening *iff, for any* $\mathbf{A} = \langle \mathcal{A}, w, \mathcal{R}, \mathcal{S} \rangle \in \mathtt{wBAG}$, *for all* $a \in \mathcal{A}$, *if* $w(a) > 0$ *and there exists an injective function* f *from* $\mathtt{Supp}_{\mathbf{A}}(a)$ *to* $\mathtt{Att}_{\mathbf{A}}(a)$ *such that:*

- $\forall x \in \mathtt{Supp}_{\mathbf{A}}(a)$, $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}}(x) \leq \mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}}(f(x))$; *and*
- $\mathtt{sAtt}_{\mathbf{A}}(a) \setminus \{f(x) \mid x \in \mathtt{Supp}_{\mathbf{A}}(a)\} \neq \emptyset$ *or* $\exists x \in \mathtt{Supp}_{\mathbf{A}}(a)$ *s.t.* $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}}(x) < \mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}}(f(x))$,

then $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}}(a) < w(a)$.

Strengthening states that if supporters overcome attackers, the argument should gain weight. Indeed, attacks are not sufficient for counter-balancing supports, however, they may mitigate the global gain due to supports.

Principle 13 *(Strengthening). A semantics* **S** *satisfies* strengthening *iff, for any* $\mathbf{A} = \langle \mathcal{A}, w, \mathcal{R}, \mathcal{S} \rangle \in \mathtt{wBAG}$, *for all* $a \in \mathcal{A}$, *if* $w(a) < 1$ *and there exists an injective function* f *from* $\mathtt{Att}_{\mathbf{A}}(a)$ *to* $\mathtt{Supp}_{\mathbf{A}}(a)$ *such that:*

- $\forall x \in \mathtt{Att}_{\mathbf{A}}(a)$, $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}}(x) \leq \mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}}(f(x))$; *and*
- $\mathtt{sSupp}_{\mathbf{A}}(a) \setminus \{f(x) \mid x \in \mathtt{Att}_{\mathbf{A}}(a)\} \neq \emptyset$ *or* $\exists x \in \mathtt{Att}_{\mathbf{A}}(a)$ *s.t.* $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}}(x) < \mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}}(f(x))$,

then $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}}(a) > w(a)$.

It is worth mentioning that weakening and strengthening generalize their corresponding axioms in [25,26]. Indeed, when the support relation is empty, bipolar version of weakening coincides with weakening axiom in [26]. However, it handles additional cases when supports exist. Similarly, when the attack relation is empty, the principle coincides with strengthening axiom in [25].

Almost all axioms are independent, i.e., they do not follow from others. Notable exceptions are Bi-variate Monotony which follows from five other principles (namely Bi-variate Independence, Bi-variate Directionality, Stability, Neutrality and Bi-variate Reinforcement) and Franklin which follows from Strict Franklin.

Proposition 2. *Let* **S** *be a semantics.*

- *If* **S** *satisfies Bi-variate Independence, Bi-variate Directionality, Stability, Neutrality and Bi-variate Reinforcement, then* **S** *satisfies Bi-variate Monotony.*
- *If* **S** *satisfies Strict Franklin, then* **S** *satisfies Franklin.*

All axioms are compatible, i.e., they can be satisfied all together by a semantics.

Proposition 3. *All the axioms are compatible.*

4. Formal analysis of existing semantics

There are several proposals in the literature for the evaluation of arguments in bipolar argumentation graphs. They can be partitioned into two families: *extension* semantics [14–18,30,31] and *weighted* semantics [22–24,32,33].

Extension semantics extend Dung's ones [6] for accounting for supports between arguments. They take as input *flat* bipolar argumentation graphs, i.e., graphs where arguments have all the *same* basic strength.

Definition 6 *(Flat Bipolar Graphs).* A *flat bipolar argumentation graph* is an element $\langle \mathcal{A}, w, \mathcal{R}, \mathcal{S} \rangle \in \mathtt{wBAG}$ such that for any $a \in \mathcal{A}$, $w(a) = 1$.

The first work on extension semantics in the bipolar context was done by Cayrol and Lagasquie in [14]. The authors argued that two kinds of attacks may emerge from a bipolar graph: supported attacks and secondary ones.

Definition 7 *(Complex Attacks).* Let $\langle \mathcal{A}, w, \mathcal{R}, \mathcal{S} \rangle \in \mathtt{wBAG}$ be a flat bipolar argumentation graph, and $a, b \in \mathcal{A}$.

- There is a *supported attack* from a to b iff there is a sequence $a_1 \mathcal{R}_1 \ldots \mathcal{R}_{n-1} a_n$, $n \geq 3$, with $a_1 = a$, $a_n = b$, for any $i = 2, \ldots, n-2$, $\mathcal{R}_i = \mathcal{S}$ and $\mathcal{R}_{n-1} = \mathcal{R}$.
- There is a *secondary attack* from a to b iff there is a sequence $a_1 \mathcal{R}_1 \ldots \mathcal{R}_{n-1} a_n$, $n \geq 3$, with $a_1 = a$, $a_n = b$, $\mathcal{R}_1 = \mathcal{R}$, and for any $i = 2, \ldots, n-2$, $\mathcal{R}_i = \mathcal{S}$.

Let $\mathcal{R}_c$ denote the set of all attacks of $\mathcal{R}$ and the supported/secondary ones; i.e., $\mathcal{R}_c = \mathcal{R} \cup \{(a, b) \mid$ there exists a supported or secondary attack from a to $b\}$.

Example 1. Consider the flat bipolar argumentation graph depicted in Fig. 1. Dashed lines represent support relations and plain lines represent attack ones.

There is a supported attack from argument e to a ($e \ \mathcal{S} \ c \ \mathcal{R} \ a$) and a secondary attack from f to b ($f \ \mathcal{R} \ d \ \mathcal{S} \ b$).

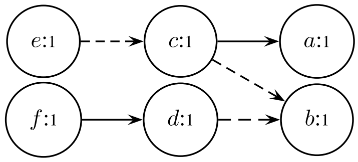

Fig. 1. Bipolar graph $\mathbf{A}_1$.

Extension semantics look for acceptable sets of arguments, called extensions in [6]. Each extension represents a coherent position, thus it should satisfy a *coherence* property, called *conflict-freeness*, and a *defence* one. The former ensures that an extension does not contain conflicting arguments, while the latter requires that an extension defends its elements against any attack. These two properties were extended in [14] for accounting for complex attacks that may emerge in flat bipolar argumentation graphs.

Definition 8 *(Conflict-freeness–Safety–Defence).* Let $\langle \mathcal{A}, w, \mathcal{R}, \mathcal{S} \rangle \in \mathtt{wBAG}$ be a flat bipolar argumentation graph, and $\mathcal{E} \subseteq \mathcal{A}$.

- $\mathcal{E}$ is *conflict-free* iff $\nexists a, b \in \mathcal{E}$ such that $a\mathcal{R}_c b$.
- $\mathcal{E}$ is *safe* iff $\nexists a, b, c \in \mathcal{A}$ such that:
  - $a, b \in \mathcal{E}$,
  - $b\mathcal{S}c$ or $c \in \mathcal{E}$, and
  - $a\mathcal{R}_c c$.
- $\mathcal{E}$ defends an argument $a \in \mathcal{A}$ iff for any $b \in \mathcal{A}$, if $b\mathcal{R}_c a$, then $\exists c \in \mathcal{E}$ such that $c\mathcal{R}_c b$.

Example 1. In the graph $\mathbf{A}_1$, the set $\{e, c\}$ is safe while the set $\{e, c, f\}$ is not since it both supports and attacks the argument b.

Definition 9 *(Extensions).* Let $\mathbf{A} = \langle \mathcal{A}, w, \mathcal{R}, \mathcal{S} \rangle \in \mathtt{wBAG}$ be a flat bipolar argumentation graph, and $\mathcal{E} \subseteq \mathcal{A}$.

- $\mathcal{E}$ is a *stable extension* iff $\mathcal{E}$ is conflict-free and for any $a \notin \mathcal{E}$, there exists $c \in \mathcal{E}$ such that $c\mathcal{R}_c a$.
- $\mathcal{E}$ is a *d-preferred extension* iff $\mathcal{E}$ is maximal (for set inclusion) among the sets that are conflict-free and defend all their elements.
- $\mathcal{E}$ is a *s-preferred extension* iff $\mathcal{E}$ is maximal (for set inclusion) among the sets that are safe and defend all their elements.

Let $\mathtt{Ext_x}(\mathbf{A})$ denote the set of all extensions of $\mathbf{A}$ under semantics x (x being stable, or d-preferred, or s-preferred).

Throughout this section, we refer to the three above semantics by *reviewed semantic.*

Example 1 *(Cont.).* The graph $\mathbf{A}_1$ has one stable and d-preferred extension: $\{e, c, f\}$. It has however two s-preferred extensions: $\{e, c\}$ and $\{f\}$.

Once extensions are computed, in [7,34–37], a three-valued qualitative overall strength is assigned to every argument as follows: an argument is *accepted* if it belongs to all extensions, *undecided* (or credulously accepted) if it belongs to some but not all extensions, and *rejected* if it does not belong to any extension. For the purpose of analyzing these semantics against the principles, we replace the three qualitative values with numerical ones as follows.

Definition 10 *(Argument's overall strength).* Let $\mathbf{A} = \langle \mathcal{A}, w, \mathcal{R}, \mathcal{S} \rangle \in \mathtt{wBAG}$ be a flat bipolar argumentation graph, $a \in \mathcal{A}$, and x is one of the reviewed semantics.

- $\mathtt{Deg}^x_{\mathbf{A}}(a) = 1$ iff $a \in \bigcap_{\mathcal{E} \in \mathtt{Ext_x}(\mathbf{A})} \mathcal{E}$. (Accepted argument)
- $\mathtt{Deg}^x_{\mathbf{A}}(a) = 0.5$ iff $\exists \mathcal{E}, \mathcal{E}' \in \mathtt{Ext_x}(\mathbf{A})$ such that $a \in \mathcal{E}$ and $a \notin \mathcal{E}'$. (Undecided argument)
- $\mathtt{Deg}^x_{\mathbf{A}}(a) = 0$ iff $a \notin \bigcup_{\mathcal{E} \in \mathtt{Ext_x}(\mathbf{A})} \mathcal{E}$. (Rejected argument)

When the attack relation is empty, any flat bipolar argumentation graph has a single extension, which contains all the arguments. Thus, all arguments have the same overall strength.

Proposition 4. *Let $\mathbf{A} = \langle \mathcal{A}, w, \mathcal{R}, \mathcal{S} \rangle \in \mathtt{wBAG}$ be a flat bipolar argumentation graph. If $\mathcal{R} = \emptyset$, then for any $x \in \{$stable, d-preferred, s-preferred$\}$,*

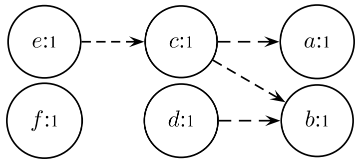

Fig. 2. Bipolar graph $\mathbf{A}_2$.

- $\texttt{Ext}_x(\mathbf{A}) = \{\mathcal{A}\}$.
- *For any* $a \in \mathcal{A}$, $\texttt{Deg}^x_{\mathbf{A}}(a) = 1$.

This means that when the attack relation is empty, the support relation does not play any role, and a supported argument is as acceptable as a non-supported one.

Example 2. Let us consider the flat bipolar argumentation graph depicted in Fig. 2. This graph has one stable, d-preferred, s-preferred extension: $\{a, b, c, d, e, f\}$. Hence, all the six arguments get value 1. Note that b which has 2 supporters is as strong as d, e, f which are not supported at all.

It was shown in [14] that when the support relation is empty, the three semantics of Definition 9 coincide with Dung's ones. Consequently, each semantics violates the same axioms as its basic version in [6]. Note that in [26], a formal analysis of Dung's semantics is done for flat attack graphs. The following result summarizes the axioms that are violated.

Proposition 5. *Stable semantics violates Stability, Bi-variate Independence, and Bi-variate Directionality. The three semantics violate Bi-variate Equivalence, Neutrality, Resilience, Strict Monotony, Strict Reinforcement, Franklin and Strengthening.*

It is worth mentioning that Inertia axiom does not apply to extension semantics since they allow only three values as possible overall strengths of arguments.

The approaches developed in [15–18] are similar to the one by Cayrol and Lagasquie. They also coincide with Dung's framework in case the support relation is empty. Furthermore, when the attack relation is empty, the approaches in [16, 18] return a single extension. The latter contains the arguments that do not belong to any cycle. Thus, they also violate strengthening and the support relation may not be fully exploited in the evaluation of arguments. They also violate the same set of axioms as the approach of Cayrol and Lagasquie.

The second family of weighted semantics was introduced for the first time in [24]. In their paper, the authors presented some properties that such semantics should satisfy (like a particular case of strengthening). However, they did not define concrete semantics. To the best of our knowledge, the first weighted semantics was introduced in [32]. Basic weights of arguments represent positive and negative votes on arguments. The semantics evaluates in the same way but separately the attackers and supporters of an argument before aggregating them.

Definition 11. Let $\mathbf{A} = \langle \mathcal{A}, w, \mathcal{R}, \mathcal{S} \rangle \in \texttt{wBAG}$ and $a \in \mathcal{A}$. Let $\texttt{Att}_{\mathbf{A}}(a) = \{b_1, \ldots, b_n\}$ and $\texttt{Supp}_{\mathbf{A}}(a) = \{s_1, \ldots, s_k\}$.

$$\texttt{Deg}^{\mathbf{S}}_{\mathbf{A}}(a) = \begin{cases} w(a) & \text{if } \texttt{Supp}_{\mathbf{A}}(a) = \texttt{Att}_{\mathbf{A}}(a) = \emptyset \\ f_a(a) & \text{if } \texttt{Supp}_{\mathbf{A}}(a) = \emptyset \text{ and } \texttt{Att}_{\mathbf{A}}(a) \neq \emptyset \\ f_s(a) & \text{if } \texttt{Supp}_{\mathbf{A}}(a) \neq \emptyset \text{ and } \texttt{Att}_{\mathbf{A}}(a) = \emptyset \\ \frac{f_a(a)+f_s(a)}{2} & \text{otherwise} \end{cases}$$

where

$$f_a(a) = w(a) \times (1 - m(b_1, \ldots, b_n))$$

and

$$f_s(a) = w(a) + (w(a) - w(a) \times (1 - m(s_1, \ldots, s_k)))$$

and

$$m(x_1, \ldots, x_j) = \begin{cases} 0 & \text{if } j = 0 \\ \texttt{Deg}^{\mathbf{S}}_{\mathbf{A}}(x_1) + m(x_2, \ldots, x_j) - \texttt{Deg}^{\mathbf{S}}_{\mathbf{A}}(x_1) \times m(x_2, \ldots, x_j) & \text{otherwise} \end{cases}$$

This semantics was proposed for any typology of graphs. However, it is easy to see that it does not handle correctly cycles. Assume a simple graph with two arguments a and b such that a attacks b and b attacks a. Assume also that $w(a) = w(b) = 1$. It is easy to check that this semantics assigns to each argument any solution of the equation $\texttt{Deg}^{\mathbf{S}}_{\mathbf{A}}(a) + \texttt{Deg}^{\mathbf{S}}_{\mathbf{A}}(b) = 1$, hence an infinite number of values. This shows that the semantics is not well-defined.

Later in [22], QuAD semantics was introduced for evaluating arguments in *acyclic weighted argumentation graphs*.

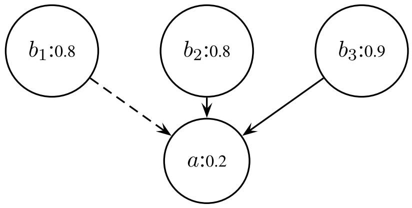

Fig. 3. Bipolar graph $\mathbf{A}_3$.

Definition 12 *(Acyclic Graphs).* A weighted bipolar argumentation graph $\mathbf{A} = \langle \mathcal{A}, w, \mathcal{R}, \mathcal{S} \rangle \in \mathtt{wBAG}$ is *acyclic* iff the following holds: for any non-empty finite sequence $\mathbf{a} = \langle a_1, a_2, \ldots, a_n \rangle$ of elements of $\mathcal{A}$, if $\forall i \in \{1, 2, \ldots, n-1\}$, $\langle a_i, a_{i+1} \rangle \in \mathcal{R} \cup \mathcal{S}$, then $\langle a_n, a_1 \rangle \notin \mathcal{R} \cup \mathcal{S}$.

Since a semantics takes as input any graph, we need to introduce the notion of restricted semantics. All notations and principles for semantics are straightforwardly adapted to restricted semantics.

Definition 13 *(Restricted semantics).* A *restricted semantics* is a function $\mathbf{S}$ transforming any acyclic $\mathbf{A} = \langle \mathcal{A}, w, \mathcal{R}, \mathcal{S} \rangle \in \mathtt{wBAG}$ into a weighting on $\mathcal{A}$.

QuAD is then a restricted semantics which assigns a numerical value to every argument on the basis of its basic strength, and the overall strengths of its attackers and supporters. It evaluates separately the supporters (by a function f_s) and the attackers (by a function f_a) before aggregating them.

Definition 14 *(QuAD).* Let $\mathbf{A} = \langle \mathcal{A}, w, \mathcal{R}, \mathcal{S} \rangle$ be an acyclic weighted bipolar argumentation graph. For any $a \in \mathcal{A}$,

$$\mathtt{Deg}_{\mathbf{A}}^{\mathtt{QuAD}}(a) = \begin{cases} f_a(a) & \text{if } \mathtt{Supp}_{\mathbf{A}}(a) = \emptyset \text{ and } \mathtt{Att}_{\mathbf{A}}(a) \neq \emptyset \\ f_s(a) & \text{if } \mathtt{Supp}_{\mathbf{A}}(a) \neq \emptyset \text{ and } \mathtt{Att}_{\mathbf{A}}(a) = \emptyset \\ w(a) & \text{if } \mathtt{Supp}_{\mathbf{A}}(a) = \emptyset \text{ and } \mathtt{Att}_{\mathbf{A}}(a) = \emptyset \\ \frac{f_a(a)+f_s(a)}{2} & \text{otherwise} \end{cases}$$

where

$$f_a(a) = w(a) \times \prod_{b_i \mathcal{R} a} (1 - \mathtt{Deg}_{\mathbf{A}}^{\mathtt{QuAD}}(b_i))$$

and

$$f_s(a) = 1 - (1 - w(a)) \times \prod_{c_i \mathcal{S} a} (1 - \mathtt{Deg}_{\mathbf{A}}^{\mathtt{QuAD}}(c_i))$$

Example 3. Consider the acyclic bipolar argumentation graph depicted in Fig. 3.
It can be checked that $\mathtt{Deg}_{\mathbf{A}_3}^{\mathtt{QuAD}}(a) = 0.422$, $\mathtt{Deg}_{\mathbf{A}_3}^{\mathtt{QuAD}}(b_1) = \mathtt{Deg}_{\mathbf{A}_3}^{\mathtt{QuAD}}(b_2) = 0.8$, and $\mathtt{Deg}_{\mathbf{A}_3}^{\mathtt{QuAD}}(b_3) = 0.9$.

The following result summarizes the principles that are satisfied (respectively violated) by QuAD.

Proposition 6. *The following properties hold.*

- *QuAD satisfies Anonymity, Bi-variate Independence, Bi-variate Directionality, Bi-variate Equivalence, Stability, Neutrality, Monotony, Reinforcement.*
- *QuAD violates Strict Monotony, Strict Reinforcement, Resilience, Franklin, Weakening, Strengthening, and Inertia.*

As a consequence of violating Weakening and Strengthening, QuAD may behave irrationally. Indeed, choosing which of support and attack should take precedence depends on the intrinsic strength of an argument.

Example 3 *(Cont.).* Consider the weighted bipolar argumentation $\mathbf{A}_3$ depicted in Fig. 3. The argument a has an attacker and a supporter of equal strengths, and an additional attacker b_3. Note that if $w(a) = 0.2$, then $\mathtt{Deg}_{\mathbf{A}_3}^{\mathtt{QuAD}}(a) = 0.422$ meaning that the single supporter is privileged to the two attackers. However, if $w(a) = 0.7$, $\mathtt{Deg}_{\mathbf{A}_3}^{\mathtt{QuAD}}(a) = 0.477$ meaning that attacks are privileged to support. More generally, we can show that if $w(a) \geq 0.5$, then $\mathtt{Deg}_{\mathbf{A}_3}^{\mathtt{QuAD}}(a) < w(a)$, else $\mathtt{Deg}_{\mathbf{A}_3}^{\mathtt{QuAD}}(a) > w(a)$.

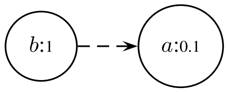

Fig. 4. Bipolar graph $\mathbf{A}_4$.

As a consequence of violating Inertia, QuAD may allow *big jumps* in gains from supports, and thus a fallacious argument may become very strong if it is supported by a strong argument. Let us illustrate the issue with the following example.

Example 4. Consider the weighted bipolar argumentation graph depicted in Fig. 4.

Note that the initial strength of a is extremely weak. It can be checked that $\texttt{Deg}^{\texttt{QuAD}}_{\mathbf{A}_4}(a) = 1$. Indeed, a strong supporter makes a very weak argument very strong.

There are two issues with such big jump: First, the gain is enormous and not reasonable. Assume that a is the argument "Tweety needs fuel, since it flies like planes". It is hard to accept a even when supported. The supporter may increase slightly the strength of the argument but does not correct the wrong premises of the argument. Second, such jump impedes the discrimination between different cases where $w(a) > 0.001$ since whatever the value of $w(a)$, the overall strength is almost 1.

QuAD was recently extended to DF-QuAD in [23]. The new semantics is restrictive since it focuses also on *acyclic graphs*. Unlike QuAD, it uses the same function for aggregating supporters and attackers separately. It satisfies Strict Franklin axiom, thus it treats equally attacks and supports. It violates Strengthening and Weakening in presence of attackers/supporters of degree 1. However, the semantics avoids the irrational behavior of QuAD.

Definition 15 *(DF-QuAD).* Let $\mathbf{A} = \langle \mathcal{A}, w, \mathcal{R}, \mathcal{S} \rangle \in \texttt{wBAG}$ be an *acyclic* weighted bipolar argumentation graph and $a \in \mathcal{A}$. Let $\texttt{Supp}_{\mathbf{A}}(a) = \{c_1, \ldots, c_n\}$ and $\texttt{Att}_{\mathbf{A}}(a) = \{b_1, \ldots, b_m\}$.

$$\texttt{Deg}^{\texttt{DF}}_{\mathbf{A}}(a) = \begin{cases} w(a) - w(a) \times |\mathcal{F}(x) - \mathcal{F}(y)| & \text{if } \mathcal{F}(y) \geq \mathcal{F}(x) \\ w(a) + (1 - w(a)) \times |\mathcal{F}(x) - \mathcal{F}(y)| & \text{if } \mathcal{F}(y) < \mathcal{F}(x) \end{cases}$$

where

$$x = \mathcal{F}(\texttt{Deg}^{\texttt{DF}}_{\mathbf{A}}(c_1), \ldots, \texttt{Deg}^{\texttt{DF}}_{\mathbf{A}}(c_n))$$
$$y = \mathcal{F}(\texttt{Deg}^{\texttt{DF}}_{\mathbf{A}}(b_1), \ldots, \texttt{Deg}^{\texttt{DF}}_{\mathbf{A}}(b_m))$$
$$\mathcal{F}(v_1, \ldots, v_k) = \begin{cases} 0 & \text{if } k = 0 \\ 1 - \prod_{i=1}^{k}(1 - v_i) & \text{otherwise} \end{cases}$$

Proposition 7. *The following properties hold.*

- *DF-QuAD satisfies Anonymity, Bi-variate Independence, Bi-variate Directionality, Bi-variate Equivalence, Stability, Neutrality, Monotony, Reinforcement, and Franklin.*
- *DF-QuAD violates Strict Monotony, Strict Reinforcement, Resilience, Weakening, Strengthening, and Inertia,.*

Like QuAD, the restricted semantics DF-QuAD suffers from the *big jump* problem. Consider the graph depicted in Fig. 4. Note that the argument a has a very low basic strength ($w(a) = 0.1$). This argument is supported by the very strong argument b. According to DF-QuAD, $\texttt{Deg}^{\texttt{DF}}_{\mathbf{A}_4}(a) = 0.991$. Thus, the value of a makes a big jump from 0.1 to 0.991. Table 1summarizes the properties of the discussed semantics.

In [33] the authors investigated weighted bipolar argumentation graphs and how arguments can be evaluated in such graphs. They defined principles which are similar to ours since they also generalized the ones proposed in [25,26]. They also provided six novel ones (neutralization, continuity, interchangeability, linearity, reverse impact, boundedness). The authors proposed also semantics that satisfy all or some principles. The first semantics, called Direct Aggregation Semantics, is a function that is based on a damping factor and that computes the values of arguments in an iterative way. The sequence of values converges in case the damping factor is greater than the in-degree of the argumentation graph. Direct Aggregation Semantics is thus graph-dependent; it changes from one graph to another since it should check the in-degree of the latter. This semantics does not thus evaluate arguments in a uniform way. In our paper, we argue that a semantics should be applied in a uniform way to any family of graphs and should not change from one graph to another. The second semantics, called Sigmoid directed aggregation semantics, is an adaptation of the first one in a way that the final values of arguments are in the interval $(0, 1)$ rather than in the set of real numbers. It is thus well-defined in a particular case. The third semantics uses a function, called Recursive Sigmoid Aggregation Function, it is based on the previous one for capturing two semantics from $[0, 1]$. This function does not converge in general. The two other semantics (recursive damped aggregation and Damped dogged) are discussed very briefly and their convergences are not shown yet.

Table 1
The symbol • (resp. ×, !) stands for satisfied (resp. violated, not applicable).

Family of semantics	Extension semantics			Gradual semantics			
	Cyclic + acyclic graphs			Acyclic graphs		Acyclic non-maximal graphs	
	Stable	s-Preferred	d-preferred	QuAD	DF-QuAD	DF-QuAD	Ebs
Anonymity	•	•	•	•	•	•	•
Bi-variate Independence	×	•	•	•	•	•	•
Bi-variate Directionality	×	•	•	•	•	•	•
Bi-variate Equivalence	×	×	×	•	•	•	•
Stability	×	•	•	•	•	•	•
Neutrality	×	×	×	•	•	•	•
Monotony	•	•	•	•	•	•	•
Strict Monotony	×	×	×	×	×	×	•
Reinforcement	•	•	•	•	•	•	•
Strict Reinforcement	×	×	×	×	×	×	•
Resilience	×	×	×	×	×	×	•
Inertia	!	!	!	×	×	×	•
Franklin	•	•	•	×	•	•	•
Strict Franklin	×	×	×	×	•	•	•
Weakening	•	•	•	×	×	•	•
Strengthening	×	×	×	×	×	•	•

5. Exponent-based semantics

As shown in the previous sections, no existing semantics satisfies all our principles together. The goal of the present section is to handle this issue. More precisely, we construct a new semantics satisfying all principles, but at the cost of a certain degree of coverage. Indeed, we only consider non-maximal and acyclic weighted argumentation graphs.

Definition 16 *(Non-maximality).* A weighted bipolar argumentation graph $\mathbf{A} = \langle \mathcal{A}, w, \mathcal{R}, \mathcal{S} \rangle \in \mathtt{wBAG}$ is *non-maximal* iff $\forall a \in \mathcal{A}$, $w(a) < 1$.

Without loss of generality, the basic strengths of arguments are less than 1. Note that few arguments are intrinsically perfect. The probability of false information, exceptions, etc., is rarely 0. In contrast, the loss of cyclic graphs is important. But, we consider that the class of all acyclic non-maximal weighted bipolar graphs is expressive enough to deserve attention.

Definition 17 *(Restricted semantics).* A *restricted semantics* is a function $\mathbf{S}$ transforming any acyclic non-maximal weighted bipolar argumentation graph $\mathbf{A} = \langle \mathcal{A}, w, \mathcal{R}, \mathcal{S} \rangle \in \mathtt{wBAG}$ into a weighting on $\mathcal{A}$.

Before presenting our semantics, we need to introduce a relation between arguments based on the longest paths to reach them (mixing support and attack arrows).

Definition 18 *(Well-founded relation).* Let $\mathbf{A} = \langle \mathcal{A}, w, \mathcal{R}, \mathcal{S} \rangle \in \mathtt{wBAG}$ be an acyclic weighted bipolar argumentation graph and $a \in \mathcal{A}$. A *path to a in* $\mathbf{A}$ is a non-empty finite sequence $\mathbf{a} = \langle a_1, a_2, \dots, a_n \rangle$ of elements of $\mathcal{A}$ such that $a_n = a$ and $\forall i \in \{1, 2, \dots, n-1\}$, $\langle a_i, a_{i+1} \rangle \in \mathcal{R} \cup \mathcal{S}$. We denote by $\mathtt{Rel}(\mathbf{A})$ the well-founded binary relation $\prec$ on $\mathcal{A}$ such that $\forall x, y \in \mathcal{A}$, $x \prec y$ iff $\max\{n \mid$ there exists a path to x of length $n\} < \max\{n \mid$ there exists a path to y of length $n\}$. Since $\mathbf{A}$ is acyclic, those maximum lengths are well-defined, so is $\mathtt{Rel}(\mathbf{A})$.

We are ready to define the *Exponent-based restricted semantics*. The general idea is to take into account supporters and attackers in an exponent E of 2 (the smallest natural number that can be effectively exponentiated). More precisely, the stronger or more-numerous the supporters, the greater and more-likely-positive that exponent. Obviously, the inverse is true with the attackers. Then, the overall strength of an argument a is naturally defined as $w(a)2^E$. Finally, we need certain tweakings (including a double polarity reversal) to make our function a restricted semantics in the first place, and to have it satisfy all our axioms. More formally:

Definition 19 *(Exponent-based restricted semantics).* We denote by $\mathtt{Ebs}$ the restricted semantics such that for any acyclic non-maximal weighted bipolar argumentation graph $\mathbf{A} = \langle \mathcal{A}, w, \mathcal{R}, \mathcal{S} \rangle \in \mathtt{wBAG}$, $\mathtt{Ebs}(\mathbf{A})$ is the weighting f on $\mathcal{A}$ recursively defined with $\mathtt{Rel}(\mathbf{A})$ as follows: $\forall a \in \mathcal{A}$,

$$f(a) = 1 - \frac{1 - w(a)^2}{1 + w(a)2^E} \quad \text{where} \quad E = \sum_{x \in \mathtt{Supp}(a)} f(x) - \sum_{x \in \mathtt{Att}(a)} f(x).$$

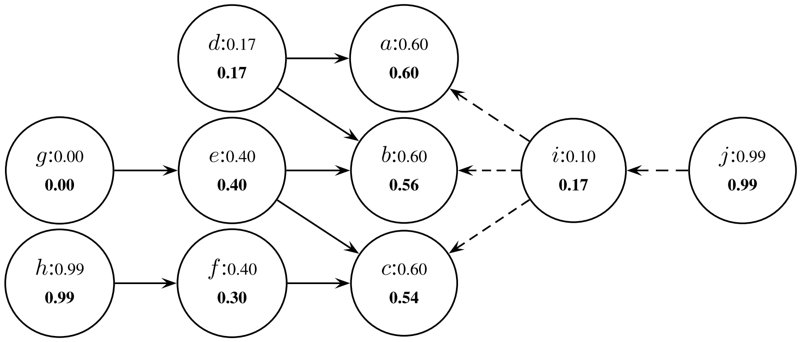

Fig. 5. Bipolar graph $\mathbf{A}_5$.

As an immediate corollary, we have:

Corollary 1. *Let $\mathbf{A} = \langle \mathcal{A}, w, \mathcal{R}, \mathcal{S} \rangle$ be an acyclic non-maximal weighted bipolar argumentation graph and $a \in \mathcal{A}$. The following holds:*

$$\mathtt{Deg}^{\mathtt{Ebs}}_{\mathbf{A}}(a) = 1 - \frac{1 - w(a)^2}{1 + w(a)2^E} \quad \textit{where} \quad E = \sum_{x \in \mathtt{Supp}(a)} \mathtt{Deg}^{\mathtt{Ebs}}_{\mathbf{A}}(x) - \sum_{x \in \mathtt{Att}(a)} \mathtt{Deg}^{\mathtt{Ebs}}_{\mathbf{A}}(x).$$

Below is an example where most principles are exemplified. Every circle contains [argument name]:[intrinsic strength] and below [**overall strength**].

Example 5. The neutrality principle can be checked with g and e, stability with e.g. d, bivariate monotony with a and b, bivariate reinforcement with b and c, Imperfection with i, Strict Franklin with a, weakening with e.g. b, and strengthening with i.

Proposition 8. Ebs *satisfies all the 13 principles.*

Note that being supported by an extremely strong argument does not cause a weak argument to become extremely strong as well, which shows that Ebs does not suffer from the big jump problem (indeed, it satisfies inertia). Note that $\mathtt{Deg}^{\mathtt{Ebs}}_{\mathbf{A}_5}(i) = 0.17$ and thus the jump is not big. Note also that by satisfying Weakening and Strengthening, the semantics avoids the irrational behavior of QuAD.

6. Conclusion

The paper presented for the first time principles that serve as guidelines for defining semantics in weighted bipolar settings. It also analyzed existing semantics with regard to the principles. The results revealed that extension-based semantics like [14–18] fail to satisfy key properties like independence and directionality. Furthermore, the role of support relation is a bit ambiguous since in case the attack relation is empty, the argumentation graph has a single extension containing all the arguments. This means that supported and non-supported arguments are all equally acceptable. Weighted semantics defined in [22,23] for the subclass of acyclic weighted bipolar graphs satisfy more but not all the principles. We proposed a novel semantics which satisfies all the 13 principles. However, this semantics deals only with acyclic graphs.

An urgent future work would be to define a semantics which considers arbitrary graphs. Note that there is no such semantics in the literature. We also plan to investigate additional properties where attacks and supports do not have the same importance. Indeed, in some applications like handling inconsistency, it is generally the case that an attack is more important than a support. Thus, Strict Franklin is not suitable for such application. Another future work consists of investigating graphs were supports are weighted. Such graphs allow a better encoding of relevance of supporters with regard their targets, and consequently the intensity of supports can be better captured.

Acknowledgements

The authors are very grateful to the reviewers for their many insightful comments. This work was supported by ANR-13-BS02-0004 and ANR-11-LABX-0040-CIMI.

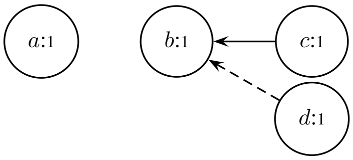

Fig. 6. Bipolar graph $\mathbf{A}_6$.

Appendix A. Proofs

Proof of Proposition 1. Let $\mathbf{S}$ be a semantics that satisfies Bi-variate Independence, Bi-variate Directionality, Stability and Strict Franklin. Let $\mathbf{A} = \langle \mathcal{A}, w, \mathcal{R}, \mathcal{S} \rangle$ and $a \in \mathcal{A}$ such that there exists a bijective function f from $\mathtt{Att}_{\mathbf{A}}(a)$ to $\mathtt{Supp}_{\mathbf{A}}(a)$ such that $\forall x \in \mathtt{Att}(a)$, $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}}(x) = \mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}}(f(x))$. Let $\mathtt{Att}_{\mathbf{A}}(a) = \{a_1, \ldots, a_n\}$ and $\mathtt{Supp}_{\mathbf{A}}(a) = \{s_1, \ldots, s_n\}$.

Let $\mathbf{A} = \langle \mathcal{A}', w', \mathcal{R}', \mathcal{S}' \rangle$ be such that $\mathcal{A}' = \mathcal{A} \cup \{y_1, \ldots, y_n\}$, with $\{y_1, \ldots, y_n\} \subseteq \mathtt{Args} \setminus \mathcal{A}$, $\forall x \in \mathcal{A}$, $w'(x) = w(x)$, $\forall i = 1, \ldots, n$, $w'(y_i) = w(a)$, $\mathcal{R}' = \mathcal{R}$ and $\mathcal{S}' = \mathcal{S}$. From Bi-variate Independence of $\mathbf{S}$, for any $x \in \mathcal{A}$, $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}'}(x) = \mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}}(x)$.

Let now $\mathbf{A} = \langle \mathcal{A}'', w'', \mathcal{R}'', \mathcal{S}'' \rangle$ be such that $\mathcal{A}'' = \mathcal{A}'$, $w'' = w'$, $\mathcal{R}'' = \mathcal{R}' \cup \{(a_i, y_j) \mid a_i \mathcal{R} a, j \in \{2, \ldots, n\}, i \in \{1, \ldots, j-1\}\}$, and $\mathcal{S}'' = \mathcal{S}' \cup \{(s_i, y_j) \mid s_i \mathcal{S} a, j \in \{2, \ldots, n\}, i \in \{1, \ldots, j-1\}\}$. Note that each y_i does not attack/support any other argument. Thus, from Bi-variate Directionality, it follows that $\forall x \in \mathcal{A}$, $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}''}(x) = \mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}'}(x)$, thus $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}''}(x) = \mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}}(x)$.

Since $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}''}(a_1) = \mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}''}(s_1)$, from Franklin, it follows that $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}''}(y_1) = \mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}''}(y_2)$. From Stability, $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}''}(y_1) = w(a)$. By applying recursively Strict Franklin, we get $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}''}(y_1) = \mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}''}(a) = \mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}}(a) = w(a)$. □

Proof of Proposition 2. Let $\mathbf{S}$ be a semantics, which satisfies Bi-variate Independence, Bi-variate Directionality, Stability, Neutrality and Bi-variate Reinforcement. Let us show that $\mathbf{S}$ satisfies also Bi-variate Monotony.

Let $\mathbf{A} = \langle \mathcal{A}, w, \mathcal{R}, \mathcal{S} \rangle$ be a weighted bipolar argumentation graph, and $a, b \in \mathcal{A}$ such that:

- $w(a) = w(b) > 0$,
- $\mathtt{Att}_{\mathbf{A}}(a) \subseteq \mathtt{Att}_{\mathbf{A}}(b)$,
- $\mathtt{Supp}_{\mathbf{A}}(b) \subseteq \mathtt{Supp}_{\mathbf{A}}(a)$.

Assume that $\mathtt{Att}_{\mathbf{A}}(b) = \mathtt{Att}_{\mathbf{A}}(a) \cup Y$, $\mathtt{Supp}_{\mathbf{A}}(a) = \mathtt{Supp}_{\mathbf{A}}(b) \cup X$, $|Y| = n$, and $|X| = m$. Let $\mathbf{A} = \langle \mathcal{A}', w', \mathcal{R}', \mathcal{S}' \rangle$ be such that $\mathcal{A}' = \mathcal{A} \cup \{a', b', y_1, \ldots, y_n, x_1, \ldots, x_m\}$ with $\{a', b', y_1, \ldots, y_n, x_1, \ldots, x_m\} \subseteq \mathtt{Args} \setminus \mathcal{A}$, $\forall z \in \mathcal{A}$, $w'(z) = w(z)$, $w'(a') = w(a)$, $w'(b') = w(b)$, $\forall i = 1, \ldots, n$, $w'(y_i) = 0$, $\forall i = 1, \ldots, m$, $w'(x_i) = 0$, $\mathcal{R}' = \mathcal{R}$ and $\mathcal{S}' = \mathcal{S}$. From Bi-variate Independence of $\mathbf{S}$, for any $x \in \mathcal{A}$, $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}'}(x) = \mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}}(x)$.

Let now $\mathbf{A} = \langle \mathcal{A}'', w'', \mathcal{R}'', \mathcal{S}'' \rangle$ be such that $\mathcal{A}'' = \mathcal{A}'$, $w'' = w'$, $\mathcal{R}'' = \mathcal{R}' \cup \{(x, a') \mid x \mathcal{R} a\} \cup \{(y_i, a') \mid i = 1, n\} \cup \{(x, b') \mid x \mathcal{R} b\}$, and $\mathcal{S}'' = \mathcal{S}' \cup \{(x, a') \mid x \mathcal{S} a\} \cup \{(x, b') \mid x \mathcal{S} b\} \cup \{(x_i, b') \mid i = 1, m\}$. Note that a' and b' do not attack/support any other argument. Thus, from Bi-variate Directionality, it follows that $\forall x \in \mathcal{A}$, $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}''}(x) = \mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}'}(x)$, thus $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}''}(x) = \mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}}(x)$. From stability, for any $i \in \{1, \ldots, n\}$, $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}''}(y_i) = 0$, and similarly, for any $i \in \{1, \ldots, m\}$, $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}''}(x_i) = 0$. Thus, from Neutrality, $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}''}(a') = \mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}''}(a) = \mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}}(a)$, and $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}''}(b') = \mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}''}(b) = \mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}}(b)$. From Reinforcement, $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}''}(a') \geq \mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}''}(b')$, hence $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}}(a) \geq \mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}}(b)$.

Let $\mathbf{S}$ satisfy Strict Franklin. Let $\mathbf{A} = \langle \mathcal{A}, w, \mathcal{R}, \mathcal{S} \rangle \in \mathtt{wBAG}$ and let $a, b, x, y \in \mathcal{A}$ be such that:

- $w(b) = w(a)$,
- $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}}(x) = \mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}}(y)$,
- $\mathtt{Att}_{\mathbf{A}}(a) = \mathtt{Att}_{\mathbf{A}}(b) \cup \{x\}$,
- $\mathtt{Supp}_{\mathbf{A}}(a) = \mathtt{Supp}_{\mathbf{A}}(b) \cup \{y\}$.

Since $\mathbf{S}$ satisfies Strict Franklin, then $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}}(a) = \mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}}(b)$. Thus, $\mathbf{S}$ satisfies Franklin. □

Proof of Proposition 3. Euler-based semantics satisfies all the axioms. □

Proof of Proposition 4. Let $\mathbf{A} = \langle \mathcal{A}, w, \mathcal{R}, \mathcal{S} \rangle$ be a flat bipolar argumentation graph such that $\mathcal{R} = \emptyset$. It follows straightforwardly from Definition 8 that for any set $\mathcal{E} \subseteq \mathcal{A}$, $\mathcal{E}$ is both conflict-free and safe. From Maximality of extensions, $\mathcal{A}$ is the only stable (resp. d-preferred and s-preferred) extension. Finally, it follows that any $a \in \mathcal{A}$, $\mathtt{Deg}^{x}_{\mathbf{G}}(a) = 1$. □

Proof of Proposition 5. Since the three semantics generalize Dung's ones with a support relation, then any axiom violated by Dung's semantics is also violated by their extended versions. Consider then the counter-examples given in [26]. From graph $\mathbf{A}_2$ (Fig. 2), it is also clear that Strengthening is violated by the three semantics. Let us consider the following simple graph $\mathbf{A}_6$ (depicted in Fig. 6) to show that the 3 semantics violate Franklin.

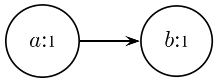

Fig. 7. Bipolar graph $\mathbf{A}_7$.

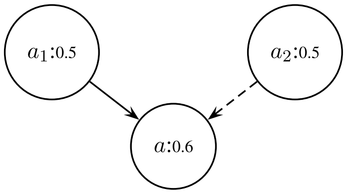

Fig. 8. Bipolar graph $\mathbf{A}_8$.

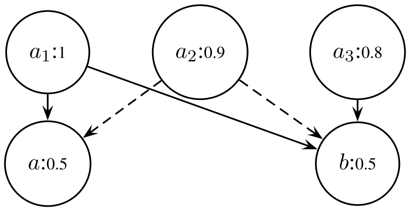

Fig. 9. Bipolar graph $\mathbf{A}_9$.

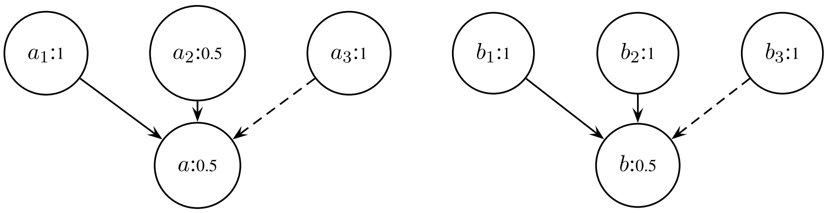

Fig. 10. Bipolar graph $\mathbf{A}_{10}$.

This graph has one stable (respectively d-preferred, s-preferred) extension $\{a, c, d\}$. Thus, $\texttt{Deg}^x_{\mathbf{G}}(a) = 1$ while $\texttt{Deg}^x_{\mathbf{G}}(b) = 0$. $\square$

Proof of Proposition 6. The satisfied axioms were proved in [22]. In order to show that QuAD violates Inertia, it is sufficient to consider Example 4.

To show that QuAD violates Resilience, consider the argumentation graph depicted in Fig. 7. It can be checked that $\texttt{Deg}^{\text{QuAD}}_{\mathbf{A}_7}(b) = 0$ while $w(b) > 0$.

To show that QuAD violates Strict Franklin principle, consider the bipolar argumentation graph depicted in Fig. 8. Note that $\texttt{Deg}^{\text{QuAD}}_{\mathbf{A}_8}(a) = 0.55 < w(a)$. Assume now that $w(a) = 0.4$. Hence, $\texttt{Deg}^{\text{QuAD}}_{\mathbf{A}_8}(a) = 0.45 > w(a)$, which shows that QuAD violates Franklin.

To show that QuAD violates Strict Monotony, consider the weighted bipolar argumentation graph depicted in Fig. 9. Note that $\texttt{Deg}^{\text{QuAD}}_{\mathbf{A}_9}(a) = \texttt{Deg}^{\text{QuAD}}_{\mathbf{A}_9}(b) = 0.475$.

To show that QuAD violates Strict Reinforcement, it is sufficient to consider the bipolar argumentation graph depicted in Fig. 10. It can be checked that $\texttt{Deg}^{\text{QuAD}}_{\mathbf{A}_{10}}(a) = \texttt{Deg}^{\text{QuAD}}_{\mathbf{A}_{10}}(b)$. $\square$

Proof of Proposition 7. The satisfied properties were already proved in [23]. Let us show that DF-QuAD violates Resilience. Consider a simple graph $\mathbf{A}$ made of two arguments a and b such that $w(a) = 1$, $w(b) = 0.5$ and $a\mathcal{R}b$. It follows that $\texttt{Deg}^{\text{DF}}_{\mathbf{A}}(b) = 0$.

To show that it violates Strict Monotony, it is sufficient to consider the counter-example given for QuAD (Fig. 9). It can be checked that $\texttt{Deg}^{\text{DF}}_{\mathbf{A}_9}(a) = \texttt{Deg}^{\text{DF}}_{\mathbf{A}_9}(b) = 0.45$.

To show that DF-QuAD violates Strict Reinforcement, it is sufficient to consider the counter-example given for QuAD (Fig. 10). It can be checked that $\texttt{Deg}^{\text{DF}}_{\mathbf{A}_{10}}(a) = \texttt{Deg}^{\text{DF}}_{\mathbf{A}_{10}}(b)$.

In order to show that DF-QuAD violates Inertia, it is sufficient to consider the graph of Fig. 5. Note that $\texttt{Deg}^{\text{DF}}_{\mathbf{A}_5}(i) = 0.991$ while $w(i) = 0.1$.

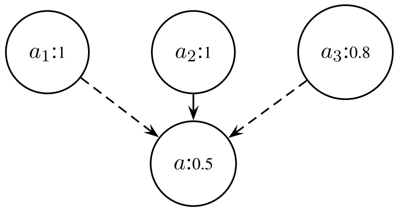

Fig. 11. Bipolar graph $\mathbf{A}_{11}$.

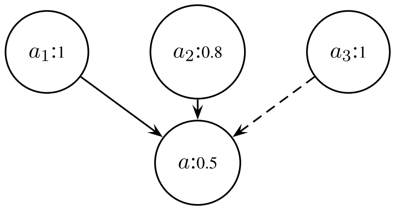

Fig. 12. Bipolar graph $\mathbf{A}_{12}$.

Let us show that it violates Strengthening. For that purpose, let us consider the graph depicted in Fig. 11. Note that $\mathtt{Deg}^{\mathrm{DF}}_{\mathbf{A}_{11}}(a) = w(a) = 0.5$ while it should be greater than 0.5.

Let us now show that DF-QuAD violates Weakening. For that purpose, consider the graph depicted in Fig. 12. Note that $\mathtt{Deg}^{\mathrm{DF}}_{\mathbf{A}_{12}}(a) = w(a)$ while it should be less than 0.5. □

Proof of Proposition 8. Anonymity, Bi-variate independence, Bi-variate equivalence are obvious.

Bi-variate directionality comes from the fact that the strength of an argument only depends on its attackers, the attackers of its attackers, an so on.

Stability is satisfied, because $\mathtt{Deg}^{\mathrm{Ebs}}_{\mathbf{A}}(a) = 1 - \frac{1-w(a)^2}{1+w(a)2^0} = 1 - \frac{1-w(a)^2}{1+w(a)} = \frac{1+w(a)-1+w(a)^2}{1+w(a)} = \frac{w(a)+w(a)^2}{1+w(a)} = \frac{w(a)(1+w(a))}{1+w(a)} = w(a)$.

Neutrality holds, because $\sum_{x \in \mathtt{Supp}(a)} \mathtt{Deg}^{\mathrm{Ebs}}_{\mathbf{A}}(x) - \sum_{x \in \mathtt{Att}(a)} \mathtt{Deg}^{\mathrm{Ebs}}_{\mathbf{A}}(x) = \sum_{x \in \mathtt{Supp}(a)} \mathtt{Deg}^{\mathrm{Ebs}}_{\mathbf{A}}(x) - \sum_{x \in \mathtt{Att}(a)} \mathtt{Deg}^{\mathrm{Ebs}}_{\mathbf{A}}(x) + 0 = \sum_{x \in \mathtt{Supp}(b)} \mathtt{Deg}^{\mathrm{Ebs}}_{\mathbf{A}}(x) - \sum_{x \in \mathtt{Att}(b)} \mathtt{Deg}^{\mathrm{Ebs}}_{\mathbf{A}}(x)$.

Monotony holds, because $\sum_{x \in \mathtt{Supp}(a)} \mathtt{Deg}^{\mathrm{Ebs}}_{\mathbf{A}}(x) - \sum_{x \in \mathtt{Att}(a)} \mathtt{Deg}^{\mathrm{Ebs}}_{\mathbf{A}}(x) \geq \sum_{x \in \mathtt{Supp}(b)} \mathtt{Deg}^{\mathrm{Ebs}}_{\mathbf{A}}(x) - \sum_{x \in \mathtt{Att}(b)} \mathtt{Deg}^{\mathrm{Ebs}}_{\mathbf{A}}(x)$.

Strict monotony holds, because $1 - w(a)^2 > 0$ (recall the graph is non-maximal) and $\sum_{x \in \mathtt{Supp}(a)} \mathtt{Deg}^{\mathrm{Ebs}}_{\mathbf{A}}(x) - \sum_{x \in \mathtt{Att}(a)} \mathtt{Deg}^{\mathrm{Ebs}}_{\mathbf{A}}(x) > \sum_{x \in \mathtt{Supp}(b)} \mathtt{Deg}^{\mathrm{Ebs}}_{\mathbf{A}}(x) - \sum_{x \in \mathtt{Att}(b)} \mathtt{Deg}^{\mathrm{Ebs}}_{\mathbf{A}}(x)$.

The proof of reinforcement and strict reinforcement are similar to those of monotony and strict monotony, respectively.

Concerning resilience, $\mathtt{Deg}^{\mathrm{Ebs}}_{\mathbf{A}}(a) \geq 1 - \frac{1-w(a)^2}{1+0} = w(a)^2 > 0$. In addition, there exists a natural number n such that $\mathtt{Deg}^{\mathrm{Ebs}}_{\mathbf{A}}(a) \leq 1 - \frac{1-w(a)^2}{n}$, but $1 - w(a)^2 > 0$ (by non-maximality), thus $\frac{1-w(a)^2}{n} > 0$, thus $1 - \frac{1-w(a)^2}{n} < 1$.

Franklin is satisfied, because $\sum_{x \in \mathtt{Supp}(a)} \mathtt{Deg}^{\mathrm{Ebs}}_{\mathbf{A}}(x) - \sum_{x \in \mathtt{Att}(a)} \mathtt{Deg}^{\mathrm{Ebs}}_{\mathbf{A}}(x) = \sum_{x \in \mathtt{Supp}(b)} \mathtt{Deg}^{\mathrm{Ebs}}_{\mathbf{A}}(x) - \sum_{x \in \mathtt{Att}(b)} \mathtt{Deg}^{\mathrm{Ebs}}_{\mathbf{A}}(x)$.

Weakening holds, because $\sum_{x \in \mathtt{Supp}(a)} \mathtt{Deg}^{\mathrm{Ebs}}_{\mathbf{A}}(x) < \sum_{x \in \mathtt{Att}(a)} \mathtt{Deg}^{\mathrm{Ebs}}_{\mathbf{A}}(x)$, thus $\sum_{x \in \mathtt{Supp}(a)} \mathtt{Deg}^{\mathrm{Ebs}}_{\mathbf{A}}(x) - \sum_{x \in \mathtt{Att}(a)} \mathtt{Deg}^{\mathrm{Ebs}}_{\mathbf{A}}(x) < 0$, thus $\mathtt{Deg}^{\mathrm{Ebs}}_{\mathbf{A}}(a) < 1 - \frac{1-w(a)^2}{1+w(a)} = w(a)$ (recall $1 - w(a)^2 > 0$ by non-maximality).

The proof of strengthening is similar to that of weakening.

Finally, we turn to inertia. We have $\sum_{x \in \mathtt{Supp}(b)} \mathtt{Deg}^{\mathrm{Ebs}}_{\mathbf{A}}(x) \leq 1 + \sum_{x \in \mathtt{Supp}(a)} \mathtt{Deg}^{\mathrm{Ebs}}_{\mathbf{A}}(x)$. Thus, $E(b) = \sum_{x \in \mathtt{Supp}(b)} \mathtt{Deg}^{\mathrm{Ebs}}_{\mathbf{A}}(x) - \sum_{x \in \mathtt{Att}(b)} \mathtt{Deg}^{\mathrm{Ebs}}_{\mathbf{A}}(x) \leq 1 + \sum_{x \in \mathtt{Supp}(a)} \mathtt{Deg}^{\mathrm{Ebs}}_{\mathbf{A}}(x) - \sum_{x \in \mathtt{Att}(a)} \mathtt{Deg}^{\mathrm{Ebs}}_{\mathbf{A}}(x) = 1 + E(a)$. So, $\mathtt{Deg}^{\mathrm{Ebs}}_{\mathbf{A}}(b) = 1 - \frac{1-w(a)^2}{1+w(a)2^{E(b)}} \leq 1 - \frac{1-w(a)^2}{1+w(a)2^{(1+E(a))}} = 1 - \frac{1-w(a)^2}{1+w(a)2^{E(a)}2}$. So, $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}}(a) + [1 - \mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}}(a)]/2 = \mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}}(a) + 1/2 - \mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}}(a)/2 = 1/2 + \mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}}(a)/2 = 1/2 + 1/2 - \frac{1-w(a)^2}{2+w(a)2^{E(a)}2} = 1 - \frac{1-w(a)^2}{2+w(a)2^{E(a)}2} > 1 - \frac{1-w(a)^2}{1+w(a)2^{E(a)}2} \geq \mathtt{Deg}^{\mathrm{Ebs}}_{\mathbf{A}}(b)$. □

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