Evaluation of Arguments in Weighted Bipolar Graphs

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Abstract. The paper tackled the issue of arguments evaluation in *weighted bipolar argumentation graphs* (i.e., graphs whose arguments have basic strengths, and may be both supported and attacked). We introduce axioms that an evaluation method (or semantics) could satisfy. Such axioms are very useful for judging and comparing semantics. We then analyze existing semantics on the basis of our axioms, and finally propose a new semantics for the class of acyclic graphs.

1 Introduction

Argumentation is a form of common-sense reasoning consisting of the justification of claims by arguments. The latter have generally basic strengths, and may be attacked and/or supported by other arguments, leading to the so-called *bipolar argumentation graphs* (BAGs). Several methods, called *semantics*, were proposed in the literature for the evaluation of arguments in such settings. They can be partitioned into two main families: *extension semantics* [1–5], and *gradual semantics* [6–8]. The former extend Dung's semantics [9] for accounting for supports, and look for acceptable sets of arguments (called extensions). The latter focus on the evaluation of individual arguments.

This paper extends our previous works on axiomatic foundations of semantics for unipolar graphs (support graphs [10] and attack graphs [11]). It defines axioms (i.e. properties) that a semantics should satisfy in a bipolar setting. Such axioms are very useful for judging and understanding the underpinnings of semantics, and also for comparing semantics of the same family, and those of different families. Some of the proposed axioms are simple combinations of those proposed in [10,11]. Others are new and show how support and attack should be aggregated. The second contribution of the paper consists of analyzing existing semantics against the axioms. The main conclusion is that extension semantics do not harness the potential of support relation. Indeed, when the attack relation is empty, the existing semantics declare all (supported, non-supported) arguments of a graph as equally accepted. Gradual semantics take into account supporters in this particular case, however they violate some key axioms. The third contribution of the paper is the definition of a novel gradual semantics for the sub-class of acyclic bipolar graphs. We show that it satisfies all the proposed axioms. Furthermore, it avoids the *big jump* problem that impedes the relevance of existing gradual semantics for practical applications, like dialogue.

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The paper is structured as follows: Sect. 2 introduces basic notions, Sect. 3 presents our list of axioms as well as some properties, Sect. 4 analyses existing semantics, and Sect. 5 introduces our new semantics and discusses its properties.

2 Main Concepts

This section introduces the main concepts of the paper. Let us begin with weightings:

Definition 1 (Weighting). *A* weighting *on a set X is a function from X to* $[0, 1]$.

Next, we introduce weighted bipolar argumentation graphs (BAGs).

Definition 2 (BAG). *A* BAG *is a quadruple $\mathbf{A} = \langle \mathcal{A}, w, \mathcal{R}, \mathcal{S} \rangle$, where $\mathcal{A}$ is a finite set of arguments, w a weighting on $\mathcal{A}$, $\mathcal{R} \subseteq \mathcal{A} \times \mathcal{A}$, and $\mathcal{S} \subseteq \mathcal{A} \times \mathcal{A}$.*

Given two arguments a and b, $a\mathcal{R}b$ (resp. $a\mathcal{S}b$) means a *attacks* (resp. *supports*) b, and $w(a)$ is the *intrinsic strength* of a. The latter may be the certainty degree of the argument's premises, trustworthiness of the argument's source, ...

We turn to the core concept of the paper. A semantics is a function transforming any weighted bipolar argumentation graph into a weighting on the set of arguments. The weight of an argument given by a semantics represents its *overall strength* or *acceptability degree*. It is obtained from the aggregation of its intrinsic strength and the overall strengths of its attackers and supporters. Arguments that get value 1 are *extremely strong* whilst those that get value 0 are *worthless*.

Definition 3 (Semantics). *A* semantics *is a function* $\mathbf{S}$ *transforming any BAG* $\mathbf{A} = \langle \mathcal{A}, w, \mathcal{R}, \mathcal{S} \rangle$ *into a weighting f on $\mathcal{A}$. Let $a \in \mathcal{A}$, we denote by $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}}(a)$ the acceptability degree of a, i.e., $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}}(a) = f(a)$.*

Let us recall the notion of *isomorphism* between graphs.

Definition 4 (Isomorphism). *Let $\mathbf{A} = \langle \mathcal{A}, w, \mathcal{R}, \mathcal{S} \rangle$ and $\mathbf{A}' = \langle \mathcal{A}', w', \mathcal{R}', \mathcal{S}' \rangle$ be two BAGs. An* isomorphism *from $\mathbf{A}$ to $\mathbf{A}'$ is a bijective function f from $\mathcal{A}$ to $\mathcal{A}'$ such that: (i) $\forall\, a \in \mathcal{A}$, $w(a) = w'(f(a))$, (ii) $\forall\, a, b \in \mathcal{A}$, $a\mathcal{R}b$ iff $f(a)\mathcal{R}'f(b)$, (iii) $\forall\, a, b \in \mathcal{A}$, $a\mathcal{S}b$ iff $f(a)\mathcal{S}'f(b)$.*

Notations: Let $\mathbf{A} = \langle \mathcal{A}, w, \mathcal{R}, \mathcal{S} \rangle$ be a BAG and $a \in \mathcal{A}$. We denote by $\mathtt{Att}_{\mathbf{A}}(a)$ the set of all attackers of a in $\mathbf{A}$ (i.e., $\mathtt{Att}_{\mathbf{A}}(a) = \{b \in \mathcal{A} \mid b\mathcal{R}a\}$), and by $\mathtt{sAtt}_{\mathbf{A}}(a)$ the set of all *significant attackers* of a, i.e., attackers x of a such that $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}}(x) \neq 0$. Similarly, we denote by $\mathtt{Supp}_{\mathbf{A}}(a)$ the set of all supporters of a (i.e., $\mathtt{Supp}_{\mathbf{A}}(a) = \{b \in \mathcal{A} \mid b\mathcal{S}a\}$) and by $\mathtt{sSupp}_{\mathbf{A}}(a)$ the *significant supporters* of a, i.e., supporters x such that $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}}(x) \neq 0$.

Let $\mathbf{A}' = \langle \mathcal{A}', w', \mathcal{R}', \mathcal{S}' \rangle$ be another BAG such that $\mathcal{A} \cap \mathcal{A}' = \emptyset$. We denote by $\mathbf{A} \oplus \mathbf{A}'$ the BAG $\langle \mathcal{A}'', w'', \mathcal{R}'', \mathcal{S}'' \rangle$ such that $\mathcal{A}'' = \mathcal{A} \cup \mathcal{A}'$, $\mathcal{R}'' = \mathcal{R} \cup \mathcal{R}'$, $\mathcal{S}'' = \mathcal{S} \cup \mathcal{S}'$, and $\forall x \in \mathcal{A}''$, the following holds: $w''(x) = w(x)$, if $x \in \mathcal{A}$; $w''(x) = w'(x)$, if $x \in \mathcal{A}'$.

3 Axioms for Acceptability Semantics

In what follows, we propose axioms that shed light on foundational principles behind semantics. In other words, properties that help us to better understand the underpinnings of semantics, and that facilitate their comparisons. The first nine axioms are simple *combinations* of axioms proposed for graphs with only one type of interactions (support in [10], attack in [11]). The three last axioms are new and show how the overall strengths of supporters and attackers of an argument should be aggregated.

The first very basic axiom, Anonymity, states that the degree of an argument is independent of its identity. It combines the two Anonymity axioms from [10,11].

Axiom 1 (Anonymity). *A semantics* $\mathbf{S}$ *satisfies* anonymity *iff, for any two BAGs* $\mathbf{A} = \langle \mathcal{A}, w, \mathcal{R}, \mathcal{S} \rangle$ *and* $\mathbf{A}' = \langle \mathcal{A}', w', \mathcal{R}', \mathcal{S}' \rangle$*, for any isomorphism* f *from* $\mathbf{A}$ *to* $\mathbf{A}'$*, the following property holds:* $\forall\, a \in \mathcal{A}, \mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}}(a) = \mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}'}(f(a))$.

Bi-variate independence axiom states the following: the acceptability degree of an argument a should be independent of any argument b that is not connected to it (i.e., there is no path from b to a, ignoring the direction of the edges). This axiom combines the two independence axioms from [10,11].

Axiom 2 (Bi-variate Independence). *A semantics* $\mathbf{S}$ *satisfies* bi-variate independence *iff, for any two BAGs* $\mathbf{A} = \langle \mathcal{A}, w, \mathcal{R}, \mathcal{S} \rangle$ *and* $\mathbf{A}' = \langle \mathcal{A}', w', \mathcal{R}', \mathcal{S}' \rangle$ *such that* $\mathcal{A} \cap \mathcal{A}' = \emptyset$*, the following property holds:* $\forall\, a \in \mathcal{A}$, $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}}(a) = \mathtt{Deg}^{\mathbf{S}}_{\mathbf{A} \oplus \mathbf{A}'}(a)$.

Bi-variate directionality axiom combines Non-Dilution from [10] and Circumscription from [11]. It states that the overall strength of an argument should depend only on its incoming arrows, and thus not on the arguments it itself attacks or supports.

Axiom 3 (Bi-variate Directionality). *A semantics* $\mathbf{S}$ *satisfies* bi-variate directionality *iff, for any two BAGs* $\mathbf{A} = \langle \mathcal{A}, w, \mathcal{R}, \mathcal{S} \rangle$, $\mathbf{A}' = \langle \mathcal{A}', w', \mathcal{R}', \mathcal{S}' \rangle$ *such that* $\mathcal{A} = \mathcal{A}'$*,* $\mathcal{R} \subseteq \mathcal{R}'$*, and* $\mathcal{S} \subseteq \mathcal{S}'$*, the following holds: for all* $a, b, x \in \mathcal{A}$*, if* $\mathcal{R}' \cup \mathcal{S}' = \mathcal{R} \cup \mathcal{S} \cup \{(a, b)\}$ *and there is no path from* b *to* x*, then* $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}}(x) = \mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}'}(x)$*. Note that a path can mix attack and support relations, but the edges must always be directed from* b *to* x*.*

Bi-variate Equivalence axiom ensures that the overall strength of an argument depends *only* on the overall strengths of its direct attackers and supporters. It combines the two equivalence axioms from [10,11].

Axiom 4 (Bi-variate Equivalence). *A semantics* $\mathbf{S}$ *satisfies* bi-variate equivalence *iff, for any BAG* $\mathbf{A} = \langle \mathcal{A}, w, \mathcal{R}, \mathcal{S} \rangle$*, for all* $a, b \in \mathcal{A}$*, if:*

- $w(a) = w(b)$,
- *there exists a bijective function* f *from* $\mathtt{Att}_{\mathbf{A}}(a)$ *to* $\mathtt{Att}_{\mathbf{A}}(b)$ *such that* $\forall x \in \mathtt{Att}_{\mathbf{A}}(a)$, $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}}(x) = \mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}}(f(x))$*, and*

– *there exists a bijective function* f' *from* $\texttt{Supp}_{\mathbf{A}}(a)$ *to* $\texttt{Supp}_{\mathbf{A}}(b)$ *such that* $\forall x \in \texttt{Supp}_{\mathbf{A}}(a)$, $\texttt{Deg}^{\mathbf{S}}_{\mathbf{A}}(x) = \texttt{Deg}^{\mathbf{S}}_{\mathbf{A}}(f(x))$,

then $\texttt{Deg}^{\mathbf{S}}_{\mathbf{A}}(a) = \texttt{Deg}^{\mathbf{S}}_{\mathbf{A}}(b)$.

Stability axiom combines Minimality [10] and Maximality [11] axioms. It says the following: if an argument is neither attacked nor supported, its overall strength should be equal to its intrinsic strength.

Axiom 5 (Stability). *A semantics* **S** *satisfies* stability *iff, for any BAG* $\mathbf{A} = \langle \mathcal{A}, w, \mathcal{R}, \mathcal{S} \rangle$, *for any argument* $a \in \mathcal{A}$, *if* $\texttt{Att}_{\mathbf{A}}(a) = \texttt{Supp}_{\mathbf{A}}(a) = \emptyset$, *then* $\texttt{Deg}^{\mathbf{S}}_{\mathbf{A}}(a) = w(a)$.

Neutrality axiom generalizes Dummy axiom [10] and Neutrality one from [11]. It states that worthless attackers or supporters have no effect.

Axiom 6 (Neutrality). *A semantics* **S** *satisfies* neutrality *iff, for any BAG* $\mathbf{A} = \langle \mathcal{A}, w, \mathcal{R}, \mathcal{S} \rangle$, $\forall a, b, x \in \mathcal{A}$, *if:*

– $w(a) = w(b)$,
– $\texttt{Att}_{\mathbf{A}}(a) \subseteq \texttt{Att}_{\mathbf{A}}(b)$,
– $\texttt{Supp}_{\mathbf{A}}(a) \subseteq \texttt{Supp}_{\mathbf{A}}(b)$,
– $\texttt{Att}_{\mathbf{A}}(b) \cup \texttt{Supp}_{\mathbf{A}}(b) = \texttt{Att}_{\mathbf{A}}(a) \cup \texttt{Supp}_{\mathbf{A}}(a) \cup \{x\}$, *and* $\texttt{Deg}^{\mathbf{S}}_{\mathbf{A}}(x) = 0$,

then $\texttt{Deg}^{\mathbf{S}}_{\mathbf{A}}(a) = \texttt{Deg}^{\mathbf{S}}_{\mathbf{A}}(b)$.

Bi-variate Monotony states the following: if an argument a is equally or less attacked than an argument b, and equally or more supported than b, then a should be equally strong or stronger than b. This axiom generalizes 4 axioms from the literature (Monotony and Counting [10] for supports, and the same axioms from [11] for attacks).

Axiom 7 (Bi-variate Monotony). *A semantics* **S** *satisfies* bi-variate monotony *iff, for any BAG* $\mathbf{A} = \langle \mathcal{A}, w, \mathcal{R}, \mathcal{S} \rangle$, *for all* $a, b \in \mathcal{A}$ *such that:*

– $w(a) = w(b) > 0$,
– $\texttt{Att}_{\mathbf{A}}(a) \subseteq \texttt{Att}_{\mathbf{A}}(b)$,
– $\texttt{Supp}_{\mathbf{A}}(b) \subseteq \texttt{Supp}_{\mathbf{A}}(a)$,

the following holds:

– $\texttt{Deg}^{\mathbf{S}}_{\mathbf{A}}(a) \geq \texttt{Deg}^{\mathbf{S}}_{\mathbf{A}}(b)$, ***(Monotony)***
– *if (*$\texttt{Deg}^{\mathbf{S}}_{\mathbf{A}}(a) > 0$ *or* $\texttt{Deg}^{\mathbf{S}}_{\mathbf{A}}(b) < 1$*) and (*$\texttt{sAtt}_{\mathbf{A}}(a) \subset \texttt{sAtt}_{\mathbf{A}}(b)$, *or* $\texttt{sSupp}_{\mathbf{A}}(b) \subset \texttt{sSupp}_{\mathbf{A}}(a)$*), then* $\texttt{Deg}^{\mathbf{S}}_{\mathbf{A}}(a) > \texttt{Deg}^{\mathbf{S}}_{\mathbf{A}}(b)$. ***(Strict Monotony)***

The next axiom combines the Reinforcement axioms of [10,11]. It states that any argument becomes stronger if the quality of its attackers is reduced and the quality of its supporters is increased.

Axiom 8 (Bi-variate Reinforcement). *A semantics* **S** *satisfies* bi-variate reinforcement *iff, for any BAG* $\mathbf{A} = \langle \mathcal{A}, w, \mathcal{R}, \mathcal{S} \rangle$, *for all* $C, C' \subseteq \mathcal{A}$, *for all* $a, b \in \mathcal{A}$, *for all* $x, x', y, y' \in \mathcal{A} \backslash (C \cup C')$ *such that*

- $w(a) = w(b) > 0$,
- $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}}(x) \leq \mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}}(y)$,
- $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}}(x') \geq \mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}}(y')$,
- $\mathtt{Att}_{\mathbf{A}}(a) = C \cup \{x\}$,
- $\mathtt{Att}_{\mathbf{A}}(b) = C \cup \{y\}$,
- $\mathtt{Supp}_{\mathbf{A}}(a) = C' \cup \{x'\}$,
- $\mathtt{Supp}_{\mathbf{A}}(b) = C' \cup \{y'\}$,

the following holds:

- $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}}(a) \geq \mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}}(b)$, ***(Reinforcement)***
- *if* ($\mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}}(a) > 0$ *or* $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}}(b) < 1$) *and* ($\mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}}(x) < \mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}}(y)$, *or* $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}}(x') > \mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}}(y')$), *then* $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}}(a) > \mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}}(b)$. ***(Strict Reinforcement)***

Our next axiom combines Imperfection axiom from [10] with Resilience axiom from [11]. Imperfection states that an argument whose basic strength is less than 1 cannot be fully rehabilitated by supports. In other words, it cannot get an acceptability degree 1 due to supports. This axiom prevents irrational behaviors, like fully accepting fallacious arguments that are supported. Below, the argument A remains fallacious even if it is supported by B.

A: Tweety needs fuel, since it flies like planes.
B: Indeed, Tweety flies. It is a bird.

Resilience in [11] states that an argument whose basic strength is positive cannot be completely destroyed by attacks. Assume that B is attacked by the argument C below. Despite the attack, the argument B is still reasonable.

C: Tweety does not fly since it is a penguin

Axiom 9 (Resilience). *A semantics* **S** *satisfies* resilience *iff, for any BAG* $\mathbf{A} = \langle \mathcal{A}, w, \mathcal{R}, \mathcal{S} \rangle$, *for all* $a \in \mathcal{A}$, *if* $0 < w(a) < 1$, *then* $0 < \mathtt{Deg}(a) < 1$.

The next three axioms are new and answer the same question: how the overall strengths of attackers and supporters of an argument are aggregated? To answer this question, it is important to specify which of the two types of interactions is more important. In this paper, we consider both relations as equally important. Hence, Franklin axiom states that an attacker and a supporter of equal strength should counter-balance each other. Thus, neither attacks nor supports will have impact on the argument.

Axiom 10 (Franklin). *A semantics* **S** *satisfies* franklin *iff, for any BAG* $\mathbf{A} = \langle \mathcal{A}, w, \mathcal{R}, \mathcal{S} \rangle$, *for all* $a, b, x, y \in \mathcal{A}$, *if*

- $w(b) = w(a)$,
- $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}}(x) = \mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}}(y)$
- $\mathtt{Att}_{\mathbf{A}}(a) = \mathtt{Att}_{\mathbf{A}}(b) \cup \{x\}$,
- $\mathtt{Supp}_{\mathbf{A}}(a) = \mathtt{Supp}_{\mathbf{A}}(b) \cup \{y\}$,

then $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}}(a) = \mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}}(b)$.

We show that attacks and supports of equal strengths eliminate each others.

Proposition 1. *Let* $\mathbf{S}$ *be a semantics that satisfies Bi-variate Independence, Bi-variate Directionality, Stability and Franklin. For any BAG* $\mathbf{A} = \langle \mathcal{A}, w, \mathcal{R}, \mathcal{S} \rangle$, *for all* $a \in \mathcal{A}$, *if there exists a bijective function* f *from* $\mathtt{Att}_{\mathbf{A}}(a)$ *to* $\mathtt{Supp}_{\mathbf{A}}(a)$ *such that* $\forall x \in \mathtt{Att}(a)$, $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}}(x) = \mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}}(f(x))$, *then* $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}}(a) = w(a)$.

Weakening states that if attackers overcome supporters, the argument should loose weight. The idea is that supports are not sufficient for counter-balancing attacks. Please note that this does not means that supports will not have an impact on the overall strength of an argument. They may mitigate the global loss due to attacks.

Axiom 11 (Weakening). *A semantics* $\mathbf{S}$ *satisfies* weakening *iff, for any BAG* $\mathbf{A} = \langle \mathcal{A}, w, \mathcal{R}, \mathcal{S} \rangle$, *for all* $a \in \mathcal{A}$, *if* $w(a) > 0$ *and there exists an injective function* f *from* $\mathtt{Supp}_{\mathbf{A}}(a)$ *to* $\mathtt{Att}_{\mathbf{A}}(a)$ *such that:*

– $\forall x \in \mathtt{Supp}_{\mathbf{A}}(a)$, $\mathtt{Deg}(x) \leq \mathtt{Deg}(f(x))$; *and*
– $\mathtt{sAtt}_{\mathbf{A}}(a) \backslash \{f(x) \mid x \in \mathtt{Supp}_{\mathbf{A}}(a)\} \neq \emptyset$ *or* $\exists x \in \mathtt{Supp}_{\mathbf{A}}(a)$ *s.t* $\mathtt{Deg}(x) < \mathtt{Deg}(f(x))$,

then $\mathtt{Deg}(a) < w(a)$.

Strengthening states that if supporters overcome attackers, the argument should gain weight. Indeed, attacks are not sufficient for counter-balancing supports, however, they may mitigate the global gain due to supports.

Axiom 12 (Strengthening). *A semantics* $\mathbf{S}$ *satisfies* strengthening *iff, for any BAG* $\mathbf{A} = \langle \mathcal{A}, w, \mathcal{R}, \mathcal{S} \rangle$, *for all* $a \in \mathcal{A}$, *if* $w(a) < 1$ *and there exists an injective function* f *from* $\mathtt{Att}_{\mathbf{A}}(a)$ *to* $\mathtt{Supp}_{\mathbf{A}}(a)$ *such that:*

– $\forall x \in \mathtt{Att}_{\mathbf{A}}(a)$, $\mathtt{Deg}(x) \leq \mathtt{Deg}(f(x))$; *and*
– $\mathtt{sSupp}_{\mathbf{A}}(a) \backslash \{f(x) \mid x \in \mathtt{Att}_{\mathbf{A}}(a)\} \neq \emptyset$ *or* $\exists x \in \mathtt{Att}_{\mathbf{A}}(a)$ *s.t.* $\mathtt{Deg}(x) < \mathtt{Deg}(f(x))$,

then $\mathtt{Deg}(a) > w(a)$.

It is worth mentioning that weakening and strengthening generalize their corresponding axioms in [10,11]. Indeed, when the support relation is empty, bipolar version of weakening coincides with weakening axiom in [11]. However, it handles additional cases when supports exist. Similarly, when the attack relation is empty, the axiom coincides with strengthening axiom in [10].

Almost all axioms are independent, i.e., they do not follow from others. A notable exception is Bivariate Monotony which follows from five axioms.

Proposition 2. *If a semantics satisfies Bi-variate Independence, Bi-variate Directionality, Stability, Neutrality and Bi-variate Reinforcement, then it satisfies Bivariate Monotony.*

All axioms are compatible, i.e., they can be satisfied all together by a semantics.

Proposition 3. *All the axioms are compatible.*

4 Formal Analysis of Existing Semantics

There are several proposals in the literature for the evaluation of arguments in bipolar argumentation graphs. They can be partitioned into two families: *extension* semantics [1–5] and *gradual* semantics [6–8].

Extension semantics extend Dung's semantics [9] for accounting for supports between arguments. They take as input an argumentation graph $\langle \mathcal{A}, w, \mathcal{R}, \mathcal{S} \rangle$ whose arguments have all the *same* basic strength, and return sets of arguments, called extensions. From the extensions, a three-valued qualitative degree is assigned to every argument. Indeed, an argument is *accepted* if it belongs to all extensions, *undecided* (or credulously accepted) if it belongs to some but not all extensions, and *rejected* if it does not belong to any extension. When the support relation is empty, the semantics proposed in [1–5] coincide with Dung's ones. Thus, they violate the axioms that are violated by Dung's semantics (see [11] for a detailed analysis of Dung's semantics). For instance, stable semantics violates Independence, Equivalence, Stability, Resilience, and strict monotony. When the attack relation is empty, the approaches from [1,2,4] return a single extension, which contains all the arguments of the BAG at hand. Thus, all arguments are equally accepted. This shows that the support relation does not play any role, and a supported argument is as acceptable as a non-supported one. To say it differently, these approaches violate strengthening axiom which captures the role of supports. The approaches developed in [3,5] return a single extension when the attack relation is empty. This extension coincides with the set of arguments when there are no cycles in the BAG. Thus, they also violate strengthening and the support relation may not be fully exploited in the evaluation of arguments.

The second family of gradual semantics was introduced for the first time in [6]. In their paper, the authors presented some properties that such semantics should satisfy (like a particular case of strengthening). However, they did not define concrete semantics. To the best of our knowledge, the first gradual semantics is QuAD, introduced in [7], for evaluating arguments in *acyclic* graphs. This semantics assigns a numerical value to every argument on the basis of its intrinsic strength, and the overall strengths of its attackers and supporters. It evaluates separately the supporters and the attackers before aggregating them. Due to lack of space, we do not provide the formal definitions.

Proposition 4. *QuAD satisfies Anonymity, Bi-variate Independence, Bi-variate Directionality, Bi-variate Equivalence, Stability, Neutrality, Monotony, Reinforcement.*

QuAD violates Strict Monotony, Strict Reinforcement, Resilience, Franklin, Weakening, and Strengthening.

As a consequence of violating Weakening and Strengthening, QuAD may behave irrationally. Consider a BAG where $\mathcal{A} = \{a, b_1, b_2, b_3\}$, $w(b_1) = w(b_2) = 0.8$, $w(b_3) = 0.9$, $\mathcal{R} = \{(b_2, a), (b_3, a)\}$, and $\mathcal{S} = \{(b_1, a)\}$. Thus, a has an attacker and a supporter of equal strengths, and an additional attacker b_3. Note that if $w(a) = 0.2$, then $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}}(a) = 0.422$ meaning that the single supporter is privileged to the two attackers. However, if $w(a) = 0.7$, $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}}(a) = 0.477$ meaning that attacks are privileged to support. More generally, we can show that if $w(a) \geq 0.5$, then $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}}(a) < w(a)$, else $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}}(a) > w(a)$. Hence, choosing which of support and attack should take precedence depends on the intrinsic strength of an argument.

QuAD was recently extended to DF-QuAD in [8]. The new semantics focuses also on *acyclic graphs*. Unlike QuAD, it uses the same function for aggregating supporters and attackers separately. It satisfies Franklin axiom, thus it treats equally attacks and supports. It violates Strengthening and Weakening in presence of attackers/supporters of degree 1. However, the semantics avoids the irrational behavior of QuAD.

Proposition 5. *DF-QuAD satisfies Anonymity, Bi-variate Independence, Bi-variate Directionality, Bi-variate Equivalence, Stability, Neutrality, Monotony, Reinforcement, and Franklin. DF-QuAD violates Strict Monotony, Strict Reinforcement, Resilience, Weakening, and Strengthening.*

Both semantics (QuAD and DF-QuAD) suffer from a *big jump* problem. Let us illustrate the problem with the BAG depicted in Fig. 1. Note that the argument i has a very low basic strength ($w(i) = 0.1$). This argument is supported by the very strong argument j. According to QuAD and DF-QuAD, $\mathtt{Deg}^{\mathbf{S}}_{\mathbf{A}}(i) = 0.991$. Thus, the value of i makes a big jump from 0.1 to 0.991. The argument i became even stronger than its supporter j. There are two issues with such jump: First, the gain is enormous and not reasonable. Assume that i is the argument "Tweety needs fuel, since it flies like planes". It is hard to accept i even when supported. The supporter may increase slightly the strength of the argument but does not correct the wrong premises of the argument. Second, such jump impedes the discrimination between different cases where $w(i) > 0.1$ since whatever the value of $w(i)$, the overall strength is almost 1.

5 Euler-Based Graded Semantics

As shown in the previous sections, no existing semantics satisfies all our 12 axioms together. The goal of the present section is to handle this issue. More precisely, we construct a new semantics satisfying all axioms, but at the cost of a certain degree of coverage. Indeed, we only consider a subclass of BAGs: acyclic non-maximal BAGs.

Definition 5 (BAG properties). *A BAG* $\mathbf{A} = \langle \mathcal{A}, w, \mathcal{R}, \mathcal{S} \rangle$ *is* acyclic *iff the following holds: for any non-empty finite sequence* $\mathbf{a} = \langle a_1, a_2, \ldots, a_n \rangle$ *of elements of* $\mathcal{A}$, *if* $\forall i \in \{1, 2, \ldots, n-1\}$, $\langle a_i, a_{i+1} \rangle \in \mathcal{R} \cup \mathcal{S}$, *then* $\langle a_n, a_1 \rangle \notin \mathcal{R} \cup \mathcal{S}$. *Next,* $\mathbf{A}$ *is* non-maximal *iff* $\forall a \in \mathcal{A}$, $w(a) < 1$.

Without loss of generality, the basic strengths of arguments are less than 1. Note that few arguments are intrinsically perfect. The probability of false information, exceptions, etc., is rarely 0. In contrast, the loss of cyclic BAGs is important. But, we consider that the class of all acyclic non-maximal BAGs is expressive enough to deserve attention.

Definition 6 (Restricted semantics). *A* restricted semantics *is a function* $\mathbf{S}$ *transforming any acyclic non-maximal BAG* $\mathbf{A} = \langle \mathcal{A}, w, \mathcal{R}, \mathcal{S} \rangle$ *into a weighting on* $\mathcal{A}$*.*

All notations and axioms for semantics are straightforwardly adapted to restricted semantics. Before presenting our semantics, we need to introduce a relation between arguments based on the longest paths to reach them (mixing support and attack arrows).

Definition 7 (Well-founded relation). *Let* $\mathbf{A} = \langle \mathcal{A}, w, \mathcal{R}, \mathcal{S} \rangle$ *be an acyclic BAG and* $a \in \mathcal{A}$*. A* path to a in $\mathbf{A}$ *is a non-empty finite sequence* $\mathbf{a} = \langle a_1, a_2, \ldots, a_n \rangle$ *of elements of* $\mathcal{A}$ *such that* $a_n = a$ *and* $\forall i \in \{1, 2, \ldots, n-1\}$, $\langle a_i, a_{i+1} \rangle \in \mathcal{R} \cup \mathcal{S}$*. We denote by* $\texttt{Rel}(\mathbf{A})$ *the well-founded binary relation* $\prec$ *on* $\mathcal{A}$ *such that* $\forall x, y \in \mathcal{A}$*,* $x \prec y$ *iff* $\max\{n \mid$ *there exists a path to* x *of length* $n\} < \max\{n \mid$ *there exists a path to* y *of length* $n\}$*. Since* $\mathbf{A}$ *is acyclic, those maximum lengths are well-defined, so is* $\texttt{Rel}(\mathbf{A})$*.*

We are ready to define the *Euler-based restricted semantics*. The general idea is to take into account supporters and attackers in an exponent E of $\mathtt{e}$ (the Euler's number). More precisely, the stronger or more-numerous the supporters, the greater and more-likely-positive that exponent. Obviously, the inverse is true with the attackers. Then, the overall strength of an argument a is naturally defined as $w(a)\mathtt{e}^E$. Finally, we need certain tweakings (including a double polarity reversal) to make our function a restricted semantics in the first place, and to have it satisfy all our axioms. More formally:

Definition 8 (Euler-based restricted semantics). *We denote by* $\texttt{Ebs}$ *the restricted semantics such that for any acyclic non-maximal BAG* $\mathbf{A} = \langle \mathcal{A}, w, \mathcal{R}, \mathcal{S} \rangle$*,* $\texttt{Ebs}(\mathbf{A})$ *is the weighting* f *on* $\mathcal{A}$ *recursively defined with* $\texttt{Rel}(\mathbf{A})$ *as follows:* $\forall a \in \mathcal{A}$*,*

$$f(a) = 1 - \frac{1 - w(a)^2}{1 + w(a)\mathtt{e}^E} \quad \textit{where} \quad E = \sum_{x \in \texttt{Supp}(a)} f(x) - \sum_{x \in \texttt{Att}(a)} f(x).$$

As an immediate corollary, we have:

Corollary 1. *Let* $\mathbf{A} = \langle \mathcal{A}, w, \mathcal{R}, \mathcal{S} \rangle$ *be an acyclic non-maximal BAG and* $a \in \mathcal{A}$*. The following holds:*

$$\texttt{Deg}^{\texttt{Ebs}}_{\mathbf{A}}(a) = 1 - \frac{1 - w(a)^2}{1 + w(a)\mathtt{e}^E} \quad \textit{where} \quad E = \sum_{x \in \texttt{Supp}(a)} \texttt{Deg}^{\texttt{Ebs}}_{\mathbf{A}}(x) - \sum_{x \in \texttt{Att}(a)} \texttt{Deg}^{\texttt{Ebs}}_{\mathbf{A}}(x).$$

Below is an example where most axioms are exemplified. Every circle contains [argument name]:[intrinsic strength] and below [**overall strength**].

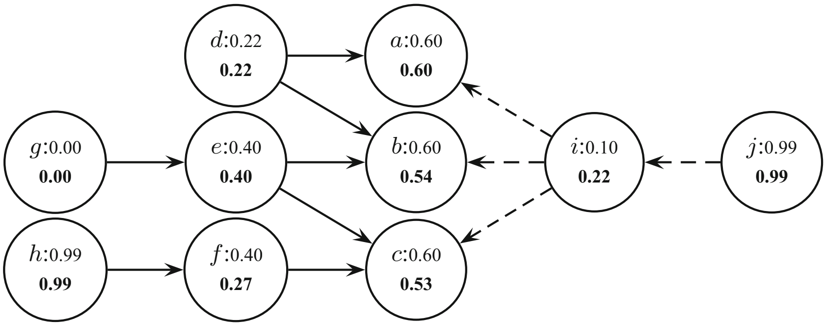

Fig. 1. Example of BAG

Example 1. The axiom neutrality can be checked with g and e, stability with e.g. d, bivariate monotony with a and b, bivariate reinforcement with b and c, Imperfection with i, Franklin with a, weakening with e.g. b, and strengthening with i.

Theorem 1. Ebs *satisfies all our 12 axioms.*

Note that being supported by an extremely strong argument does not cause a weak argument to become extremely strong as well, which shows that Ebs does not suffer from the big jump problem. Note that $\texttt{Deg}_{\mathbf{A}}^{\texttt{Ebs}}(i) = 0.22$ and thus the jump is not big. Note also that by satisfying Weakening and Strengthening, the semantics avoids the irrational behavior of QuAD.

6 Conclusion

The paper presented for the first time axioms that serve as guidelines for defining acceptability semantics in weighted bipolar settings. It also analyzed existing semantics with regard to the axioms. The results revealed that extension-based semantics like [1–5] fail to satisfy key properties. Furthermore, the role of support relation is a bit ambiguous since in case the attack relation of a BAG is empty, the argumentation graph has a single extension containing all the arguments. This means that supported and non-supported arguments are all equally acceptable. Gradual semantics defined in [7,8] satisfy more but not all the axioms. We proposed a novel semantics which satisfies all the 12 axioms. However, this semantics deals only with acyclic graphs. An urgent future work would be to prove whether the sequence of values it returns converges in case of arbitrary graphs. We also plan to investigate additional properties where attacks and supports do not have the same importance.

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References

1. Cayrol, C., Lagasquie-Schiex, M.C.: On the acceptability of arguments in bipolar argumentation frameworks. In: Godo, L. (ed.) ECSQARU 2005. LNCS, vol. 3571, pp. 378–389. Springer, Heidelberg (2005). doi:10.1007/11518655_33
2. Oren, N., Norman, T.: Semantics for evidence-based argumentation. In: Proceedings of COMMA, pp. 276–284 (2008)
3. Brewka, G., Woltran, S.: Abstract dialectical frameworks. In: Proceedings of KR (2010)
4. Boella, G., Gabbay, D.M., van der Torre, L., Villata, S.: Support in abstract argumentation. In: Proceedings of COMMA, pp. 111–122 (2010)
5. Nouioua, F., Risch, V.: Bipolar argumentation frameworks with specialized supports. In: International Conference on Tools with Artificial Intelligence, ICTAI 2010, pp. 215–218 (2010)
6. Cayrol, C., Lagasquie-Schiex, M.C.: Gradual valuation for bipolar argumentation frameworks. In: Godo, L. (ed.) ECSQARU 2005. LNCS, vol. 3571, pp. 366–377. Springer, Heidelberg (2005). doi:10.1007/11518655_32
7. Baroni, P., Romano, M., Toni, F., Aurisicchio, M., Bertanza, G.: Automatic evaluation of design alternatives with quantitative argumentation. Argum. Comput. **6**(1), 24–49 (2015)
8. Rago, A., Toni, F., Aurisicchio, M., Baroni, P.: Discontinuity-free decision support with quantitative argumentation debates. In: Proceedings of KR, pp. 63–73 (2016)
9. Dung, P.M.: On the acceptability of arguments and its fundamental role in nonmonotonic reasoning, logic programming and n-person games. Artif. Intell. **77**, 321–357 (1995)
10. Amgoud, L., Ben-Naim, J.: Evaluation of arguments from support relations: axioms and semantics. In: Proceedings of IJCAI, pp. 900–906 (2016)
11. Amgoud, L., Ben-Naim, J.: Axiomatic foundations of acceptability semantics. In: Proceedings of KR, pp. 2–11 (2016)