Wavelet p-leader non Gaussian multiscale expansions

for Heart rate variability analysis in congestive Heart failure patients

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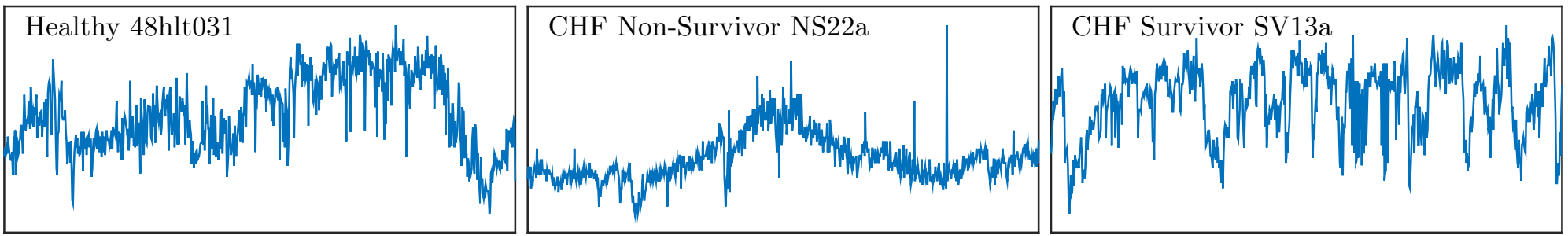
Motivation: Heart rate variability (HRV)

- Statistical characterization of temporal dynamics of HRV
  - power spectrum (bands, slope)
  - entropy (rate)
  - long-memory, scale-free and multifractal
  - non Gaussianity
  - . . . [Akselrod81,Costa05,Ivanov99,Yamamoto94,. . .] . . .



Data courtesy Y. Yamamoto (U Tokyo), K. Kiyono (U Osaka)

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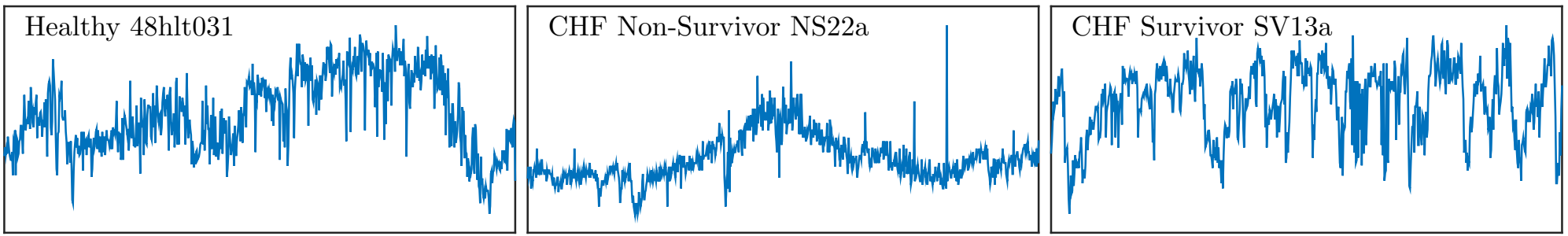
- Non Gaussian multiscale representations
  - large continuum of time scales
  - no a priori scale-free dynamics
  - combining range of statistical orders



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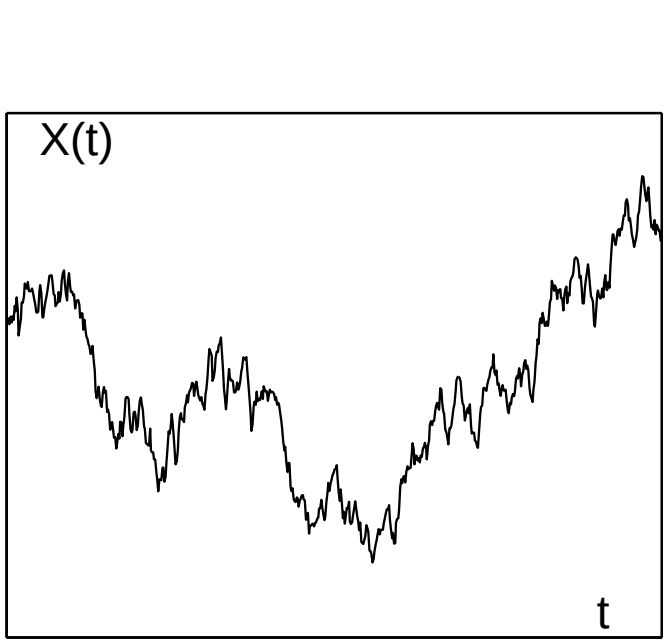
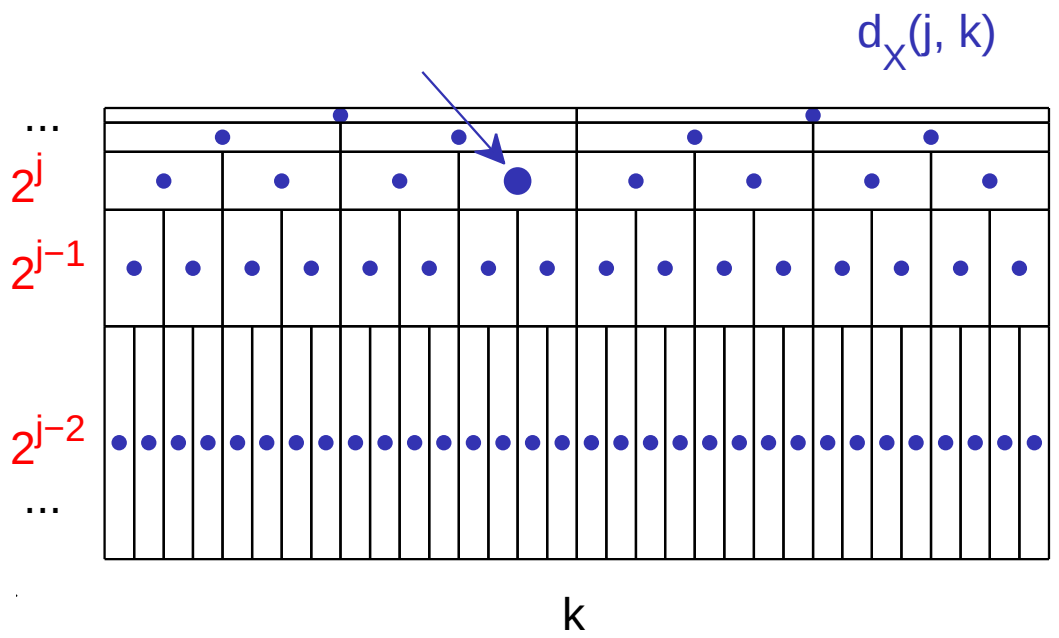
Wavelet coefficients and wavelet p-leaders

- Discrete wavelet transform:
  - mother wavelet $\psi(t)$ with vanishing moments N_ψ:
    $\int_{\mathcal{R}} t^k \psi(t) dt \equiv 0, \quad \forall k \in \mathbb{N}, k < N_\psi; \quad \int_{\mathcal{R}} t^{N_\psi} \psi(t) dt \neq 0$
  - wavelet basis $\{\psi_{j,k}(t) = 2^{-j/2}\psi(2^{-j}t - k)\}_{(j,k)\in\mathbb{N}^2}$
  - wavelet coefficients (L^1-normalized)

$$d(j,k) = \langle 2^{-j/2}\psi_{j,k} | X \rangle$$



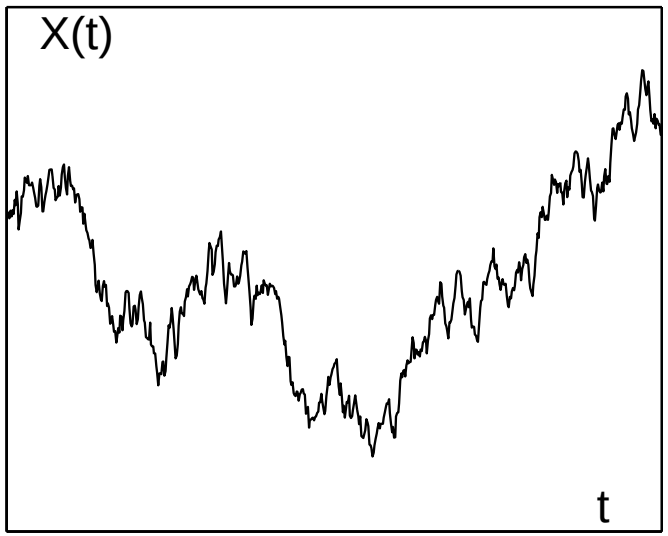


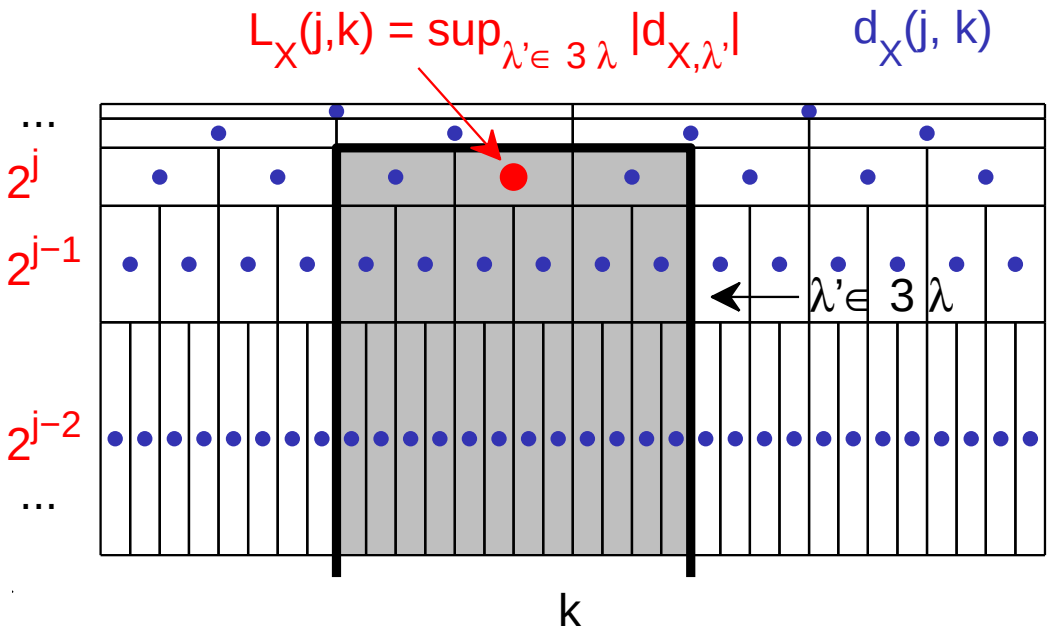
Wavelet coefficients and wavelet p-leaders

- Discrete wavelet transform: DWT coefficients $d(j,k)$
- wavelet p-leaders $\{\ell(j\cdot,\cdot)\}$ [Jaffard16]

$$\ell^{(p)}(j,k) \triangleq \left(\sum_{\lambda_{j',k'}\subset 3\lambda_{j,k}} |d(j',k')|^p 2^{-(j-j')}\right)^{1/p}$$

shown: wavelet leaders $p = +\infty$





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$\longrightarrow$ link with scale invariance

$\longrightarrow$ link with multifractals

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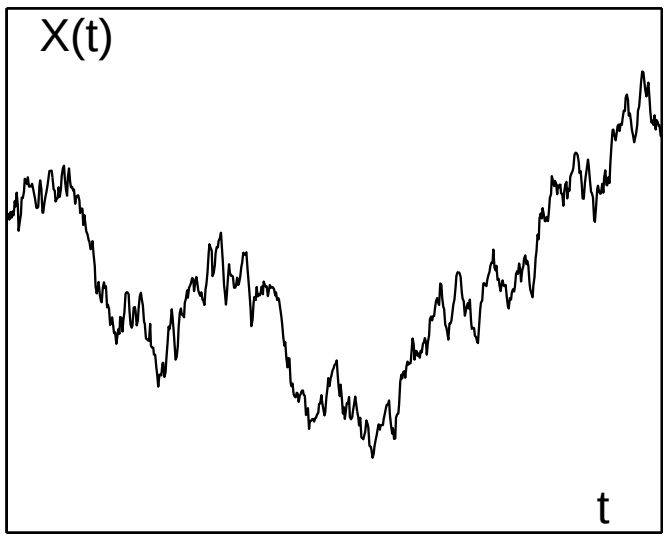

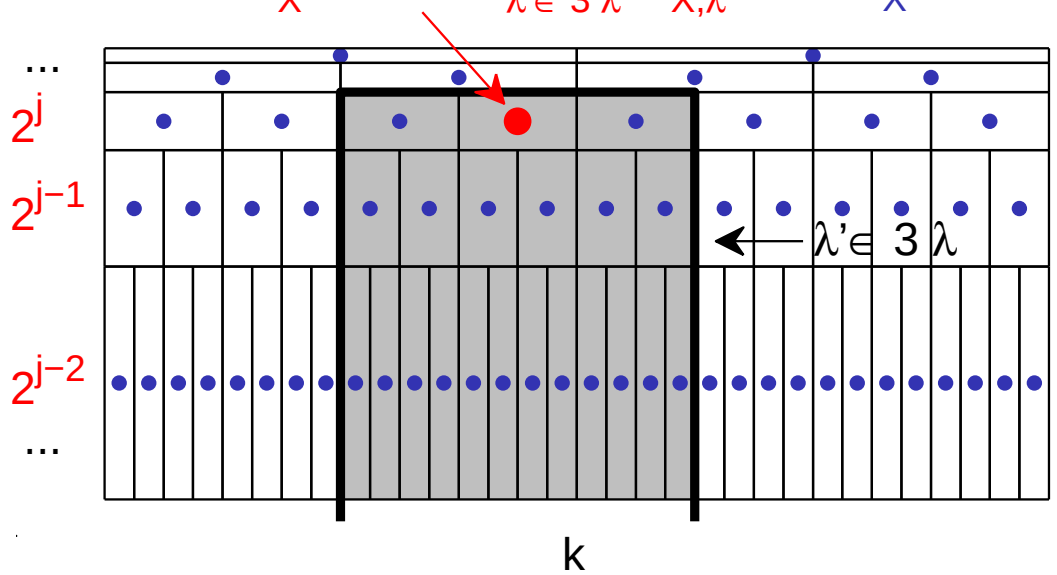

Moments, cumulants and scale invariance

- $\ell_s(k)$: p-leader coefficient $\ell^{(p)}(j,k)$ at scale $s = 2^j$
  - $L_q(s)$: log of q-th moments $\mathbb{E}\ell_s^q$, assumed finite ($\mathbb{E}\ell_s^q < \infty$)

$$L_q(s) \triangleq \log(\mathbb{E}\ell_s^q)$$

  - $C_m(s)$: m-th cumulant of $\log(\ell_s)$

$$C_m(s) \triangleq \mathrm{Cum}_m \log(\ell_s)$$

- Standard generating function expansion argument links them:

$$L_q(s) = \log(\mathbb{E}\ell_s^q) = \log \mathbb{E} e^{q \log(\ell_s)} = \sum_{m=1}^{\infty} C_m(s) \frac{q^m}{m!} \qquad (1)$$

- Scale-free temporal dynamics $\Longrightarrow$ specific scale-dependence

$$L_q(s) = \kappa_q + \zeta(q)\log(s) \qquad \text{scaling exponent } \zeta(q) \qquad (2)$$

$$C_m(s) = c_m^0 + c_m \log(s) \qquad \text{log-cumulant } c_m \qquad (3)$$

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Multifractal spectrum

e.g., [Jaffard16,Wendt07]

- Local regularity of $X(t)$ at t_0

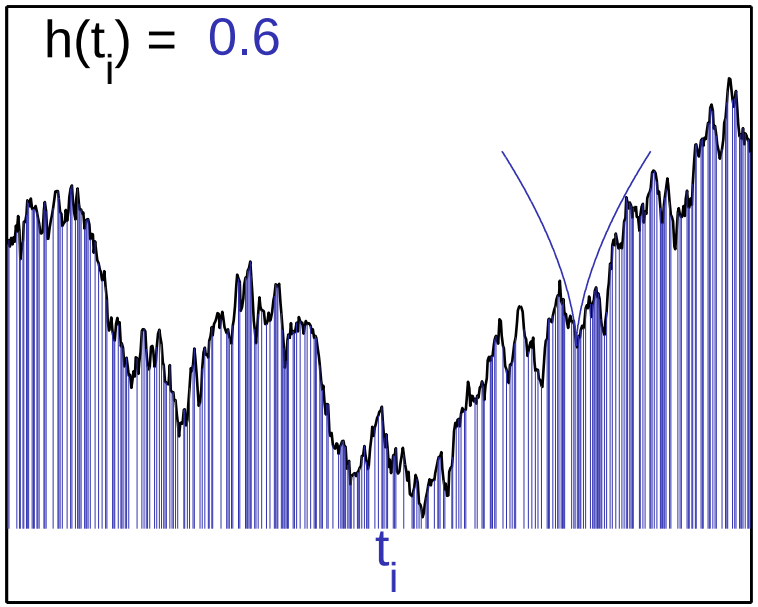
  p-exponent (Hölder exponent: $p = +\infty$)

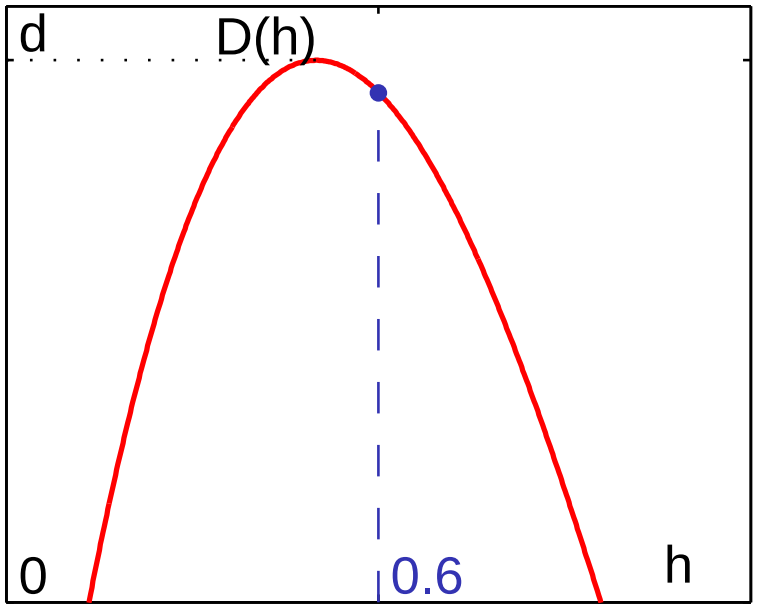
  $$h_p(t_0) \triangleq \sup_\alpha \{\alpha : \left(\tfrac{1}{s}\int_{B(0,s)} |X(t) - X(t_0)|^p dt\right)^{\frac{1}{p}} < Cs^\alpha\}, \quad s > 0$$

- Multifractal Spectrum $\mathcal{D}(h_p)$: Fluctuations of regularity $h_p(t)$

  - Set of points that share same regularity $\{t_i | h_p(t_i) = h\}$
  - Fractal (or Haussdorf) Dimension of each set:

  $$\mathcal{D}(h) \triangleq \dim_H \{t : h_p(t) = h\}$$





Multifractal spectrum and scale invariance

Key property of p-leaders ℓ_s: (under uniform regularity conditions)

$$h_p(t_0) = \liminf_{j \to -\infty} \frac{\log\left(\ell_{s=2^j}(k(t_0))\right)}{\log(2^j)}$$

Moments $\mathbb{E}\ell_s^q \longrightarrow$ empirical moments $S_n(s,q) \triangleq \frac{n}{s}\sum_k \ell_s^q(k)$

$$\begin{aligned} \Longrightarrow S_n(s,q) &\simeq \sum_h s^{1-\mathcal{D}(h)} s^{hq}, \\ &\simeq \sum_h s^{1-\mathcal{D}(h)+hq}, \\ &\sim_{s\to 0} \kappa_q s^{\zeta(q)}, \qquad \text{scaling exponents } \zeta(q) \end{aligned}$$

Saddle-point argument: $\Rightarrow$ Legendre transform

$$\zeta(q) = \inf_q (1 + hq - \mathcal{D}(h))$$

Multifractal formalism:

$$\mathcal{D}(h) \leq \inf_q (d + hq - \zeta(q)) \qquad (4)$$

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Departures from Gaussian: Cumulants $C_m(s)$?

- Cumulants $C_m(s)$ provide description of statistics of $\log \ell_s$
  - $C_1(s)$ - mean of $\log \ell_s$
    $\longrightarrow$ linear, Gaussian (mean & variance/covariance) properties
  - $C_2(s)$ - variance
  - $C_3(s)$ - skewness
  - $C_4(s)$ - kurtosis, etc ...

- Illustration for Gaussian / non Gaussian scale-free models
  - Fractional Brownian motion (fBm):
    Gaussian, self-similar
    $C_1(s) = f(s)$, $C_m(s) = const$ for $m \geq 2$
  - Multifractal random walk (MRW):
    non Gaussian multifractal of a special log-normal type
    $C_1(s), C_2(s) = f(s)$, $C_m(s) = const$ for $m \geq 3$;
  - Compound Poisson cascade (CPC):
    non Gaussian multifractal of more general form
    $C_m(s) = f(s) \neq const \; \forall m \geq 1$

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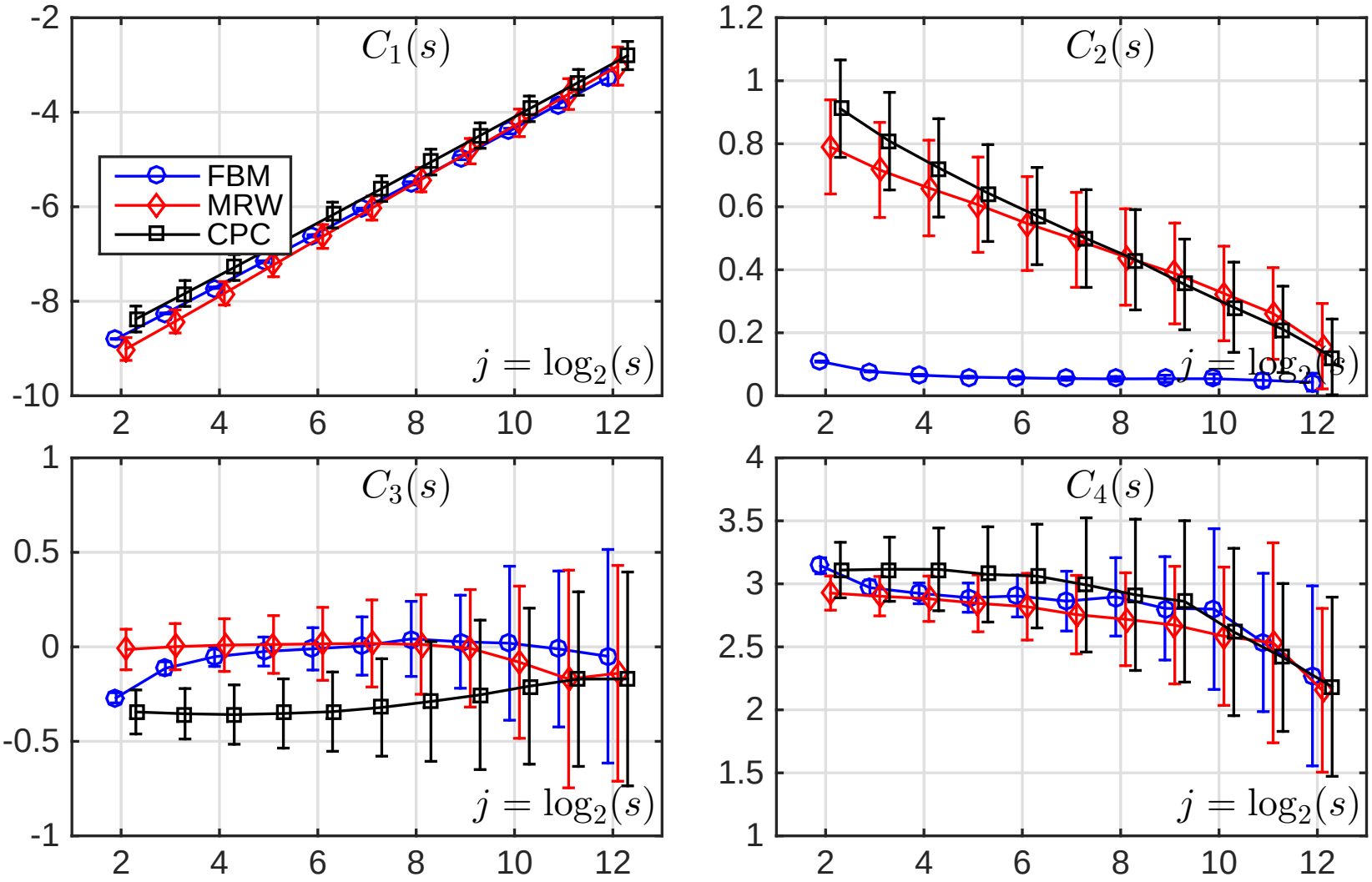

$\rightarrow$ essentially, $C_2(s) = const$ (fBm) vs. $C_2(s) = f(s)$ (MRW, CPC)

Non Gaussian multiscale expansion: Definition

Definition. Let $P \in \mathbb{N}^+$, $\boldsymbol{q} = (q_1, \ldots, q_{2P})$ with $q_i \neq 0$ and $q_i \neq q_j$:

$$\mathbf{L}_{\boldsymbol{q}}^{(2P)}(s) \triangleq \log \left(\prod_{i=1}^{P} \frac{\left(\mathbb{E}\ell_s^{q_{2i-1}}\right)^{\frac{1}{q_{2i-1}}}}{\left(\mathbb{E}\ell_s^{q_{2i}}\right)^{\frac{1}{q_{2i}}}} \right) \tag{5}$$

[Wendt18]

- Making use of (1) $\quad (L_q(s) = \log(\mathbb{E}\ell_s^q) = \sum_{m=1}^{\infty} C_m(s) \frac{q^m}{m!})$

$$\mathbf{L}_{\boldsymbol{q}}^{(2P)}(s) = \sum_{i=1}^{P} \frac{L_s(q_{2i-1})}{q_{2i-1}} - \frac{L_s(q_{2i})}{q_{2i}}$$

$$= \sum_{m=2}^{\infty} C_m(s) \frac{\sum_{i=1}^{P} q_{2i-1}^{m-1} - q_{2i}^{m-1}}{m!} \tag{6}$$

$\Longrightarrow$ infinite weighted sum of the cumulants $C_m(s)$

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Non Gaussian multiscale expansion: Properties [Wendt18]

Property 0: *Evolution across scales s of* $\mathbf{L}_{\boldsymbol{q}}^{(2P)}(s)$ indicator for non Gaussian properties.

$\Longrightarrow$ Gaussian data: $\mathbf{L}_{\boldsymbol{q}}^{(2P)}(s)$ equals a constant.

Property 1: $\mathbf{L}_{\boldsymbol{q}}^{(2P)}(s)$ *independent of* $C_1(s)$

$\Longrightarrow$ only quantifies nonlinear properties, departures from Gaussian

$(q_{2i}^{m-1} - q_{2i+1}^{m-1} = 0 \text{ for } m = 1)$

Property 2: Relative weighting of $C_m(s)$ in $\mathbf{L}_{\boldsymbol{q}}^{(2P)}(s)$ can be tuned

$\Longrightarrow$ quantify the strength or nature of deviation from Gaussian

Property 3: $\mathbf{L}_{\boldsymbol{q}}^{(2P)}(s)$ permits to probe *cumulants of high order*

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Non Gaussian multiscale expansion: Examples

notation	moments $\boldsymbol{q}$	cumulants C_m in (6) and *interpretation*
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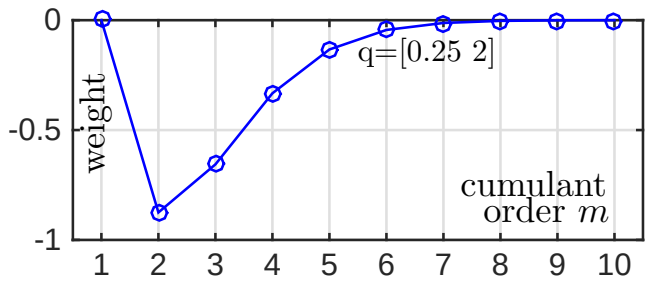

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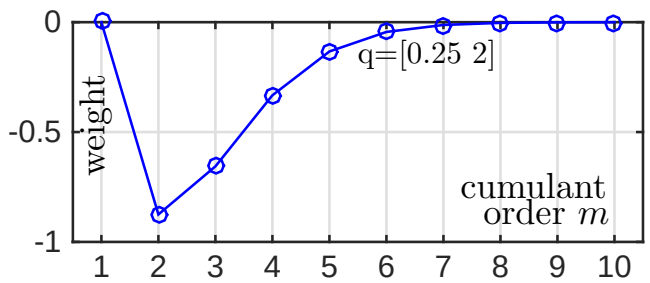

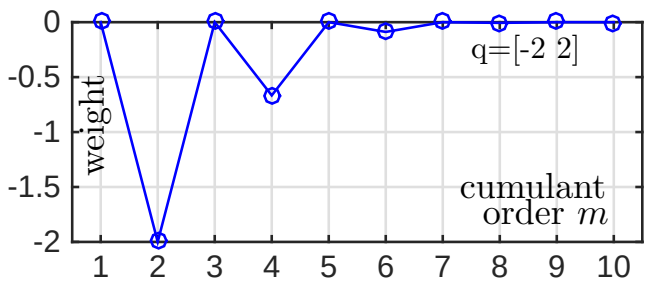

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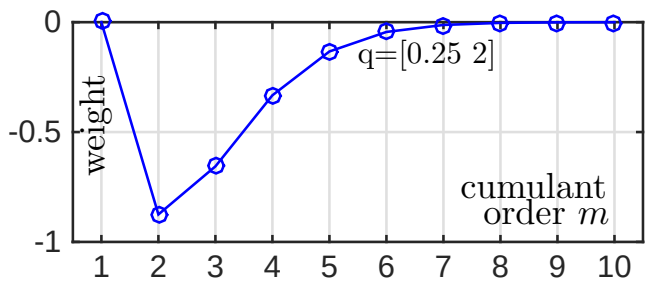
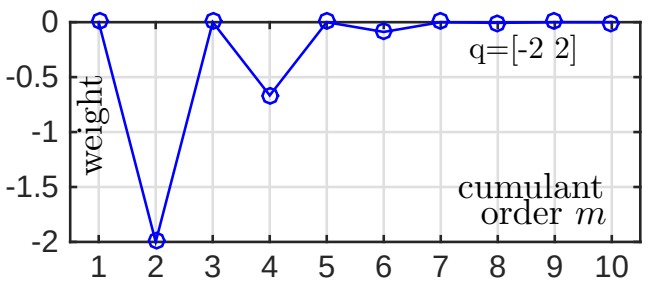
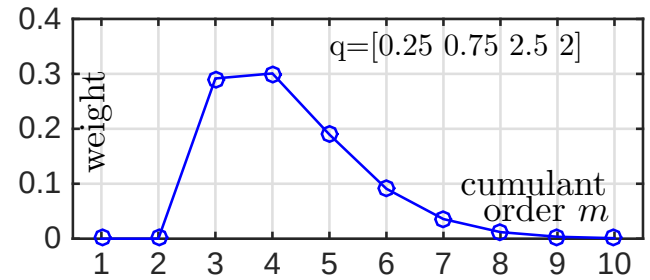
$\mathbf{L}_{\boldsymbol{q}}^{(4)}(s)$	$(0.25, 0.75, 2.5, 2)$	$m \geq 3$ *non log-normal nature of non Gaussian*

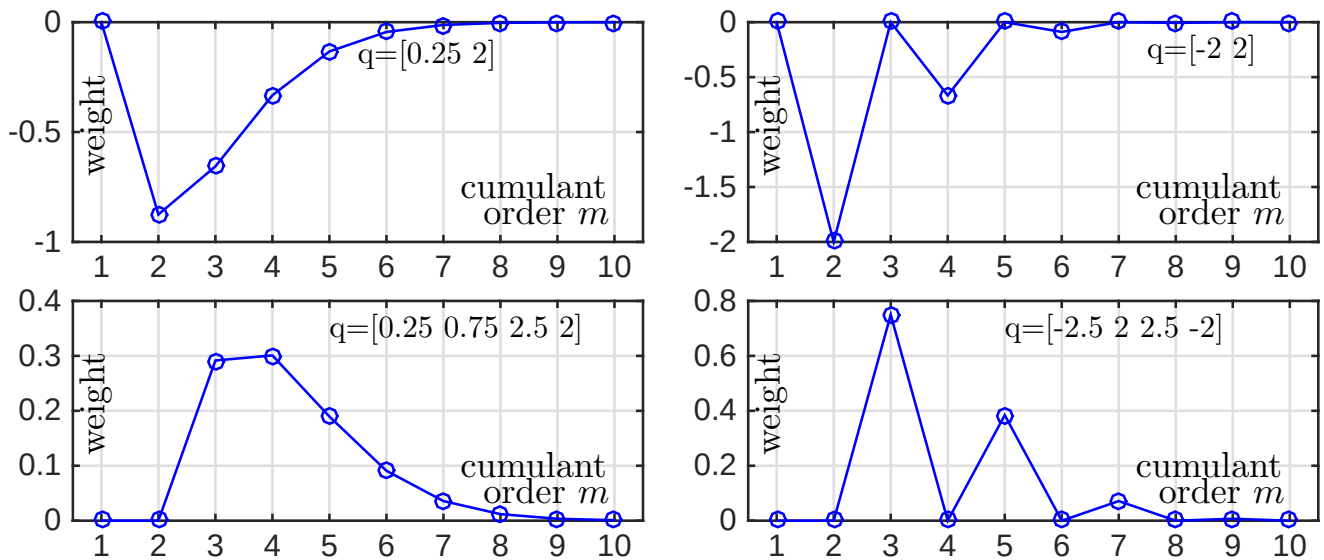




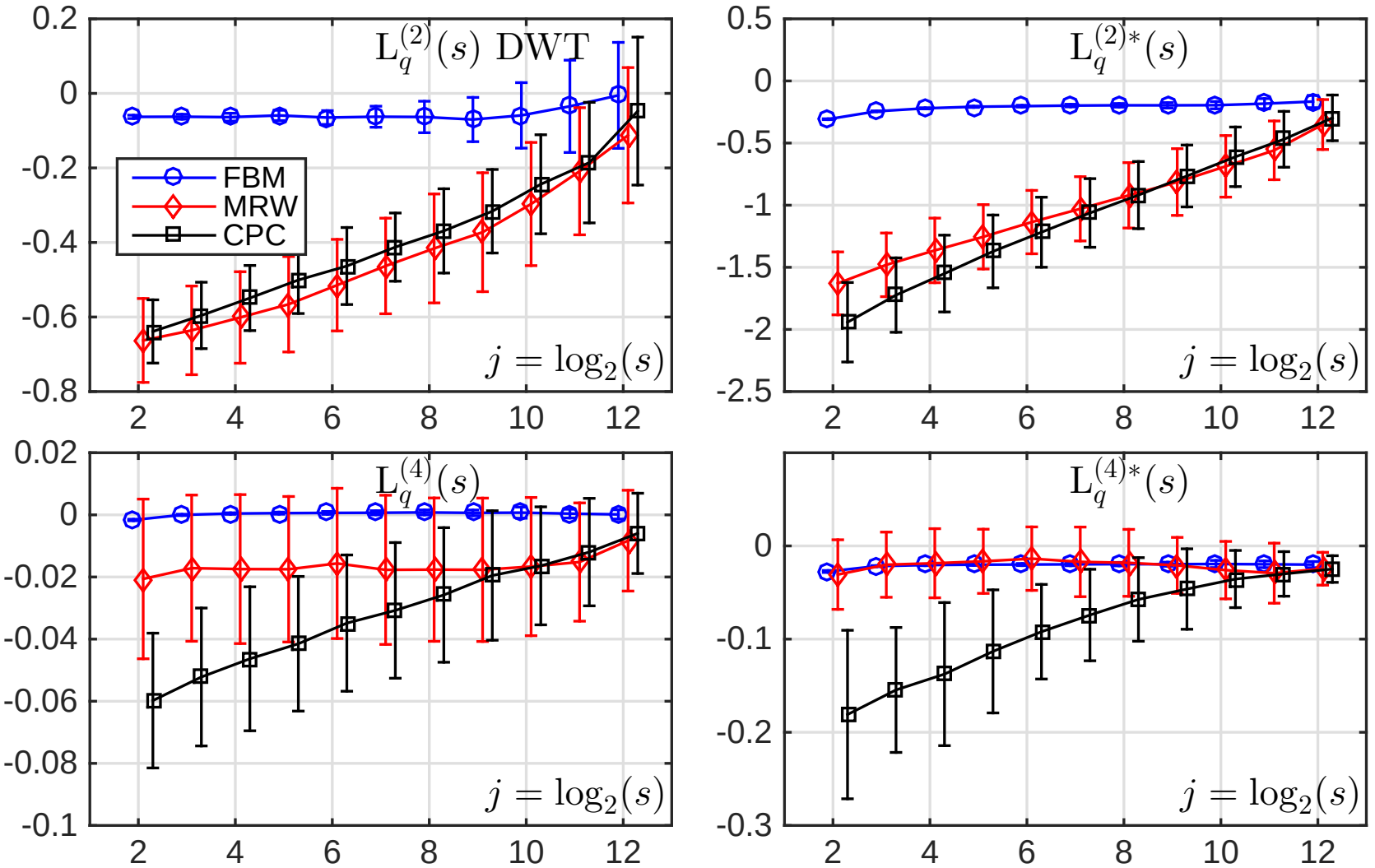


Non Gaussian multiscale expansion: Examples

notation	moments $\boldsymbol{q}$	cumulants C_m in (6) and *interpretation*
$\mathbf{L}_{\boldsymbol{q}}^{(2)}(s)$	$(0.25, 2)$	$m \geq 2$ *any deviation from Gaussian*
$\mathbf{L}_{\boldsymbol{q}}^{(2)*}(s)$	$(-2, 2)$	even order $m = 2, 4, \cdots$ *mainly symmetric properties of non Gaussian*
$\mathbf{L}_{\boldsymbol{q}}^{(4)}(s)$	$(0.25, 0.75, 2.5, 2)$	$m \geq 3$ *non log-normal nature of non Gaussian*
$\mathbf{L}_{\boldsymbol{q}}^{(4)*}(s)$	$(-2.5, 2, 2.5, -2)$	$m \geq 3$ of odd order: $m = 3, 5, \cdots$ *asymmetry of non Gaussian*



Non Gaussian multiscale expansion: Illustration



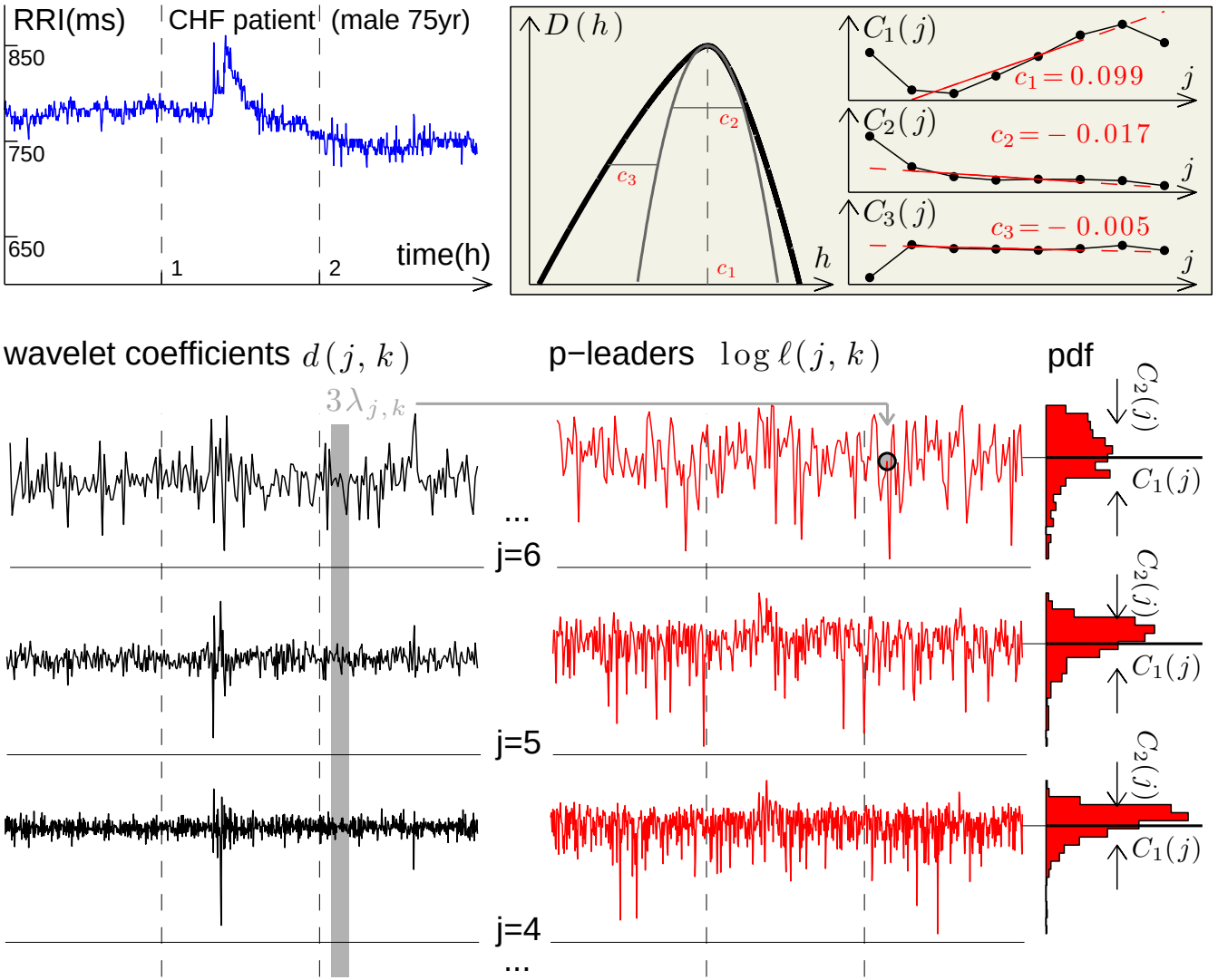
$\Longrightarrow$ discriminate: Gaussian (fBm) - non G LN (MRW) - non G (CPC)

Congestive Heart failure (CHF) patient HRV data

- cohort of 108 CHF patients
  - Fujita Health University Hospital, 2000-2001
  - 61 male and 47 female
  - age 21-92 (66.1 ± 14.8) years
  - similar health status before hospital discharge
  - 39 (36.1%) dead in follow-up of 33 ± 17 (range $1 - 59$) months

    clinical details: [Kiyono 2008]
- age-matched healthy (HM) control group of 90 subjects
- 24-hour Holter ECG recording
  - recorded prior to Hospital discharge
  - outlier / detection error corrected, no sustained tachyarrhythmias
- analyzed time series:
  - primitive $y_n = \sum_{i=1}^{n} x_n$ of RR inter-arrival time values
    $X \equiv \{x_n, n = 1, \ldots, N\}$
    interpolated at $4Hz$ (cubic spline)

Estimation



$N_\psi = 3$, octaves $j_1 = 4 \leq j \leq j_2 = 9$ (5.3s to 170.7s)

Spectral, Entropy and multifractal features

Estimates	LF	HF	LF/HF	α_{PSD}	sampEN	apEN
HM	0.057	0.028	3.001	2.314	0.267	0.324
CHF	0.064	0.160	0.713	1.362	0.431	0.600
NS	0.064	0.132	0.692	1.476	0.378	0.538
SV	0.064	0.176	0.725	1.300	0.463	0.637

WRS p-value	*LF*	*HF*	*LF/HF*	α_{PSD}	sampEN	apEN
CHF vs. HM	0.435	$< 1e{-}8$	$< 1e{-}8$	$< 1e{-}8$	$< 1e{-}8$	$< 1e{-}8$
NS vs. SV	0.406	0.173	0.927	0.327	0.062	0.127

Estimates	c_1	c_2	c_3	c_4
HM	0.255	−0.032	0.005	−0.005
CHF	0.079	−0.058	−0.008	−0.022
NS	0.053	−0.060	0.009	−0.070
SV	0.095	−0.057	−0.017	0.006

WRS p-value	c_1	c_2	c_3	c_4
CHF vs. HM	$< 1e{-}8$	0.276	$3.0e{-}4$	0.111
NS vs. SV	0.156	0.720	0.932	0.068

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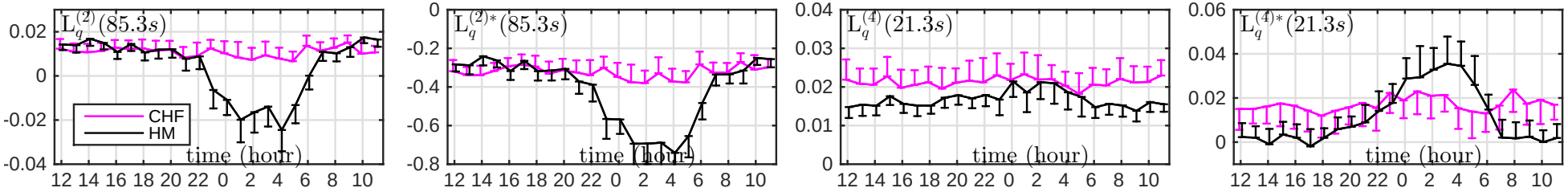
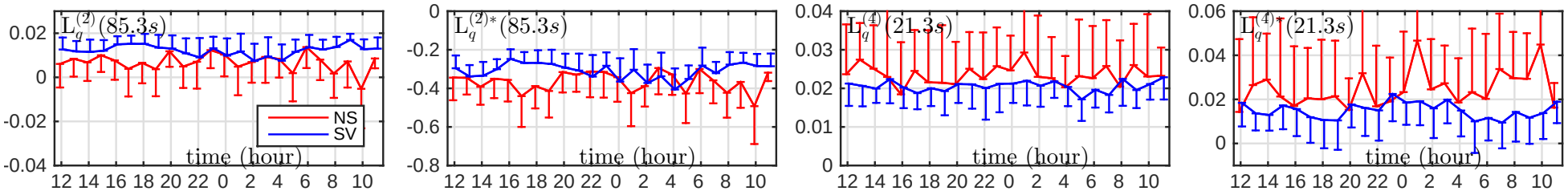
Non Gaussian multiscale expansion: Circadian evolution

CHF vs. HM

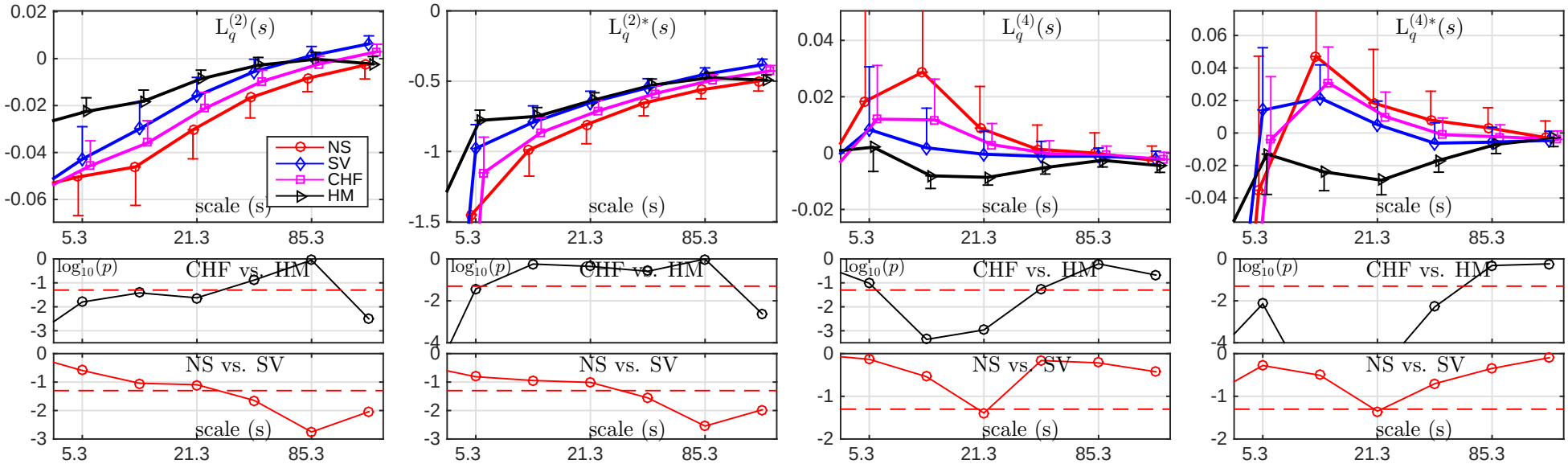


NS vs. SV



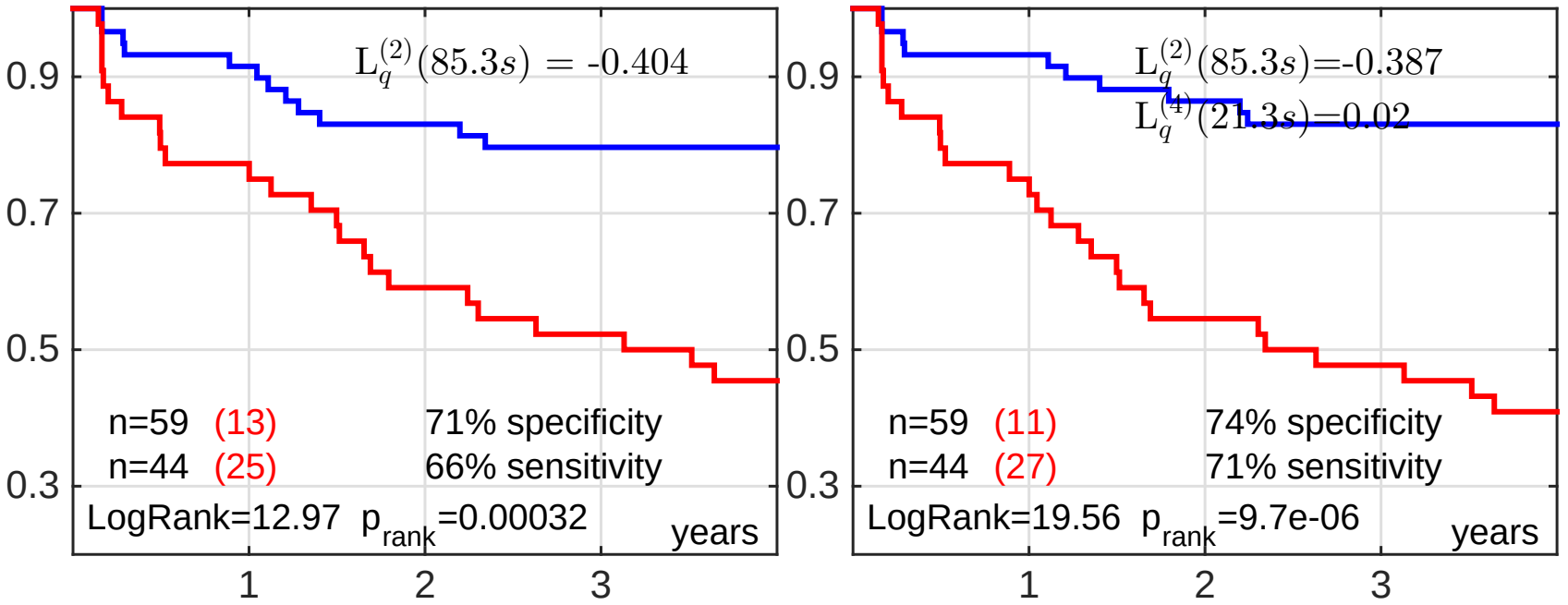
- HM: clear day-night evolution for all non Gaussian indices
- HM nighttime increase in non Gaussianity driven by:
  - purely asymmetric properties ($\mathbf{L}_q^{(4)*}$)
  - even order properties ($\mathbf{L}_q^{(4)}$)
  - stronger deviation from log-Normal
- No circadian modulation for CHF patients

Non Gaussian multiscale expansion: 6h daytime



- Degree of non Gaussianity ($\mathbf{L_q^{(2)}}$)
  - stronger non G for CHF than for HM, and NS than SV
  - significant for NS vs. SV (scale 85.3s)
- Nature of non Gaussianity ($\mathbf{L_q^{(4)}}$)
  - non log-Normal type
  - stronger non LN for CHF than for HM, and NS than SV
  - significant for NS vs. SV (scale 21.3s)

Non Gaussian multiscale expansion: discrimination



- Best results for spectral and entropy based analyses:
  - specificity $\leq 40\%$, $p > 0.01$ (sampEN, apEN)
- Degree of non Gaussianity ($\mathbf{L_q^{(2)}}$) consistent with [Kiyono2008]
  - sensitivity (specificity) of 66% (71%), $p = 0.0003$
- Combined use of non Gaussian expansion indices ($\mathbf{L_q^{(2)}}$ & $\mathbf{L_q^{(4)}}$)
  - sensitivity (specificity) of 71% (74%), $p < 10^{-5}$

Conclusions

- Non Gaussian multiscale representations
  - use of continuum of scales without a priori scale-free dynamic assumptions
  - joint analysis of customizable ranges of (higher) statistical orders
  - e.g., degree and nature of departure from Gaussian

- Nonlinear dynamics of HRV (90 healthy subjects, 108 CHF patients)
  - circadian evolution of non Gaussian indices for healthy subjects only
  - specific non Gaussian indices possess high discriminative abilities between NS and SV CHF patients
  - for specific time scales ($21.3s$ and $85.3s$)
  - substantially improving upon spectral, entropy and multifractal based indices

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