SCALING (OR MULTIFRACTAL) ANALYSIS

Scaling Invariance

Multiresolution Quantities $T_X(n,k)$

Discrete Wavelet Coefficients (vanishing moments $N_c$)

Wavelet Leaders [Affholder95]

Scale Exponents $D(n,k)$ - Multifractal spectrum

Estimation:

- Log-Cumulants $[\text{CastaingGagneMarchand09}]

- Bootstrap Test Performance

- Monte Carlo Simulations

- Scalability and Non Stationarity

- Bootstrap Time Constancy Test

- Null Distribution Estimation

- References

SCALING AND NON STATIONARITY

CONTRIVENY: Scale Invariance $\Rightarrow$ Non Stationary

- Discrimination of true scaling against various forms of non stationary variability for multifractal processes

- Heuristic

- Objective

- Summary

- Problem

- Bootstrapping Leaders

- Procedure for obtaining $T_F$ and $T_P$

- Bootstrap Test Statistic

- Bootstrap Test Performance

- Test performance under $H_0$

- Test performance under $H_1$

- Critical Value $T_c$ under $H_0$

- Null Distribution Estimation

- Performance Assessment and Results

- References

REFERENCES

- Fractal Geometry and Applications

- Null Distribution Estimation

- Bootstrap Test Performance

- Monte Carlo Simulations

- BOOTSTRAP TESTS FOR THE TIME CONSTANCY OF MULTIFRACTAL ATTRIBUTES

- Monte Carlo Random Walk (MRW) $[\text{Mandelbrot99}]

- Summary

- On open and controversial issue in empirical data analysis is to decide whether scaling and multifractal properties observed in empirical data actually exist, or whether they are induced by intrinsic non stationarities. To contribute to answering this question, we propose here a non parametric bootstrap and wavelet Leaders based procedure aiming at testing the constancy along time of multifractal attributes estimated over adjacent non overlapping time windows.

- Bootstrap Tests for the Time Constancy of Multifractal Attributes

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- BOOTSTRAP TIME CONSTANCY TEST

- Test Principle

- Multifractal attribute under test: $\theta \in \{c_2\}$

- Test identical mean for independent estimates:

- $H_0: \theta = \theta_0 \Rightarrow \theta_1 \Rightarrow B$.

- Bootstrap Test Statistic

- $L_X(j,k)$ from adjacent non overlapping subsets $X_{j,k}$

- Resample from Leaders $(L_X(j,k))$ corresponding to subsets $X_{j,k} \Rightarrow \theta_0^{(n)}$

- Under $H_0$, distribution of $T_F$ independent of precise mean/variances of $\theta_0$

- $T_F = \sum_{j=1}^{J} \sum_{k=1}^{K} \frac{\hat{\theta}_0^{(n)}}{\sum_{j=1}^{J} \sum_{k=1}^{K} \hat{\theta}_0^{(n)}}$

- Bootstrap Null Distribution Estimation

- 1. Resample from complete set $(L_X(j,k))$ of Leaders

- 2. Then cut into $M$ subsets $(L_X^{(m)},m=1,\cdots,M)$

- $\theta_0^{(m)}$ have same conditional distribution

- $T_F$ always reproduces null distribution of $T_F$

- $\theta_0^{(m)}$ from double bootstrap:

- $T_F = \sum_{j=1}^{J} \sum_{k=1}^{K} \frac{\hat{\theta}_0^{(m)}}{\sum_{j=1}^{J} \sum_{k=1}^{K} \hat{\theta}_0^{(m)}}$

- Performance Assessment and Results

- Monte Carlo Simulations

- Test performance under $H_0$

- $\text{Constant multifractal attributes } (c_1,c_2)$

- $H_1$: Simplest alternative: piecewise constant multifractal attributes

- $H_1$: $c_2^0 = 0.75$

- $H_1$: $c_2^0 = 0.01$

- $H_1$: $N_{\text{sub}} = 10^3$

- Null Distribution Estimation

- Critical value $T_c$ under $H_0$

- independent of $c_2^0 = c_2^0$

- equal $T_c$ under $H_1(c_2)$

- REFERENCES


