

Spatially regularized multifractal analysis for fMRI Data

P. Ciuciu¹, H. Wendt², S. Combrexelle², P. Abry³

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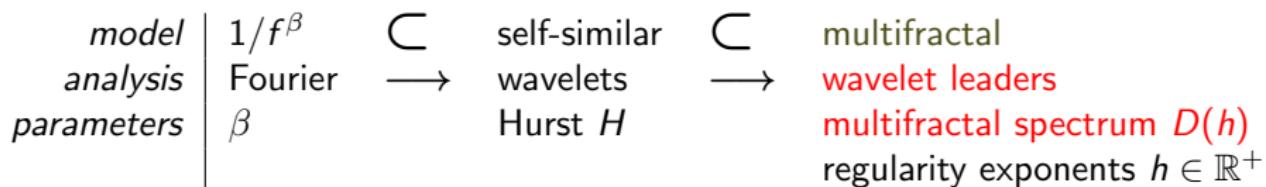
³ CNRS, Univ Lyon, ENS de Lyon, Univ Claude Bernard, France

IEEE EMBS Conference, Jeju, Korea, 14 July 2017

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Scale-free dynamics and infraslow macroscopic brain activity



[He JNS'11, Shimizu NeuroImage'04, Ciuciu FPhys'12]

- ▶ Challenges in fMRI data scale-free dynamics analysis
 - ▶ Short length/limited time resolution of voxels
→ questioning accurate estimation of multifractal parameters?
 - ▶ Tens of thousands of voxels recorded jointly
→ use neighborhood to regularize estimation? [Pellé ISBI'16]
- ▶ Contributions
 - ▶ Novel spatially regularized multifractal parameter estimation
 - ▶ First application to fMRI data from one subject (rest/task)

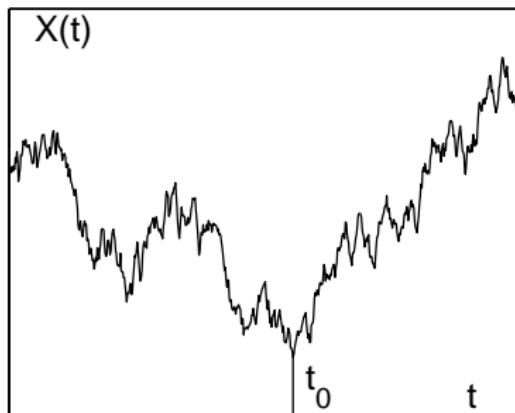
Multifractal spectrum

- Local regularity of $X(t)$: Hölder exponent $h(t)$

$$h(t_0) = \sup_{\alpha} \{ \alpha : |X(t) - X(t_0)| < C|t - t_0|^{\alpha} \} \quad 0 < \alpha$$

$h(t_0) \rightarrow 1 \Rightarrow$ smooth, very regular

$h(t_0) \rightarrow 0 \Rightarrow$ rough, very irregular



Multifractal model

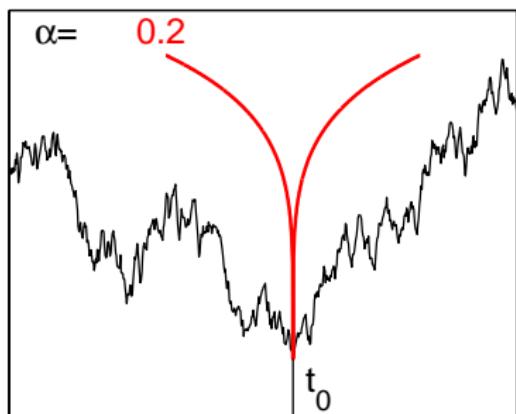
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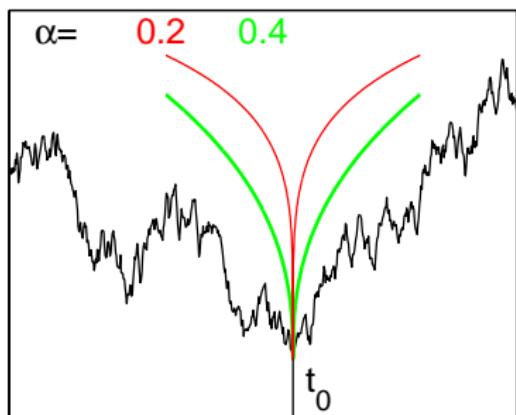
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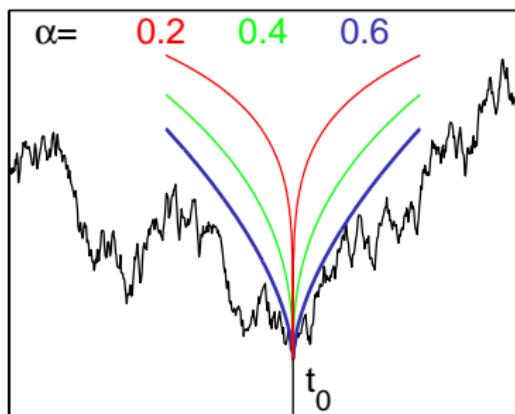
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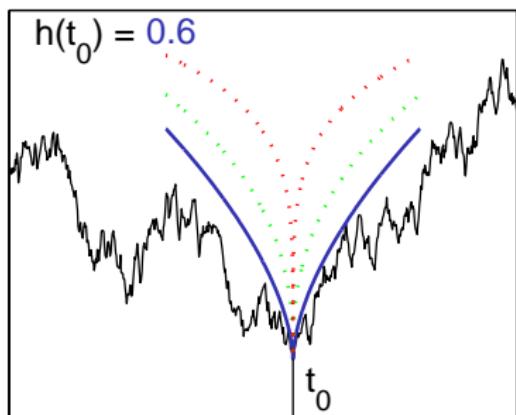
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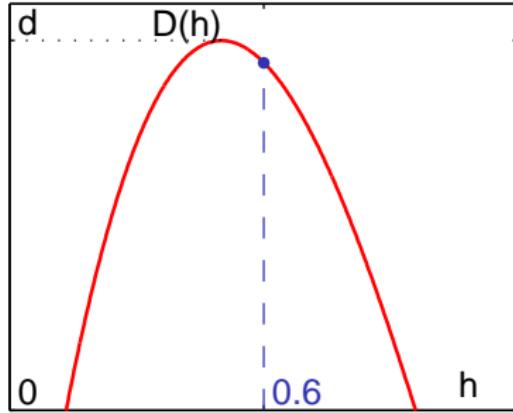
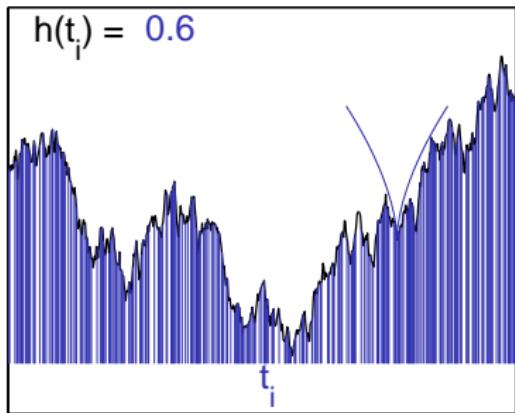
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- Multifractal Spectrum $\mathcal{D}(h)$: Fluctuations of regularity $h(t)$
 - Haussdorff dimension of set of points with same regularity

$$\mathcal{D}(h) = \dim_H \{ t : h(t) = h \}$$



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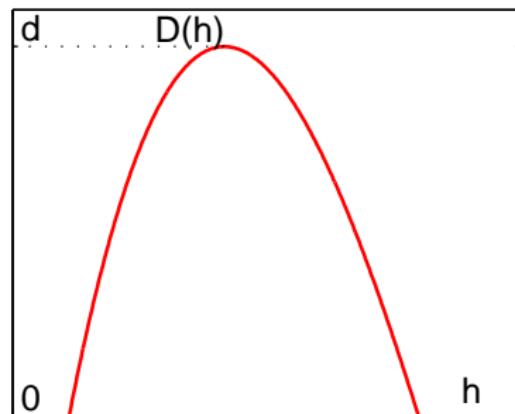
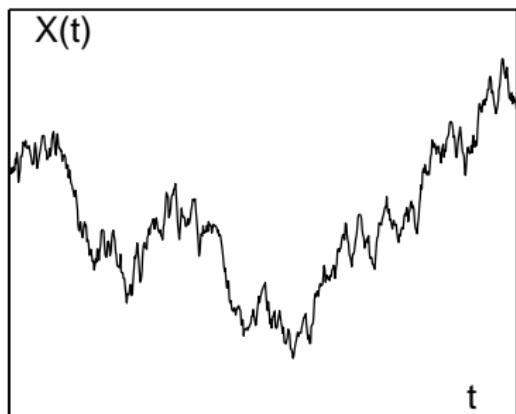
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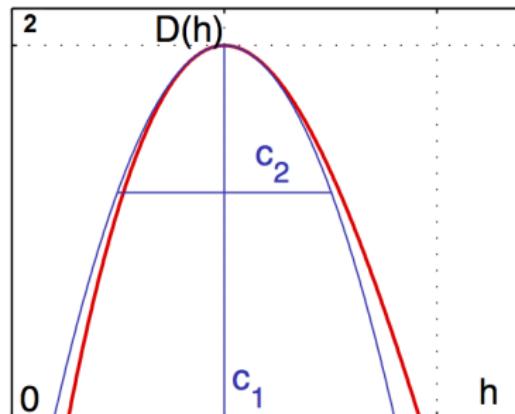
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$$\begin{aligned}\mathcal{D}(h) &= \dim_H \{t : h(t) = h\} \\ &\approx 1 + (h - c_1)^2 / (2c_2)\end{aligned}$$

c_1 : self-similarity

c_2 : multifractality parameter



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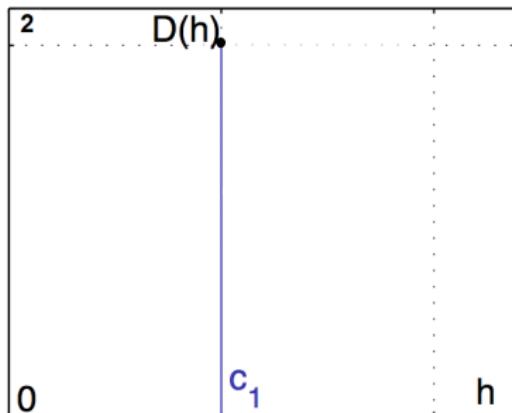
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c_2 : multifractality parameter

e.g., fractional Brownian motion (fBm):

$$c_1 = H, \quad c_2 = 0$$



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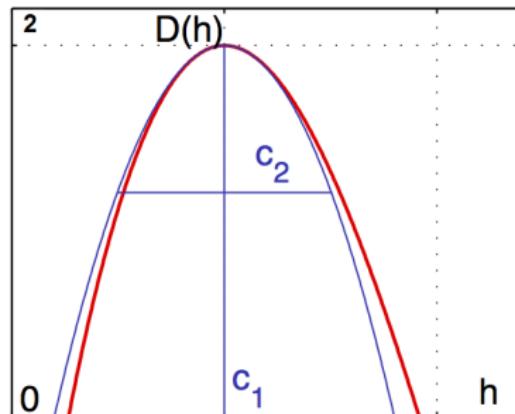
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$c_2 < 0$:

complex temporal dynamics

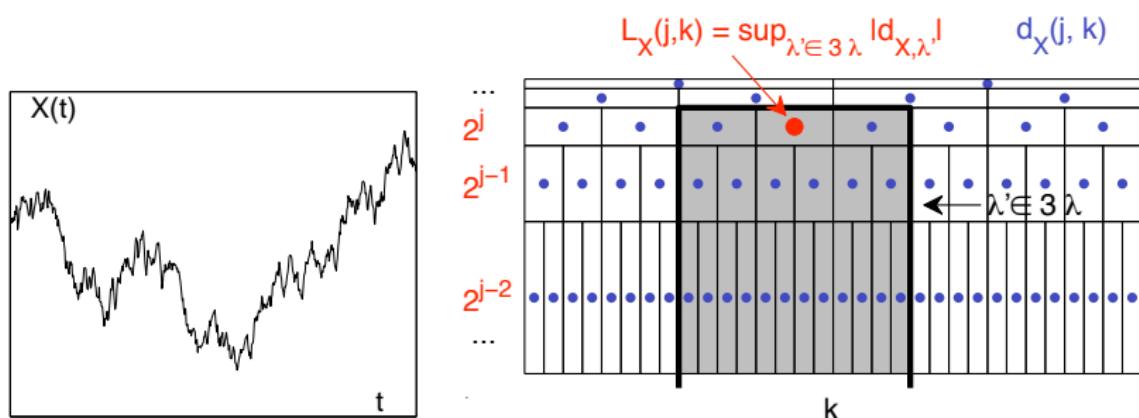
non-Gaussian



Multifractality parameter estimation

Univariate estimation for c_2 by linear regression (LR)

- Wavelet leader multiresolution coefficients $\ell(j, \cdot)$ [Jaffard04]



- c_1 & c_2 tied to mean & variance of $\log \ell(j, \cdot)$ [Castaing PhysD'93]

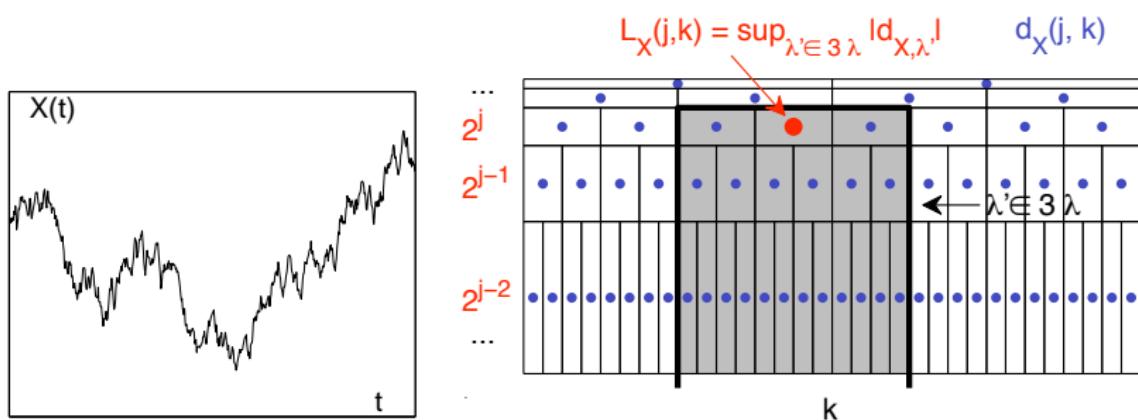
$$\text{Var}(\log \ell(j, \cdot)) = c_2^0 + c_2 \log 2^j$$

⇒ estimate for c_2 : linear regression of $\text{Var}(\log \ell(j, \cdot))$ vs. scale j

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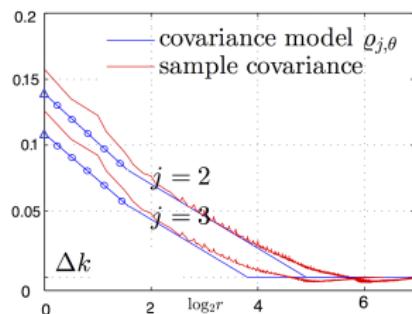
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- ▶ Log-wavelet leaders $\log \ell(j, \cdot)$ are

- ▶ Gaussian
- ▶ stationary
- ▶ with covariance $\text{Cov}(r)$, $r = |k' - k|$:
function $\varrho_{j,\theta}$ depending on $\theta = (c_2^0, c_2)$

[Combrexelle TIP'15]



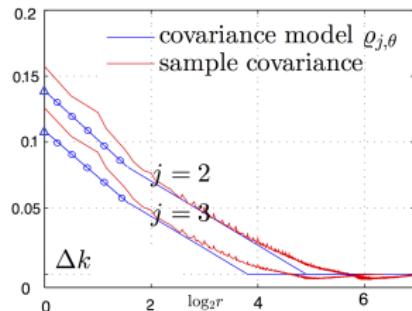
- ▶ Bayesian model and estimation for c_2 [Combrexelle TIP'15,16]
 1. Whittle approximation → avoid inversion of covariance matrix
 2. data augmentation → conjugate inverse Gamma priors for c_2

⇒ efficient MCMC estimation: cost comparable to LR [Roberts05]
 ⇒ greatly improved performance

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Multifractality parameter estimation

Multivariate Bayesian estimation for c_2 (GaMRF)

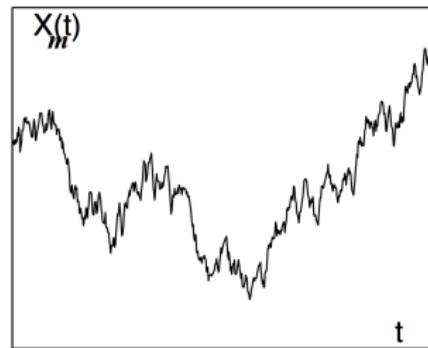
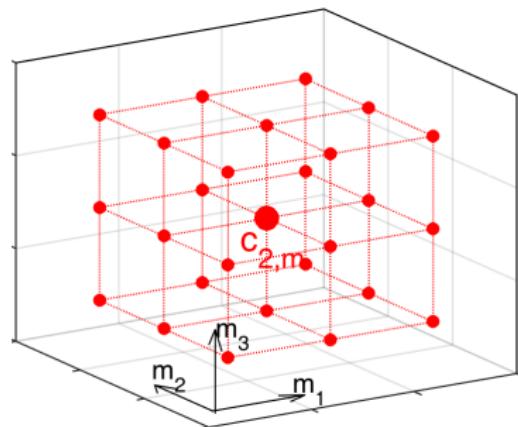
Spatial regularization for parameters $c_{2,m}$ of voxels X_m , $\mathbf{m} \triangleq (m_1, m_2, m_3)$

- ▶ Induce positive dependence via auxiliary variables $z_{m'}$ [Dikmen TASLP'10]

hidden gamma Markov random field (GaMRF) joint prior for (c_2, z)

- ⇒ Regularized multivariate multifractal estimation [Combrexelle EUSIPCO'16]

- ▶ essentially same cost as univariate Bayesian (UniB) estimator



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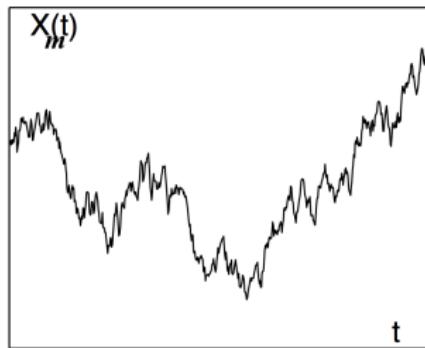
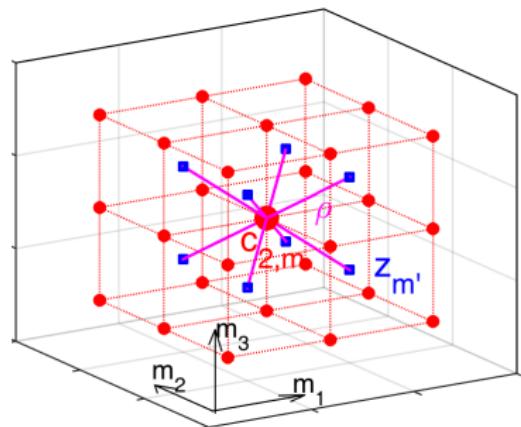
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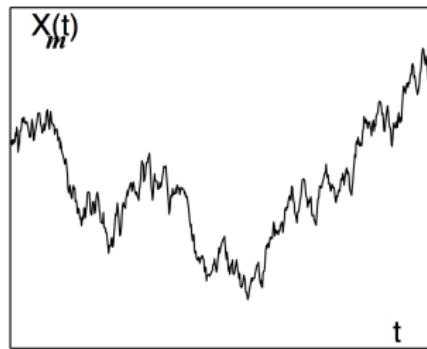
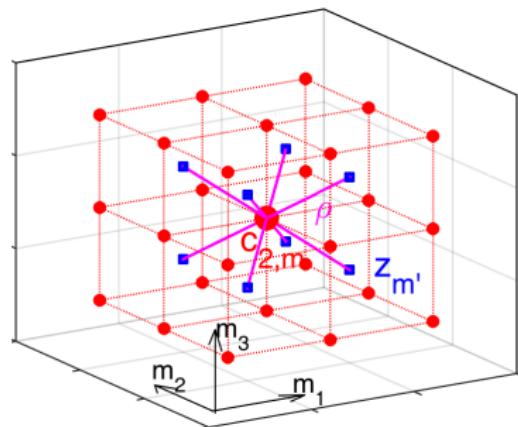
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Experimental design and acquisition

Verbal n -back working memory task ($n = 3$).

- serially presented upper-case letters (displayed 1s, separation 2s)
→ Is letter same as that presented 3 stimuli before?
- each run: alternating sequence of 8 blocks



Data acquisition.

- resting-state fMRI images collected first: participant at rest, with eyes closed
- 543 scans (9min10s) / 512 scans (8min39s) for rest / task
- fMRI data acquisition at 3 Tesla (Siemens Trio, Germany).
- multi-band GE-EPI (TE=30ms, TR=1s, FA=61, MB=2) sequence (CMRR, USA), 3mm isotropic resolution, FOV of $192 \times 192 \times 144 \text{ mm}^3$

Shown results: ($-c_2$) maps.

- for single subject (arbitrarily chosen from 40 participants).

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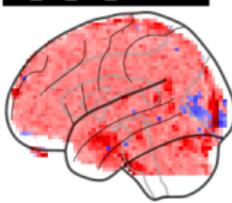
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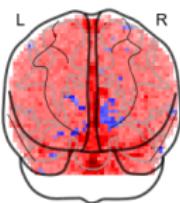
Regularized multifractal analysis for fMRI data

Resting-state analysis

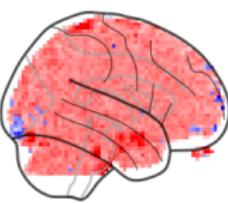
Left sagittal
c2_LF_rs1_Nmc1600



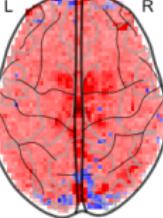
Coronal



Right sagittal



Axial



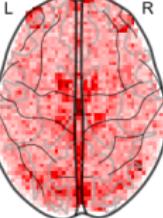
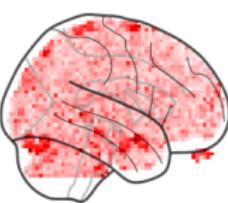
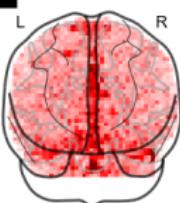
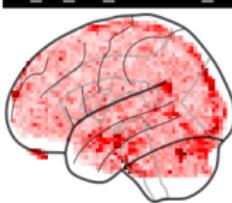
LR:

- poor estimation (var!)

UniB & GaMRF:

- estimation var decrease

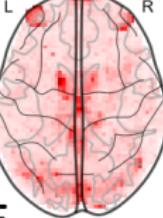
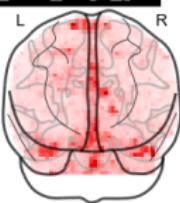
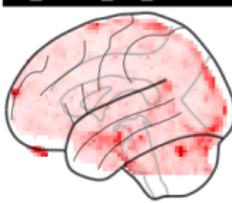
c2_IG_rs1_Nmc1600_IS10



UniB:

- enhanced MF contrast

c2_GMRF_rs1_Nmc1600_IS10_Reg1_g05



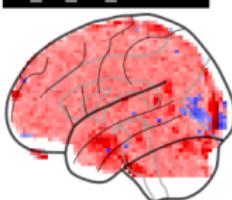
GaMRF

→ significant MF in default mode network (DMN)

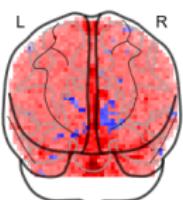
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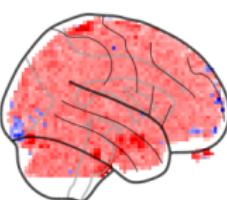
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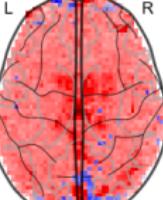
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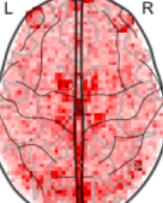
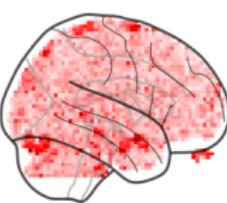
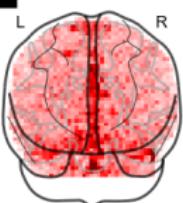
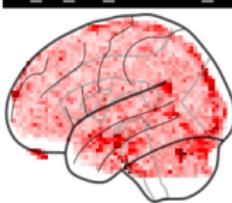
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UniB & GaMRF:

- estimation var decrease
- increase of MF in DMN

c2_IG_rs1_Nmc1600_IS10



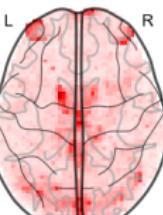
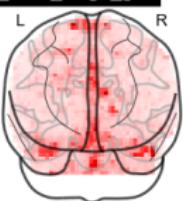
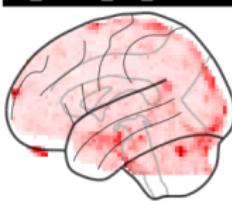
GaMRF:

- enhanced MF contrast

scale-free dynamics in DMN for resting-state fMRI reported before, but for H only [He JNS'11].

→ evidence for richer, MF resting state brain dynamics

c2_GMRF_rs1_Nmc1600_IS10_Reg1_g05



GaMRF

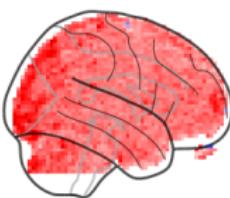
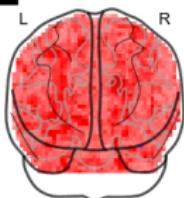
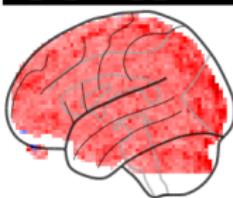
→ significant MF in default mode network (DMN)

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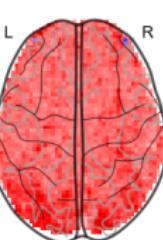
Task analysis (3-back run)

Left sagittal Coronal Right sagittal

c2_LF_nback_3_Nmc1600



Axial



0.24

0.12

0

-0.12

-0.24

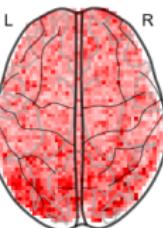
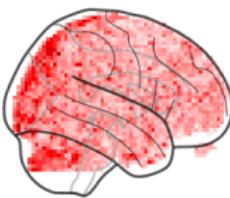
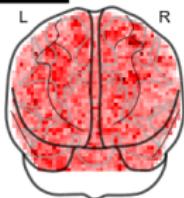
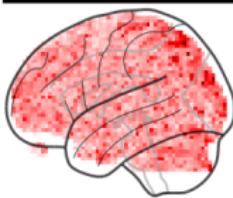
LR:

- poor estimation (var!)

UniB & GaMRF:

- estimation var decrease

c2_IG_nback_3_Nmc1600_IS10



0.11

0.083

0.055

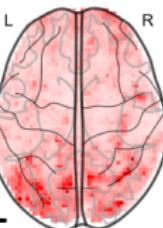
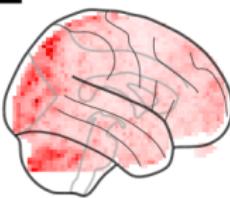
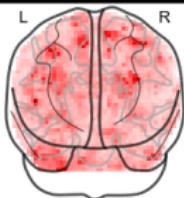
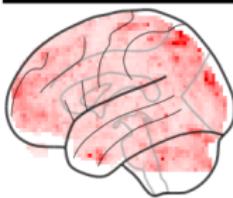
0.028

0

LR

UniB

c2_GMRF_nback_3_Nmc1600_IS10_Reg1_g05



0.11

0.083

0.055

0.028

0

GaMRF

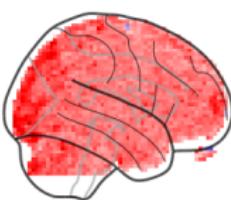
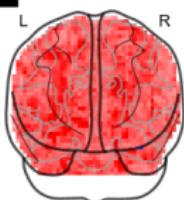
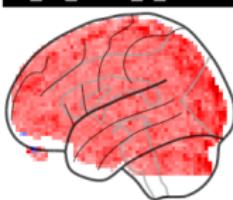
→ overall MF increase; working memory network (WMN), visual, sensory.

Regularized multifractal analysis for fMRI data

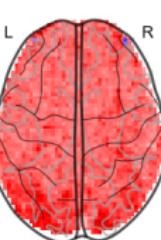
Task analysis (3-back run)

Left sagittal Coronal Right sagittal

c2_LF_nback_3_Nmc1600



Axial



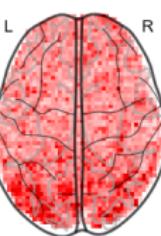
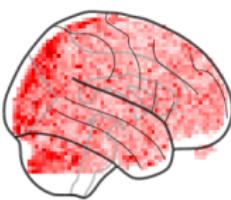
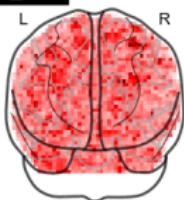
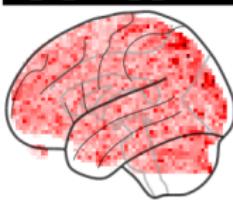
LR:

- poor estimation (var!)

UniB & GaMRF:

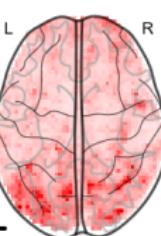
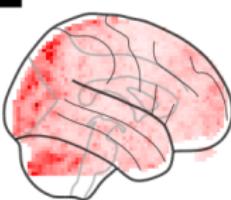
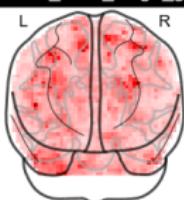
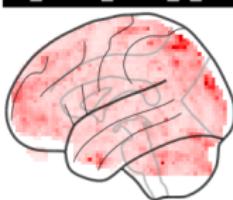
- estimation var decrease

c2_IG_nback_3_Nmc1600_IS10



overall increase in MF
during task

c2_GMRF_nback_3_Nmc1600_IS10_Reg1_g05



GaMRF

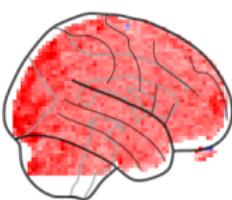
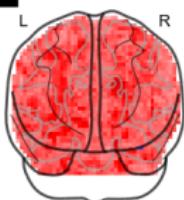
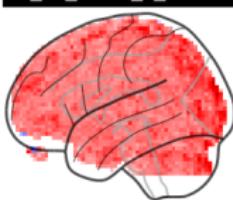
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Regularized multifractal analysis for fMRI data

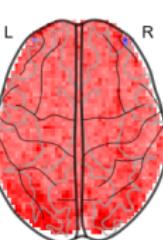
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c2_LF_nback_3_Nmc1600



Axial



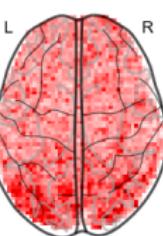
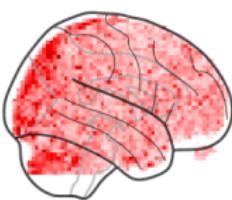
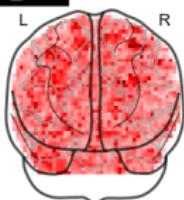
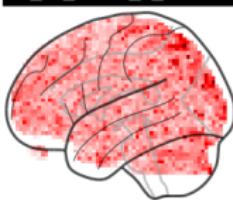
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c2_IG_nback_3_Nmc1600_IS10



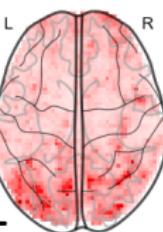
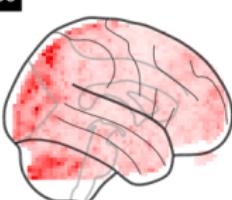
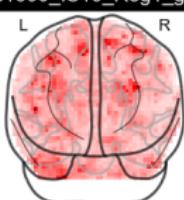
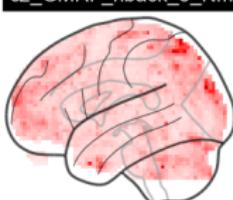
overall increase in MF during task

GaMRF:

significant MF in

- bilateral parietal regions belonging to WMN
- occipital cortex (visual)
- cerebellum (sensory) involved in task

c2_GMRF_nback_3_Nmc1600_IS10_Reg1_g05



GaMRF

→ overall MF increase; working memory network (WMN), visual, sensory.

- ▶ Novel multivoxel regularization for multifractality estimation
 - decrease in variance w.r.t. state-of-the-art voxel-wise estimation
 - characterization of scale-free temporal dynamics in fMRI
- ▶ Multifractality in fMRI (single subject data)
 - ▶ working memory task:
 - increased multifractality in brain regions involved in task
 - more irregular, bursty temporal dynamics
 - [Ciuciu FPhys'12]
 - ▶ at rest:
 - larger multifractality in Default Mode Network
 - predominance of scale-free dynamics in this circuit
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- N-Back paradigm

- Retrieve repeated letters among a sequence
- Letters serially presented (1s apart one another)
- 4 conditions of increasing difficulty:



- Stimulus sequence:

