

BRUEGEL'S DRAWINGS UNDER THE MULTIFRACTAL MICROSCOPE

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ABSTRACT

Recently, a growing interest in the exploration of the potential of signal or image processing tools for the purposes of art analysis has emerged. The wavelet leader based multifractal analysis consists of a mathematical tool recently introduced in image processing for the characterization of homogeneous textures based on their regularity properties. Here, this novel tool is applied to a set of digitized versions of drawings, made available by the NY Metropolitan Museum of Art, consisting of authentic Bruegel drawings and several imitations. Multifractal attributes are estimated from several patches of each of these drawings, and their ability to discriminate authentic drawings from impostors is investigated by means of subspace projections and quadratic discriminant analysis. Besides showing very satisfactory performance, the achieved discrimination provides interesting insights into the differences between the regularity of the textures of authentic Bruegel drawings versus imitations, potentially relating the fractal properties of the drawings to the artist's drawing style.

Index Terms— Bruegel's Drawings, Forgery Detection, Texture Analysis, Multifractal Analysis, Wavelet Leaders

1. INTRODUCTION

In a recent past, there has been a growing interest in the investigation of the potential benefits of the use of image and signal processing techniques for art analysis. This development has been driven not only by the ever increasing power of digital and computational devices, but also by a significant motivation on the part of art historians and conservators along with experts in image and signal processing, to bridge the gaps between these scientific fields.

Notably, an international research program, the *Image Processing for Art Investigation* (IP4AI) project, now gathers several international teams from the fields of Image Processing and of art history, as well as museum curators (cf. www.digitalpaintinganalysis.org/) towards this goal. The project includes several museums who have made available, under certain reproduction restrictions, partial high-resolution digitized copies of artwork for study and technology development and testing. The objective

of such a project is to explore how classical or new image processing tools may help to measure and quantify various features of artwork, sometimes called “stylometry,” which may then be involved in or partially automate an art historical or conservation task such as classification (e.g., for forgery detection), or restoration tracking, or dating (e.g., [1] for an early contribution and [2] for a review article on the topic).

Among the many ways in which computational techniques can be directed toward art analysis, texture classification represents a particularly relevant approach. Often in paintings or drawings, small patches consist of homogeneous parts representing the texture of a specific subject (e.g., landscape, forest, crowds, or skin) rather than its specific geometry. Interestingly, art experts expect that such patches of texture may reveal the *artist's hand*, that is the character of the stroke created by the artist (cf. [3] and references therein for thorough discussions of these matters).

Texture analysis can be conducted in many ways. Here, we investigate the potential for artwork analysis of texture characterization via their regularity (or fractal or scale invariance) properties. This can be effectively performed using a novel formulation of multifractal analysis referred to as the wavelet leader multifractal formalism (WLMF) [4]. Multifractal analysis for artwork analysis has been previously explored in e.g., [5], and has recently been revisited on Van Gogh's paintings [6] in a dating and forgery detection experiment.

In this contribution, we demonstrate the WLMF at work on the task of discriminating authentic Bruegel drawings from imitations. The 2D-WLMF is introduced in Section 2. The drawing data set, made available by the NY Metropolitan Museum of Art, is described in Section 3, where the fractal properties of the drawings are also investigated. In Section 4, discrimination performance based on wavelet leader multifractal attributes, obtained using projections and Quadratic Discriminant Analysis are reported and commented with respect to art expertise.

2. MULTIFRACTAL ANALYSIS

Hölder Exponent and Multifractal Spectrum. Let $X(\mathbf{x})$ (with $\mathbf{x} = (x_1, x_2)$) denote the gray level image representing the homogeneous texture of interest. It is now

well-known that its scaling and regularity properties are well-characterized by its so-called multifractal spectrum $\mathcal{D}(h)$, $0 \leq \mathcal{D}(h) \leq 2$. It consists of the Hausdorff dimensions \mathcal{D} of the set of points \mathbf{x} in the image whose local regularity takes an identical value, labeled h [7, 4]. This local regularity is mathematically defined as the Hölder exponent of X at \mathbf{x} . It is crucial to emphasize that, though based on local regularity measurements, $\mathcal{D}(h)$ conveys a global and geometrical information about the fluctuations along space of the regularity of X . To measure practically $\mathcal{D}(h)$ from a given image, one has to recourse to a multifractal formalism. Here, we make use of the wavelet leader based formalism, recently introduced and shown to have robust and favorable theoretical and practical qualities compared to previous formulations [4].

Wavelet coefficients and leaders. Let $d_X^{(m)}(j, \mathbf{k})$ denote the L^1 normalized 2D-Discrete Wavelet Transform (DWT) coefficients computed from image X using the standard recursive pyramidal algorithm, with 2D orthonormal wavelet basis obtained as tensor product of 1D wavelet basis (cf. e.g., [8, 7, 4]). Let λ_{j,k_1,k_2} denote the dyadic square $\lambda_{j,k_1,k_2} = [k_1 2^j, (k_1 + 1) 2^j) \times [k_2 2^j, (k_2 + 1) 2^j)$, and $3\lambda_{j,k_1,k_2}$ the union of λ_{j,k_1,k_2} and its 8 closest neighbours, i.e., $3\lambda_{j,k_1,k_2} = [(k_1 - 1) 2^j, (k_1 + 2) 2^j) \times [(k_2 - 1) 2^j, (k_2 + 2) 2^j)$. The wavelet leaders L_X are defined as [4]:

$$L_X(j, k_1, k_2) = \sup_{m, \lambda' \subset 3\lambda_{j,k_1,k_2}} |d_X^{(m)}(\lambda')|. \quad (1)$$

The leader $L_X(j, k_1, k_2)$ located on the node of the dyadic grid (j, k_1, k_2) is hence obtained by replacing the wavelet coefficient $d_X^{(m)}(j, k_1, k_2)$ by the largest of all the $|d_X^{(m)}(\lambda')|$ that are located at scales finer or equal to 2^j within a small neighborhood around location $(x_1 = 2^j k_1, x_2 = 2^j k_2)$.

Multifractal Formalism. Multifractal analysis is deeply related to the scale invariance properties of X in so far as its structure functions, i.e., the space averages of the q -th power of its wavelet leaders (where n_j denotes the number of wavelet coefficients actually computed at scale $a = 2^j$), behave asymptotically as power-laws with respect to the analysis scale $a = 2^j$ (in the limit of fine scales $j \rightarrow -\infty$):

$$S(2^j, q) = \frac{1}{n_j} \sum_{k_1, k_2} L_X(j, k_1, k_2)^q \sim \lambda_q 2^{j\zeta(q)}. \quad (2)$$

Such power-law behaviors and the corresponding scaling exponents $\zeta(q)$ measured from Bruegel drawings and imitations are shown in Fig. 1 (2nd and 3rd rows). Furthermore, it can be shown theoretically that the Legendre transform of the scaling function $\zeta(q)$ provides an upper bound of $\mathcal{D}(h)$: $\mathcal{D}(h) \leq \mathcal{L}(h) = \inf_{q \in \mathbb{R}} (2 + qh - \zeta(q))$. Real-life images are never available with infinite resolution. Therefore, the spectrum $\mathcal{D}(h)$ can not be computed exactly. In practice, $\mathcal{L}(h)$ is the only quantity that can actually be estimated and will (with slight abuse of language) subsequently be referred to as the multifractal spectrum. Of practical importance, the

use of the Legendre transform indicates that the full curve $\mathcal{L}(h)$ can be obtained only if both positive and negative qs are used. Estimated multifractal spectra from a Bruegel drawing and an imitation are illustrated in Fig. 1 (bottom row).

Global regularity. The wavelet coefficients $d_X^{(m)}(j, \mathbf{k})$ enable to define and measure another important regularity property of X , its global regularity h_m defined as [4]:

$$h_m = \liminf_{2^j \rightarrow 0} \log \left(\sup_{m, k_1, k_2} |d_X^{(m)}(j, k_1, k_2)| \right) / \log(2^j). \quad (3)$$

When positive, h_m corresponds to the smallest value of h that exists in X or, i.e., the leftmost point of $\mathcal{L}(h)$.

The WLMF described above applies only to functions X with $h_m > 0$. For images whose h_m is negative, the wavelet leaders need to be modified according to [4]:

$$L_X^{(\gamma)}(j, k_1, k_2) = \sup_{m, \lambda' \subset 3\lambda_{j,k_1,k_2}} |2^{\gamma j} d_X^{(m)}(\lambda')|, \quad (4)$$

with $\gamma > -h_m$. It has been shown that the $L_X^{(\gamma)}$ actually correspond to the wavelet leaders of the image X fractionally integrated, with fractional order γ (the reader is referred to [4] for details). The multifractal formalism is applied as above by replacing $L_X(j, k_1, k_2)$ with $L_X^{(\gamma)}(j, k_1, k_2)$. For thorough introductions to and details on multifractal analysis, the reader is referred to e.g., [7, 4].

Interpretation. Practically measured multifractal spectra usually consist of bell-shaped functions, often well-approximated by their parabolic expansions: $\mathcal{D}(h) = 2 - (h - c_1)^2 / (2|c_2|)$, where c_1 and $|c_2|$ are parameters modeling the position of the maximum and the typical width of $\mathcal{D}(h)$. Qualitatively, the larger c_1 the more regular the image X is globally, or, in average. When $|c_2|$ is very small, it essentially indicates that the regularity of the image is almost the same over all its pixels. Conversely, a large $|c_2|$ reveals large fluctuations of the local regularity h from one pixel to another, indicating that the texture, though globally homogeneous, yields the visual impression of densely intertwined pieces with smoothness ranging from more regular to very irregular.

Estimation. It has been shown elsewhere that the multifractal parameters $\zeta(q)$, $\mathcal{L}(h)$, h_m , c_1 , c_2 can be accurately estimated by linear regressions in suitable coordinates (cf. [4]). This collection of multifractal attributes can then be involved in usual image processing tasks, such as classification.

3. DRAWINGS: FRACTAL AND SCALING

Database. Two sets of digitized slides of drawings (2592 \times 3894 pixels, RGB Channels) consisting of 8 of Bruegel drawings and 4 known imitations (referenced in Tab. 1, not shown here for reproduction restrictions) were made available to research teams by the NY Metropolitan Museum of Art, via D. Rockmore (Dartmouth College). They are very gratefully acknowledged. A more complete description of the drawings is available in [1].

Scaling range. Because colors seem to convey a priori only little information with respect to the textures of the available drawings, only grey level intensity images are analyzed. Patches of size 1024×1024 pixels are selected in the drawings and the WLMF described above applied independently to each of them. The scaling properties and multifractal spectra are illustrated in Fig. 1 for an authentic drawing and an imitation. It is consistently found that power law behaviors, as in Eq. (2), hold, for all drawings, over 3 octaves (1 decade) covering scales 2^4 to 2^7 , i.e. 16×16 to 128×128 pixels (cf. Fig. 1, 3rd row). This corresponds to fine scales of the artwork compared to its global size and hence indicates that these fractal or scaling properties are not related to the geometry or shape of the object or subject drawn but rather stem from the *hand style* of the artist.

Fractal properties. Plots (available upon request), equivalent to those shown in Fig. 1, are obtained for all 12 drawings for at least 3 patches in each drawing. Furthermore, it is observed that estimates stemming from different patches of a single drawing are consistent (see also Fig. 2). The textures in the drawings are therefore relevantly described by their multifractal spectra $\mathcal{L}(h)$ and related attributes. Fig. 1 further suggests that imitations have multifractal spectra systematically shifted to the right compared to authentic drawings, revealing significantly more regularity in their textures.

label	MMA. cat. no.	Title	Artist
1	3	Pastoral Landscape	Bruegel
2	4	Mountain Landscape with Ridge and Valley	Bruegel
3	5	Path through a Village	Bruegel
4	6	Mule Caravan on Hillside	Bruegel
5	11	Landscape with Saint Jerome	Bruegel
6	13	Italian Landscape	Bruegel
7	20	Rest on the Flight into Egypt	Bruegel
8	9	Mountain Landscape with Ridge and Travelers	Bruegel
9	7	Mule Caravan on Hillside	unknown
10	120	Mountain Landscape with a River, Village, and Castle	unknown
11	121	Alpine Landscape	unknown
12	125	Sollicitudo Rustica	unknown

Table 1. Database. Metropolitan Museum of Art catalog number (MMA cat. no.), title, and artist. The first column gives the labels used in this work: Bruegel drawings (labels 1 to 8) and imitations (labels 9 to 12).

4. TRUE BRUEGEL'S VERSUS FORGERIES

Projections. To validate the above observation, for each drawing, 3 patches are selected to which the WLMF is applied independently. We set γ systematically to $\gamma = 0.75$ and use Daubechies Wavelet with $N_\psi = 2$ vanishing mo-

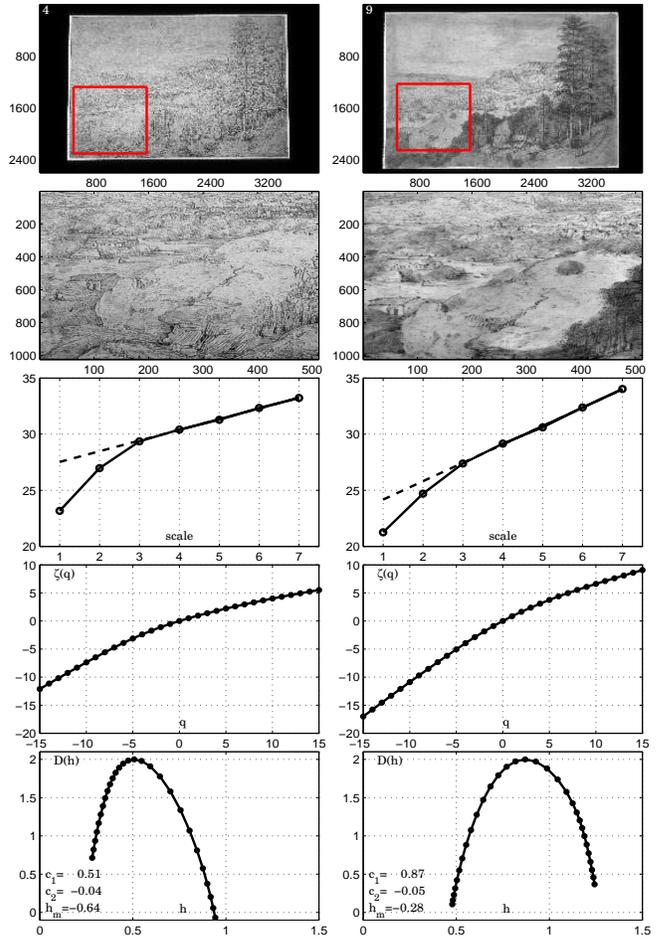


Fig. 1. Multifractal spectra of Bruegel drawing and imitation. Left column: Bruegel drawing ; Right: imitation. From top to bottom, drawings and selected patch, structure functions, scaling exponents and multifractal spectra.

ments. The patches are non overlapping and located near the bottom of the drawings, where the drawings exhibit consistently the richest textures. Projections on subspaces of pairs of multifractal attributes are reported in Fig. 2. Each patch is labeled by the number, printed in small, of the drawing identifier it belongs to, while the numbers printed in large correspond to values obtained, for each drawing, as averages of the estimates from the patches. Fig. 2 shows that h_m and c_1 globally take larger values for the imitations compared to the authentic Bruegels. Furthermore, it also suggests, with less clarity though, that the $|c_2|$ of authentic Bruegels is larger than those of imitations. Together, these results indicate that imitations show significantly more global regularity (or less variability) than the authentic Bruegels, and also display less regularity fluctuations along space. This suggests that impostors trying to reproduce Bruegel's hand style failed to replicate the strong irregularities and variabilities of the textures that characterize his drawings and likely his hand style. Interestingly, similar results were reported on differences

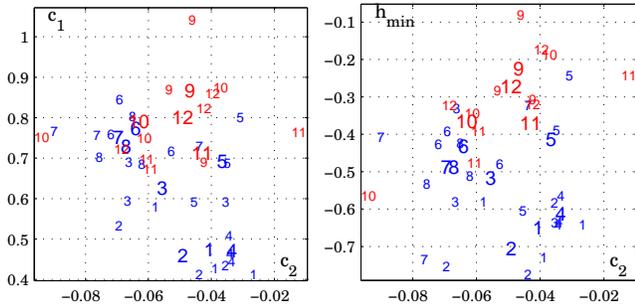


Fig. 2. Multifractal attributes. Projections of the two sets of drawings into subspaces spanned by pairs of multifractal attributes. Numbers printed in small correspond to each patch while numbers printed in large stand for the average per drawing over the 3 patches. Numbers correspond to the labels given in Tab. 1. Both projections clearly indicate that copies are globally more regular than true Bruegel's (larger c_1 and h_m). Also, copies tend to have smaller c_2 , further indication for less variability in the texture.

between original drawings and their replica, produced by one same artist in a scientific experiment (cf. [6]).

Quadratic Discriminant Analysis (QDA). To further quantify the discrimination potential of the estimated multifractal attributes, we employ a classification procedure. Given the low number of elements in each class, and the objective here not being the classification procedure itself, we chose to use QDA to perform a simple discrimination. Assuming joint Gaussian distributions with a priori different and estimated means and covariance for the two classes, QDA assigns each patch to a class according to the ratio between the corresponding log-likelihood functions. Applied to the vectors of multifractal attributes c_1, c_2, h_m , QDA yields the discrimination reported in Fig. 3. Applied to each patch independently, a perfect detection of the forgeries at the price of the misclassification of 7 out of the $24 = 8 \cdot 3$ patches of authentic Bruegels is obtained. Averaging over the patches of each drawing significantly improves the results as all imitations are detected at the price of a single false detection (Drawing 5) only. Interestingly, the use of any pair of multifractal attributes compared to the joint 3-tuple c_1, c_2, h_m decreases the classification performance, clearly showing the relevance of each parameter in fully characterizing the drawing textures.

Conclusions and perspectives. Besides showing very satisfactory discrimination performance, the WLMF based analysis motivates interesting interactions with Art experts to address more systematic questions such as: Should specific patches corresponding to particular art characteristics of the drawing receive more refined investigations? How can the scales where fractal properties are found to hold be related more precisely to the artist's hand? These questions are under current investigation and discussion with Art experts (cf. [6]). *D. Rockmore is gratefully acknowledge for making the drawings available to us, and for valuable comments on this work.*

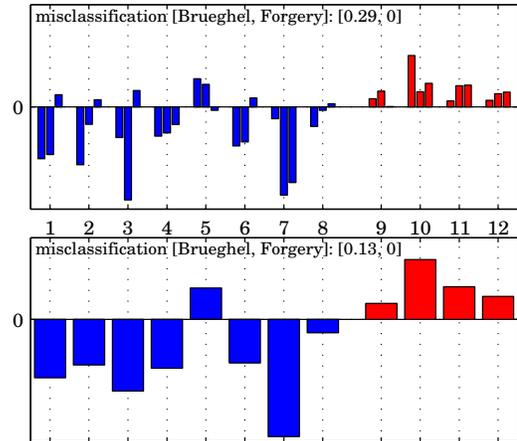


Fig. 3. Quadratic Discriminant Analysis. QDA applied to the 3D vectors of multifractal attributes c_1, c_2, h_m for all patches (top) and average over patches per drawing (bottom). Bruegel's drawings (in blue, left), Forgeries (in red, right). QDA reveals very satisfactory discrimination between true Bruegel's drawings and forgeries from their multifractal attributes, with one single true Bruegel being misclassified.

5. REFERENCES

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