Research team and supervision.

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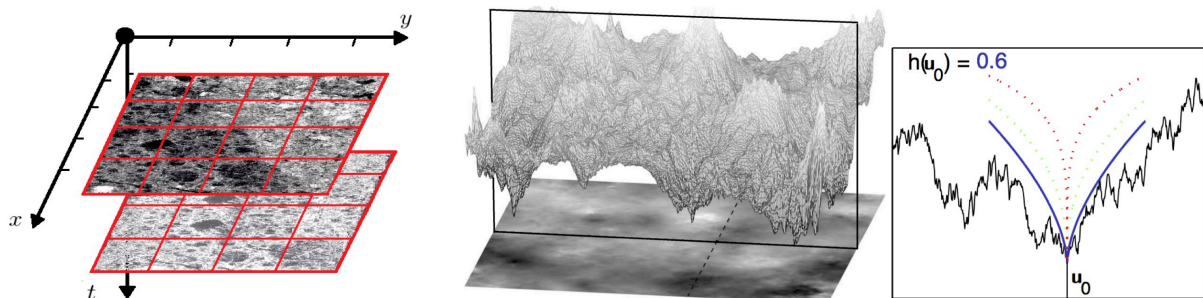CR CNRS, Signal & Communication team, IRIT Lab, INPT-ENSEEIHT, Toulouse

Scientific context.

Texture characterization and anomaly detection are ubiquitous and challenging issues in image processing and have been envisaged through many different concepts (filtering, texton, morphology, wavelets…). However, over the decades following the seminal works of Benoît Mandelbrot, an overwhelming number of signals and images have been shown to be well-characterized by scale invariance. This constitutes a major change in paradigm since the characterization can no longer rely on one specific analysis scale: Scale invariance implies that all scales play an equivalent role, and the classical signal/image processing tools which rely on the identification of characteristic scales must thus be replaced with tools that evidence and quantify the mechanisms relating scales one to the other.

Multifractal analysis has recently matured to become one of the most powerful tools for this purpose. It benefits from a well-grounded theoretical framework and a robust practical implementation which rely on the use of local suprema of wavelet coefficients in a time/space versus scale neighborhood. Multifractal analysis has been extremely successful in a large panel of applications of very different natures, ranging from biomedical over remote sensing and to man-made signals and images. However, these successes relied on the hypotheses that

     i)   data are univariate (independent analysis of one image at a time)
     ii)   data are isotropic (all directions in an image are equivalent)
     iii)   data are homogeneous (multifractal properties are the same everywhere in the image)

while these requirements are not met any longer in many modern real-world applications. Indeed, data are often naturally multivariate (dependent measurements, captured by different imaging sensors, jointly convey the information of interest), anisotropic (certain directions in the image have privileged roles) and often consist of zones, to be detected, whose properties differ from that of the rest of the data. To date, most theoretical concepts underlying multivariate anisotropic multifractality in heterogeneous data are still in their early phases of developments.

Left: a multivariate image consisting of a spatial (x,y) and temporal (t) collection of patches (in red). Multifractal analysis characterizes scale invariance of the image intensities (schematized in the center) via the fluctuation if their pointwise regularity, measured by the Hölder exponent h(u) (right).

Missions and Activities. Research program and targeted contributions.

In this context, this Ph.D has the overall double objective of i) contributing theoretically to the multifractal analysis of multivariate, anisotropic, heterogeneous images and ii) developing the practical tools that implement these formal contributions and applying them in two major fields of applications: remote sensing and biomedical imaging. This can be organized along the following lines.

Multivariate data. The coupling of data components can be as important as the information conveyed by each component individually, yet the possible formal definitions for a multifractal theory and conceptual tools for this purpose remain barely explored. Also, the actual meaning and interest of such potential multivariate definitions of multifractal in applications remain to be studied. To this end, the design of multivariate stochastic processes with controlled scale invariance properties can be considered.

Heterogeneity. To relax the assumption of identical multifractal properties throughout the image and detect heterogeneities and anomalies, a fundamental obstruction needs to be addressed: scale invariance is a non-local property that requires the use of spatial neighborhoods. The overall challenge here is thus to associate multifractal properties to regions of the image where they can be assumed homogeneous, and to estimate jointly the borders between such regions and the multifractal properties of these regions which are also unknown.

Anisotropy. Scale invariance and anisotropy are commonly encountered in real-world images, yet can not easily be modeled jointly. In essence, scale invariance implies that while zooming in on the data, one sees the same statistical properties, while anisotropy implies that statistical properties are not direction invariant and hence must change when changing the analysis scale. This apparent paradox has been partially solved when assuming a single anisotropy direction and simple scale invariant models, yet the theoretical and practical framework for multifractal models and space dependent anisotropy remains to be developed.

Selected references.

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S. Combrexelle, H. Wendt, N. Dobigeon, J.-Y. Tourneret, S. McLaughlin, P. Abry, *Bayesian Estimation of the Multifractality Parameter for Image Texture Using a Whittle Approximation*, IEEE Trans. Image Proces., vol. 24, no. 8, pp. 2540-2551, 2015.

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the .pdf of these articles are available at the web pages of the advisors.

Application.
All applications must be sent electronically to the advisors (minimum: motivation letter, CV). Only candidates with nationality from EU countries can be considered. The deadline is 31 March 2017.