

```

1 THEORY
2 IMPORT THEORY PROJECTS
3 /WaterTankTheory THEORIES /WaterTankTheory/Valves.dtf | org.eventb.theory.core.deployedTheoryRoot
4 #Valves
5 DATA TYPES
6 TankState
7 CONSTRUCTORS
8 Normal()
9 Emptying()
10 Filling()
11 Stable()
12 OPERATORS
13 isFlow <predicate> (state: TankState, DR:  $\mathbb{P}(\mathbb{RReal})$ , Phi:  $\mathbb{RRealPlus} \rightarrow \mathbb{RReal}$ , Qmin:  $\mathbb{RReal}$ , Qmax:
14  $\mathbb{RReal}$ )
15 well-definedness  $DR \subseteq \text{dom}(\text{Phi})$ 
16 isFlowODE <predicate> (state: TankState, DR:  $\mathbb{P}(\mathbb{RReal})$ , Phip:  $\mathbb{RRealPlus} \times \mathbb{RReal} \rightarrow \mathbb{RReal}$ , Qmin:  $\mathbb{RReal}$ ,
17 Qmax:  $\mathbb{RReal}$ )
18 well-definedness  $DR \times \text{Closed2Closed}(Qmin, Qmax) \subseteq \text{dom}(\text{Phip})$ 
19 isFlowEq <predicate> (state: TankState, DR:  $\mathbb{P}(\mathbb{RReal})$ , eq:  $\text{DE}(\mathbb{RReal})$ , Qmin:  $\mathbb{RReal}$ , Qmax:  $\mathbb{RReal}$ )
20 well-definedness  $\text{Solvable}(DR, \text{eq})$ 
21 direct definition
22  $\forall Q \cdot Q \in \mathbb{RRealPlus} \rightarrow \mathbb{RReal} \wedge DR \subseteq \text{dom}(Q) \wedge \text{solutionOf}(DR, Q, \text{eq}) \Rightarrow \text{isFlow}(state, DR, Q, Qmin,$ 
23  $Qmax)$ 
24 Delta <expression> (delta_in:  $\mathbb{RReal}$ , delta_out:  $\mathbb{RReal}$ )
25 well-definedness  $\text{Rzero} \mapsto \text{delta\_in} \in \text{lt}, \text{Rzero} \mapsto \text{delta\_out} \in \text{lt}$ 
26 direct definition
27  $(\lambda \text{io\_} \cdot \text{io\_} \in \text{InOutValve} \mid \text{minus}(\text{times}(\text{delta\_in} \mapsto \text{in\_rstatus}(\text{io\_})) \mapsto \text{times}(\text{delta\_out} \mapsto$ 
28  $\text{out\_rstatus}(\text{io\_})))$ 
29 flowIO <expression> (Qmin:  $\mathbb{RReal}$ , Qmax:  $\mathbb{RReal}$ , delta_in:  $\mathbb{RReal}$ , delta_out:  $\mathbb{RReal}$ )
30 well-definedness  $\text{Rzero} \mapsto Qmin \in \text{leq}, \text{Rzero} \mapsto Qmax \in \text{lt}, Qmin \mapsto Qmax \in \text{leq}, \text{Rzero} \mapsto \text{delta\_in} \in$ 
31  $\text{lt}, \text{Rzero} \mapsto \text{delta\_out} \in \text{lt}$ 
32 direct definition
33  $(\lambda \text{io\_} \cdot \text{io\_} \in \text{InOutValve} \mid$ 
34  $(\lambda \text{t0\_} \mapsto \text{q0\_} \cdot \text{t0\_} \in \mathbb{RRealPlus} \wedge \text{q0\_} \in \mathbb{RReal} \mid$ 
35  $(\lambda \text{t\_} \mapsto \text{q\_} \cdot$ 
36  $\text{t\_} \in \mathbb{RRealPlus} \wedge \text{q\_} \in \mathbb{RReal} \wedge$ 
37  $\text{plus}(\text{times}(\text{Delta}(\text{delta\_in}, \text{delta\_out})(\text{io\_}) \mapsto \text{minus}(\text{t\_} \mapsto \text{t0\_})) \mapsto \text{q0\_}) \in \text{Closed2Closed}$ 
38  $(Qmin, Qmax) \mid$ 
39  $\text{Delta}(\text{delta\_in}, \text{delta\_out})(\text{io\_})$ 
40  $) \cup$ 
41  $(\lambda \text{t\_} \mapsto \text{q\_} \cdot$ 
42  $\text{t\_} \in \mathbb{RRealPlus} \wedge \text{q\_} \in \mathbb{RReal} \wedge$ 
43  $\text{plus}(\text{times}(\text{Delta}(\text{delta\_in}, \text{delta\_out})(\text{io\_}) \mapsto \text{minus}(\text{t\_} \mapsto \text{t0\_})) \mapsto \text{q0\_}) \notin \text{Closed2Closed}$ 
44  $(Qmin, Qmax) \mid$ 
45  $\text{Rzero}$ 
46  $)$ 
47  $)$ 
48  $)$ 
49 flowIOParts <expression> (Qmin:  $\mathbb{RReal}$ , Qmax:  $\mathbb{RReal}$ , delta_in:  $\mathbb{RReal}$ , delta_out:  $\mathbb{RReal}$ , iov:
50  $\text{InOutValve}$ , t0:  $\mathbb{RRealPlus}$ , q0:  $\mathbb{RReal}$ )
51 well-definedness  $\text{Rzero} \mapsto Qmin \in \text{lt}, \text{Rzero} \mapsto Qmax \in \text{lt}, \text{Rzero} \mapsto \text{delta\_in} \in \text{lt}, \text{Rzero} \mapsto$ 
52  $\text{delta\_out} \in \text{lt}, \text{Rzero} \mapsto \text{q0} \in \text{leq}, \text{q0} \mapsto Qmax \in \text{leq}$ 
53 direct definition
54  $\{$ 
55  $\{ \text{t\_} \mid \text{t\_} \in \mathbb{RRealPlus} \wedge \text{plus}(\text{times}(\text{Delta}(\text{delta\_in}, \text{delta\_out})(\text{iov}) \mapsto \text{minus}(\text{t\_} \mapsto \text{t0})) \mapsto \text{q0}$ 
56  $) \in \text{Closed2Closed}(Qmin, Qmax) \},$ 
57  $\{ \text{t\_} \mid \text{t\_} \in \mathbb{RRealPlus} \wedge \text{plus}(\text{times}(\text{Delta}(\text{delta\_in}, \text{delta\_out})(\text{iov}) \mapsto \text{minus}(\text{t\_} \mapsto \text{t0})) \mapsto \text{q0}$ 
58  $) \mapsto Qmin \in \text{lt} \},$ 
59  $\{ \text{t\_} \mid \text{t\_} \in \mathbb{RRealPlus} \wedge Qmax \mapsto \text{plus}(\text{times}(\text{Delta}(\text{delta\_in}, \text{delta\_out})(\text{iov}) \mapsto \text{minus}(\text{t\_} \mapsto \text{t0}$ 
60  $)) \mapsto \text{q0}) \in \text{lt} \}$ 
61  $\}$ 
62 FlowIOODE <expression> (Qmin:  $\mathbb{RReal}$ , Qmax:  $\mathbb{RReal}$ , delta_in:  $\mathbb{RReal}$ , delta_out:  $\mathbb{RReal}$ )
63 well-definedness  $\text{Rzero} \mapsto Qmin \in \text{leq}, \text{Rzero} \mapsto Qmax \in \text{lt}, Qmin \mapsto Qmax \in \text{leq}, \text{Rzero} \mapsto \text{delta\_in} \in$ 
64  $\text{lt}, \text{Rzero} \mapsto \text{delta\_out} \in \text{lt}$ 
65 direct definition
66  $(\lambda \text{io\_} \cdot \text{io\_} \in \text{InOutValve} \mid$ 
67  $(\lambda \text{t0\_} \mapsto \text{q0\_} \cdot \text{t0\_} \in \mathbb{RRealPlus} \wedge \text{q0\_} \in \mathbb{RReal} \mid$ 
68  $\text{ode}(\text{flowIO}(Qmin, Qmax, \text{delta\_in}, \text{delta\_out})(\text{io\_})(\text{t0\_} \mapsto \text{q0\_}), \text{q0\_}, \text{t0\_})$ 
69  $)$ 
70  $)$ 
71 SingleTankPolicy <expression> (mode: TankState)
72 NoFlow <expression> ()
73 direct definition

```

($\lambda t \mapsto Q \cdot t \in \mathbb{R}\text{RealPlus} \wedge Q \in \mathbb{R}\text{Real} \mid \mathbb{R}\text{zero}$)

THEOREMS

flowode_yields_flow:

$\forall DR, Q_{\min}, Q_{\max}, \text{Phip}, st, t_0, q_0 \cdot$

$DR \subseteq \mathbb{R}\text{RealPlus} \wedge$

$Q_{\min} \in \mathbb{R}\text{Real} \wedge \mathbb{R}\text{zero} \mapsto Q_{\min} \in \text{leq} \wedge$

$Q_{\max} \in \mathbb{R}\text{Real} \wedge \mathbb{R}\text{zero} \mapsto Q_{\max} \in \text{lt} \wedge$

$Q_{\min} \mapsto Q_{\max} \in \text{leq} \wedge$

$st \in \text{TankState} \wedge$

$t_0 \in DR \wedge q_0 \in \mathbb{R}\text{Real} \wedge$

$\text{Phip} \in \mathbb{R}\text{RealPlus} \times \mathbb{R}\text{Real} \mapsto \mathbb{R}\text{Real} \wedge$

$DR \times \text{Closed2Closed}(Q_{\min}, Q_{\max}) \subseteq \text{dom}(\text{Phip}) \wedge$

$\text{isFlowODE}(st, DR, \text{Phip}, Q_{\min}, Q_{\max}) \wedge$

$\text{Solvable}(DR, \text{ode}(\text{Phip}, q_0, t_0))$

\Rightarrow (

$\forall \text{Phi} \cdot \text{Phi} \in \mathbb{R}\text{RealPlus} \mapsto \mathbb{R}\text{Real} \wedge DR \subseteq \text{dom}(\text{Phi}) \wedge \text{solutionOf}(DR, \text{Phi}, \text{ode}(\text{Phip}, q_0, t_0)) \Rightarrow$
 $\text{isFlow}(st, DR, \text{Phi}, Q_{\min}, Q_{\max})$

)

flow_type:

$\forall Q_{\min}, Q_{\max}, \text{delta_in}, \text{delta_out}, \text{iov}, t_0, q_0 \cdot$

$Q_{\min} \in \mathbb{R}\text{Real} \wedge \mathbb{R}\text{zero} \mapsto Q_{\min} \in \text{leq} \wedge$

$Q_{\max} \in \mathbb{R}\text{Real} \wedge \mathbb{R}\text{zero} \mapsto Q_{\max} \in \text{lt} \wedge$

$Q_{\min} \mapsto Q_{\max} \in \text{leq} \wedge$

$\text{delta_in} \in \mathbb{R}\text{Real} \wedge \mathbb{R}\text{zero} \mapsto \text{delta_in} \in \text{lt} \wedge$

$\text{delta_out} \in \mathbb{R}\text{Real} \wedge \mathbb{R}\text{zero} \mapsto \text{delta_out} \in \text{lt} \wedge$

$\text{iov} \in \text{InOutValve} \wedge$

$t_0 \in \mathbb{R}\text{RealPlus} \wedge$

$q_0 \in \mathbb{R}\text{Real} \Rightarrow$ (

$\text{flowIO}(Q_{\min}, Q_{\max}, \text{delta_in}, \text{delta_out})(\text{iov})(t_0 \mapsto q_0) \in \mathbb{R}\text{RealPlus} \times \mathbb{R}\text{Real} \rightarrow \mathbb{R}\text{Real}$

)

flow_lipschitz:

$\forall Q_{\min}, Q_{\max}, \text{delta_in}, \text{delta_out}, \text{iov}, t_0, q_0 \cdot$

$Q_{\min} \in \mathbb{R}\text{Real} \wedge \mathbb{R}\text{zero} \mapsto Q_{\min} \in \text{leq} \wedge$

$Q_{\max} \in \mathbb{R}\text{Real} \wedge \mathbb{R}\text{zero} \mapsto Q_{\max} \in \text{lt} \wedge$

$Q_{\min} \mapsto Q_{\max} \in \text{leq} \wedge$

$\text{delta_in} \in \mathbb{R}\text{Real} \wedge \mathbb{R}\text{zero} \mapsto \text{delta_in} \in \text{lt} \wedge$

$\text{delta_out} \in \mathbb{R}\text{Real} \wedge \mathbb{R}\text{zero} \mapsto \text{delta_out} \in \text{lt} \wedge$

$\text{iov} \in \text{InOutValve} \wedge$

$t_0 \in \mathbb{R}\text{RealPlus} \wedge$

$q_0 \in \mathbb{R}\text{Real} \wedge \mathbb{R}\text{zero} \mapsto q_0 \in \text{leq} \wedge q_0 \mapsto Q_{\max} \in \text{leq} \Rightarrow$ (

$\forall t \cdot t \in \mathbb{R}\text{RealPlus} \Rightarrow$

$\text{lipschitzContinuous}(\mathbb{R}\text{Real}, \mathbb{R}\text{Real}, \text{partial2}(\text{flowIO}(Q_{\min}, Q_{\max}, \text{delta_in}, \text{delta_out})(\text{iov})(t_0 \mapsto$
 $q_0), t))$

)

flow_piecewise_continuous:

$\forall Q_{\min}, Q_{\max}, \text{delta_in}, \text{delta_out}, \text{iov}, t_0, q_0 \cdot$

$Q_{\min} \in \mathbb{R}\text{Real} \wedge \mathbb{R}\text{zero} \mapsto Q_{\min} \in \text{leq} \wedge$

$Q_{\max} \in \mathbb{R}\text{Real} \wedge \mathbb{R}\text{zero} \mapsto Q_{\max} \in \text{lt} \wedge$

$Q_{\min} \mapsto Q_{\max} \in \text{leq} \wedge$

$\text{delta_in} \in \mathbb{R}\text{Real} \wedge \mathbb{R}\text{zero} \mapsto \text{delta_in} \in \text{lt} \wedge$

$\text{delta_out} \in \mathbb{R}\text{Real} \wedge \mathbb{R}\text{zero} \mapsto \text{delta_out} \in \text{lt} \wedge$

$\text{iov} \in \text{InOutValve} \wedge$

$t_0 \in \mathbb{R}\text{RealPlus} \wedge$

$q_0 \in \mathbb{R}\text{Real} \wedge \mathbb{R}\text{zero} \mapsto q_0 \in \text{leq} \wedge q_0 \mapsto Q_{\max} \in \text{leq} \Rightarrow$ (

$\text{partialPiecewiseContinuous}$ (

$\text{flowIOParts}(Q_{\max}, Q_{\min}, \text{delta_in}, \text{delta_out}, \text{iov}, t_0, q_0),$

$\mathbb{R}\text{Real}, \mathbb{R}\text{Real},$

$\text{flowIO}(Q_{\min}, Q_{\max}, \text{delta_in}, \text{delta_out})(\text{iov})(t_0 \mapsto q_0)$

)

)

flow_pw_CL_cond:

$\forall Q_{\min}, Q_{\max}, \text{delta_in}, \text{delta_out}, \text{iov}, t_0, q_0 \cdot$

$Q_{\min} \in \mathbb{R}\text{Real} \wedge \mathbb{R}\text{zero} \mapsto Q_{\min} \in \text{leq} \wedge$

$Q_{\max} \in \mathbb{R}\text{Real} \wedge \mathbb{R}\text{zero} \mapsto Q_{\max} \in \text{lt} \wedge$

$Q_{\min} \mapsto Q_{\max} \in \text{leq} \wedge$

$\text{delta_in} \in \mathbb{R}\text{Real} \wedge \mathbb{R}\text{zero} \mapsto \text{delta_in} \in \text{lt} \wedge$

$\text{delta_out} \in \mathbb{R}\text{Real} \wedge \mathbb{R}\text{zero} \mapsto \text{delta_out} \in \text{lt} \wedge$

$\text{iov} \in \text{InOutValve} \wedge$

$t_0 \in \mathbb{R}\text{RealPlus} \wedge$

$q_0 \in \mathbb{R}\text{Real} \wedge \mathbb{R}\text{zero} \mapsto q_0 \in \text{leq} \wedge q_0 \mapsto Q_{\max} \in \text{leq} \Rightarrow$ (

$\text{PiecewiseCauchyLipschitzCondition}$ (

$\text{flowIOParts}(Q_{\min}, Q_{\max}, \text{delta_in}, \text{delta_out}, \text{iov}, t_0, q_0),$

$\mathbb{R}\text{Real},$

$\text{FlowIOODE}(Q_{\min}, Q_{\max}, \text{delta_in}, \text{delta_out})(\text{iov})(t_0 \mapsto q_0)$

)

```

133     )
134   )
135   flowio_is_flowode_single_tank_policy:
136   ∀ DR, Qmin, Qmax, delta_in, delta_out, state ·
137     DR ⊆ RRealPlus ∧
138     Qmin ∈ RReal ∧ Rzero ⇒ Qmin ∈ leq ∧
139     Qmax ∈ RReal ∧ Rzero ⇒ Qmax ∈ lt ∧
140     Qmin ⇒ Qmax ∈ leq ∧
141     delta_in ∈ RReal ∧ Rzero ⇒ delta_in ∈ lt ∧
142     delta_out ∈ RReal ∧ Rzero ⇒ delta_out ∈ lt ∧
143     state ∈ TankState
144   ⇒ (
145     ∀ iov, t0, q0 ·
146       iov ∈ SingleTankPolicy(state) ∧
147       t0 ∈ DR ∧ q0 ∈ RReal ∧
148       Qmin ⇒ q0 ∈ leq ∧ q0 ⇒ Qmax ∈ leq
149     ⇒
150       isFlowODE(state, DR, flowIO(Qmin, Qmax, delta_in, delta_out)(iov)(t0 ⇒ q0), Qmin, Qmax)
151   )
152   flow_LC:
153   ∀ DR,
154     Phi1, l, Q1min, Q1max,
155     Phi2, m, Q2min, Q2max ·
156     DR ⊆ RReal ∧
157     l ∈ RReal ∧ Rzero ⇒ l ∈ lt ∧
158     Q1min ∈ RReal ∧ Q1max ∈ RReal ∧ Q1min ⇒ Q1max ∈ leq ∧
159     Rzero ⇒ Q1min ∈ leq ∧ Rzero ⇒ Q1max ∈ lt ∧
160     Phi1 ∈ RRealPlus ⇒ RReal ∧ DR ⊆ dom(Phi1) ∧
161     m ∈ RReal ∧ Rzero ⇒ m ∈ lt ∧
162     Q2min ∈ RReal ∧ Q2max ∈ RReal ∧ Q2min ⇒ Q2max ∈ leq ∧
163     Rzero ⇒ Q2min ∈ leq ∧ Rzero ⇒ Q2max ∈ lt ∧
164     Phi2 ∈ RRealPlus ⇒ RReal ∧ DR ⊆ dom(Phi2) ⇒ (
165       (∀ st · st ∈ TankState ⇒ isFlow(st, DR, Phi1, Q1min, Q1max) ∧ isFlow(st, DR, Phi2, Q2min, Q2max))
166     ⇔
167       (∀ st · st ∈ TankState ⇒ isFlow(
168         st,
169         DR,
170         LinComb2(1, Phi1, m, Phi2),
171         sLinComb2(1, Q1min, m, Q2min),
172         sLinComb2(1, Q1max, m, Q2max)
173       )
174     )
175   )
176   flowODE_LC:
177   ∀ DR,
178     Phip1, l, Q1min, Q1max,
179     Phip2, m, Q2min, Q2max ·
180     DR ⊆ RReal ∧
181     Q1min ∈ RReal ∧ Q1max ∈ RReal ∧ Q1min ⇒ Q1max ∈ leq ∧
182     Rzero ⇒ Q1min ∈ leq ∧ Rzero ⇒ Q1max ∈ lt ∧
183     Phip1 ∈ RRealPlus × RReal ⇒ RReal ∧ DR × Closed2Closed(Q1min, Q1max) ⊆ dom(Phip1) ∧
184     m ∈ RReal ∧ Rzero ⇒ m ∈ lt ∧
185     Q2min ∈ RReal ∧ Q2max ∈ RReal ∧ Q2min ⇒ Q2max ∈ leq ∧
186     Rzero ⇒ Q2min ∈ leq ∧ Rzero ⇒ Q2max ∈ lt ∧
187     Phip2 ∈ RRealPlus × RReal ⇒ RReal ∧ DR × Closed2Closed(Q2min, Q2max) ⊆ dom(Phip2) ⇒ (
188       (∀ st · st ∈ TankState ⇒ isFlowODE(st, DR, Phip1, Q1min, Q1max) ∧ isFlowODE(st, DR, Phip2, Q2min, Q2max))
189     ⇔
190       (∀ st · st ∈ TankState ⇒ isFlowODE(
191         st,
192         DR,
193         LinComb2(1, Phip1, m, Phip2),
194         sLinComb2(1, Q1min, m, Q2min),
195         sLinComb2(1, Q1max, m, Q2max)
196       )
197     )
198   )
199   flowode_is_floweq:
200   ∀ DR, Qmin, Qmax, Phip, st, t0, q0 ·
201     DR ⊆ RRealPlus ∧
202     Qmin ∈ RReal ∧ Rzero ⇒ Qmin ∈ leq ∧
203     Qmax ∈ RReal ∧ Rzero ⇒ Qmax ∈ lt ∧
204     Qmin ⇒ Qmax ∈ leq ∧
205     st ∈ TankState ∧

```

```
206 t0 ∈ DR ∧ q0 ∈ RReal ∧
207 Phip ∈ RRealPlus×RReal → RReal ∧ DR×Closed2Closed(Qmin,Qmax) ⊆ dom(Phip) ∧
208 isFlowODE(st,DR,Phip,Qmin,Qmax) ∧
209 Solvable(DR,ode(Phip,q0,t0))
210 ⇒
211 isFlowEq(st,DR,ode(Phip,q0,t0),Qmin,Qmax)
212 END
```