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1 THEORY Ring
2 IMPORT THEORY PROJECTS
3 / SimpleDEq THEORIES / SimpleDEq/Group . dtf | org . eventb . theory . core . deployedTheoryRoot#Group
4 TYPE PARAMETERS A
5 OPERATORS
6 distributive <predicate> (oplus: (A × A) → A, otimes: (A × A) → A)
7   well-definedness A ≠ ∅
8   direct definition
9     ∀ x, y, z · x ∈ A ∧ y ∈ A ∧ z ∈ A ⇒ (
10       otimes(x ↪ oplus(y ↪ z)) = oplus(otimes(x ↪ y) ↪ otimes(x ↪ z)) ∧
11       otimes(oplus(y ↪ z) ↪ x) = oplus(otimes(y ↪ x) ↪ otimes(z ↪ x))
12     )
13 integral <predicate> (otimes: (A × A) → A, azero: A)
14   well-definedness A ≠ ∅ ∧ A ≠ {azero}
15   direct definition
16     ∀ x, y · x ∈ A ∧ y ∈ A ⇒ (otimes(x ↪ y) = azero ⇒ (x = azero ∨ y = azero))
17 Ring <predicate> (oplus: (A × A) → A, otimes: (A × A) → A, azero: A, aunit: A)
18   well-definedness A ≠ ∅
19   direct definition
20     AbelianGroup(oplus, azero) ∧ Monoid(otimes, aunit) ∧ distributive(oplus, otimes)
21 CommutativeRing <predicate> (oplus: (A × A) → A, otimes: (A × A) → A, azero: A, aunit: A)
22   well-definedness A ≠ ∅
23   direct definition
24     Ring(oplus, otimes, azero, aunit) ∧ commutative(otimes)
25 nonZeroInvertible <predicate> (op: (A × A) → A, azero: A, aunit: A)
26   well-definedness A ≠ ∅ ∧ (azero ≠ aunit)
27   direct definition
28     ∀ x · x ∈ A ∧ x ≠ azero ⇒ (∃ y · y ∈ A ∧ y ≠ azero ⇒ (op(x ↪ y) = aunit ∧ op(y ↪ x) = aunit))
29 nonZeroInverses <predicate> (op: (A × A) → A, azero: A, aunit: A, x: A, y: A)
30   well-definedness A ≠ ∅ ∧ (azero ≠ aunit)
31   direct definition
32     nonZeroInvertible(op, azero, aunit) ∧ x ≠ azero ⇒ ((op(x ↪ y) = aunit) ∧ (op(y ↪ x) = aunit))
33 DivisionRing <predicate> (oplus: (A × A) → A, otimes: (A × A) → A, azero: A, aunit: A)
34   well-definedness A ≠ ∅
35   direct definition
36     (azero ≠ aunit) ∧ Ring(oplus, otimes, azero, aunit) ∧ nonZeroInvertible(otimes, azero, aunit)
37 Field <predicate> (oplus: (A × A) → A, otimes: (A × A) → A, azero: A, aunit: A)
38   well-definedness A ≠ ∅
39   direct definition
40     (azero ≠ aunit) ∧
41     Ring(oplus, otimes, azero, aunit) ∧
42     nonZeroInvertible(otimes, azero, aunit) ∧
43     integral(otimes, azero) ∧
44     commutative(otimes)
45 absorbing <predicate> (op: (A × A) → A, azero: A)
46   direct definition
47     ∀ x · x ∈ A ⇒ (op(x ↪ azero) = azero ∧ op(azero ↪ x) = azero)
48 THEOREMS
49 zeroAbsorbing:
50   ∀ oplus, otimes, azero, aunit · oplus ∈ ((A × A) → A) ∧ otimes ∈ ((A × A) → A) ∧ azero ∈ A ∧ aunit
51     ∈ A ∧
52     Ring(oplus, otimes, azero, aunit) ⇒ absorbing(otimes, azero)
53 plusInverseLeftDistribution:
54   ∀ oplus, otimes, azero, aunit · oplus ∈ ((A × A) → A) ∧ otimes ∈ ((A × A) → A) ∧ azero ∈ A ∧ aunit
55     ∈ A ∧ Ring(oplus, otimes, azero, aunit) ⇒ (
56     ∀ a, b, a1 · a ∈ A ∧ b ∈ A ∧ a1 ∈ A ∧ inverses(oplus, azero, a, a1) ⇒ (
57       inverses(oplus, azero, otimes(a ↪ b), otimes(a1 ↪ b))
58     )
59 plusInverseRightDistribution:
60   ∀ oplus, otimes, azero, aunit · oplus ∈ ((A × A) → A) ∧ otimes ∈ ((A × A) → A) ∧ azero ∈ A ∧ aunit
61     ∈ A ∧ Ring(oplus, otimes, azero, aunit) ⇒ (
62     ∀ a, b, b1 · a ∈ A ∧ b ∈ A ∧ b1 ∈ A ∧ inverses(oplus, azero, b, b1) ⇒ (
63       inverses(oplus, azero, otimes(a ↪ b), otimes(a ↪ b1))
64     )
65 ringLeftCancellation:
66   ∀ oplus, otimes, azero, aunit · oplus ∈ ((A × A) → A) ∧ otimes ∈ ((A × A) → A) ∧ azero ∈ A ∧ aunit
67     ∈ A ∧ azero ≠ aunit ∧ Ring(oplus, otimes, azero, aunit) ∧ integral(otimes, azero) ⇒ (
68     ∀ a, b, c · a ∈ A ∧ b ∈ A ∧ c ∈ A ∧ a ≠ azero ⇒
69       ((otimes(a ↪ b) = otimes(a ↪ c)) ⇔ (b = c)))

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69   ringRightCancellation:
70      $\forall \text{oplus}, \text{otimes}, \text{azero}, \text{aunit} . \text{oplus} \in ((A \times A) \rightarrow A) \wedge \text{otimes} \in ((A \times A) \rightarrow A) \wedge \text{azero} \in A \wedge \text{aunit}$ 
71      $\in A \wedge \text{azero} \neq \text{aunit} \wedge \text{Ring}(\text{oplus}, \text{otimes}, \text{azero}, \text{aunit}) \wedge \text{integral}(\text{otimes}, \text{azero}) \Rightarrow ($ 
72      $\forall a, b, c . a \in A \wedge b \in A \wedge c \in A \wedge a \neq \text{azero} \Rightarrow$ 
73      $((\text{otimes}(b \mapsto a)) = \text{otimes}(c \mapsto a)) \Leftrightarrow (b = c))$ 
74   nonZeroInverseNotZero:
75      $\forall \text{oplus}, \text{otimes}, \text{azero}, \text{aunit}, x, y .$ 
76      $\text{oplus} \in ((A \times A) \rightarrow A) \wedge \text{otimes} \in ((A \times A) \rightarrow A) \wedge \text{azero} \in A \wedge \text{aunit} \in A \wedge$ 
77      $\text{Ring}(\text{oplus}, \text{otimes}, \text{azero}, \text{aunit}) \wedge \text{nonZeroInvertible}(\text{otimes}, \text{azero}, \text{aunit}) \wedge$ 
78      $x \in A \wedge y \in A \wedge x \neq \text{azero} \wedge \text{nonZeroInverses}(\text{otimes}, \text{azero}, \text{aunit}, x, y) \Rightarrow ($ 
79      $y \neq \text{azero}$ 
80   )
81 END

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