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1 THEORY Reals
2 IMPORT THEORY PROJECTS
3 /SimpleDEq THEORIES /SimpleDEq/Relation.dtf|org.eventb.theory.core.deployedTheoryRoot#Relation
4 AXIOMATIC DEFINITIONS Rdef:
5 TYPES RReal
6 OPERATORS
7 Rzero <expression> () : RReal
8 Rone <expression> () : RReal
9 Rtwo <expression> () : RReal
10 AXIOMS
11 oneIsNotZero:
12 Rone ≠ Rzero
13 twoIsNotOne:
14 Rtwo ≠ Rone
15 twoIsNotZero:
16 Rtwo ≠ Rzero Rorder:
17 OPERATORS
18 leq <expression> () : RReal ↔ RReal
19 lt <expression> () : RReal ↔ RReal
20 geq <expression> () : RReal ↔ RReal
21 gt <expression> () : RReal ↔ RReal
22 Rmax <expression> (A: ℙ(RReal)) : RReal
23 Rmin <expression> (A: ℙ(RReal)) : RReal
24 Rsup <expression> (A: ℙ(RReal)) : RReal
25 Rinf <expression> (A: ℙ(RReal)) : RReal
26 AXIOMS
27 leqDef:
28 order(leq)
29 leqTotal:
30 total(leq)
31 ltDef:
32 lt = strict(leq)
33 geqDef:
34 geq = converse(leq)
35 gtDef:
36 gt = strict(geq)
37 supremumProperty:
38 ∀ P · P ∈ ℙ(RReal) ∧ P ≠ ∅ ⇒ (
39 (∃ m · m ∈ RReal ∧ upperBound(leq, P, m)) ⇒
40 (∃ B · B ∈ RReal ∧ supremum(leq, P, B))
41 )
42 zeroLtOne:
43 (Rzero ↦ Rone) ∈ lt
44 oneLtTwo:
45 (Rone ↦ Rtwo) ∈ lt
46 RmaxDef:
47 ∀ S · S ⊆ RReal ∧ hasMaximum(leq, S) ⇒ (Rmax(S) = Gmax(leq, S))
48 RminDef:
49 ∀ S · S ⊆ RReal ∧ hasMinimum(leq, S) ⇒ (Rmin(S) = Gmin(leq, S))
50 RsupDef:
51 ∀ S · S ⊆ RReal ∧ upperBounded(leq, S) ⇒ (Rsup(S) = Gsup(leq, S))
52 RinfDef:
53 ∀ S · S ⊆ RReal ∧ lowerBounded(leq, S) ⇒ (Rinf(S) = Ginf(leq, S)) Rparts:
54 OPERATORS
55 RRealStar <expression> () : ℙ(RReal)
56 RRealPlus <expression> () : ℙ(RReal)
57 RRealMinus <expression> () : ℙ(RReal)
58 RRealPlusStar <expression> () : ℙ(RReal)
59 RRealMinusStar <expression> () : ℙ(RReal)
60 AXIOMS
61 realStarDef:
62 RRealStar = { x | x ∈ RReal ∧ x ≠ Rzero }
63 realPlusDef:
64 RRealPlus = { x | Rzero ↦ x ∈ leq }
65 realMinusDef:
66 RRealMinus = { x | x ↦ Rzero ∈ leq }
67 realPlusStarDef:
68 RRealPlusStar = { x | Rzero ↦ x ∈ lt }
69 realMinusStarDef:
70 RRealMinusStar = { x | x ↦ Rzero ∈ lt } Roperators:
71 OPERATORS
72 plus <expression> () : ℙ(RReal×RReal×RReal)
73 times <expression> () : ℙ(RReal×RReal×RReal)

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74 **uminus** <expression> () : $\mathbb{P}(\mathbb{RReal} \times \mathbb{RReal})$
75 **inverse** <expression> () : $\mathbb{P}(\mathbb{RReal} \times \mathbb{RReal})$
76 **minus** <expression> () : $\mathbb{P}(\mathbb{RReal} \times \mathbb{RReal} \times \mathbb{RReal})$
77 **divide** <expression> () : $\mathbb{P}(\mathbb{RReal} \times \mathbb{RReal} \times \mathbb{RReal})$
78 **abs** <expression> () : $\mathbb{P}(\mathbb{RReal} \times \mathbb{RReal})$
79 **sqrt** <expression> () : $\mathbb{P}(\mathbb{RReal} \times \mathbb{RReal})$

AXIOMS

81 *plusType*:
82 $\text{plus} \in ((\mathbb{RReal} \times \mathbb{RReal}) \rightarrow \mathbb{RReal})$
83 *timesType*:
84 $\text{times} \in ((\mathbb{RReal} \times \mathbb{RReal}) \rightarrow \mathbb{RReal})$
85 *uminusType*:
86 $\text{uminus} \in (\mathbb{RReal} \Rightarrow \mathbb{RReal})$
87 *inverseType*:
88 $\text{inverse} \in (\mathbb{RRealStar} \Rightarrow \mathbb{RRealStar})$
89 *minusType*:
90 $\text{minus} \in ((\mathbb{RReal} \times \mathbb{RReal}) \rightarrow \mathbb{RReal})$
91 *divideType*:
92 $\text{divide} \in ((\mathbb{RReal} \times \mathbb{RRealStar}) \rightarrow \mathbb{RReal})$
93 *uminusDef*:
94 $\forall x \cdot x \in \mathbb{RReal} \Rightarrow (\text{inverses}(\text{plus}, \text{Rzero}, x, \text{uminus}(x)))$
95 *minusDef*:
96 $\forall x, y \cdot x \in \mathbb{RReal} \wedge y \in \mathbb{RReal} \Rightarrow ($\text{minus}(x \mapsto y) = \text{plus}(x \mapsto \text{uminus}(y))$)$
97)
98 *inverseDef*:
99 $\forall x \cdot x \in \mathbb{RReal} \wedge x \neq \text{Rzero} \Rightarrow ($\text{times}(x \mapsto \text{inverse}(x)) = \text{Rone}$)$
100)
101 *divideDef*:
102 $\forall x, y \cdot x \in \mathbb{RReal} \wedge y \in \mathbb{RReal} \wedge y \neq \text{Rzero} \Rightarrow ($\text{divide}(x \mapsto y) = \text{times}(x \mapsto \text{inverse}(y))$)$
103)
104 *absType*:
105 $\text{abs} \in (\mathbb{RReal} \rightarrow \mathbb{RRealPlus})$
106 *absPos*:
107 $\forall x \cdot x \in \mathbb{RReal} \Rightarrow (\text{Rzero} \mapsto \text{abs}(x) \in \text{leq})$
108 *absZero*:
109 $\forall x \cdot x \in \mathbb{RReal} \Rightarrow ((x = \text{Rzero}) \Leftrightarrow (\text{abs}(x) = \text{Rzero}))$
110 *absDef*:
111 $\forall x \cdot x \in \mathbb{RReal} \Rightarrow ($(x \mapsto \text{Rzero} \in \text{lt}) \Rightarrow (\text{abs}(x) = \text{uminus}(x)) \wedge$
112 $(\text{Rzero} \mapsto x \in \text{lt}) \Rightarrow (\text{abs}(x) = x)$)$
113)
114 *absTriangular*:
115 $\forall x, y \cdot x \in \mathbb{RReal} \wedge y \in \mathbb{RReal} \Rightarrow ($\text{abs}(\text{plus}(x \mapsto y)) \mapsto \text{plus}(\text{abs}(x) \mapsto \text{abs}(y)) \in \text{leq}$)$
116)
117 *absMinus*:
118 $\forall x \cdot x \in \mathbb{RReal} \Rightarrow (\text{abs}(\text{uminus}(x)) = \text{abs}(x))$
119 *absMult*:
120 $\forall x, y \cdot x \in \mathbb{RReal} \wedge y \in \mathbb{RReal} \Rightarrow ($\text{abs}(\text{times}(x \mapsto y)) = \text{times}(\text{abs}(x) \mapsto \text{abs}(y))$)$
121)
122 *sqrtType*:
123 $\text{sqrt} \in (\mathbb{RRealPlus} \Rightarrow \mathbb{RRealPlus})$
124 *sqrtDef*:
125 $\forall x \cdot x \in \mathbb{RRealPlus} \Rightarrow (\text{times}(\text{sqrt}(x) \mapsto \text{sqrt}(x)) = x)$
126 *twoDef*:
127 $\text{Rtwo} = \text{plus}(\text{Rone} \mapsto \text{Rone})$ *Rstructure*:

AXIOMS

128 *realField*:
129 $\text{Field}(\text{plus}, \text{times}, \text{Rzero}, \text{Rone})$
130 *realIntegrity*:
131 $\text{integral}(\text{times}, \text{Rzero})$
132 *ringCompatibility*:
133 $\text{ringCompatible}(\text{plus}, \text{times}, \text{Rzero}, \text{Rone}, \text{leq})$

THEOREMS

134 *RzeroLtRtwo*:
135 $\text{Rzero} \mapsto \text{Rtwo} \in \text{lt}$
136 *uminusOrderInversion*:
137 $\forall x \cdot x \in \mathbb{RReal} \wedge (\text{Rzero} \mapsto x \in \text{leq}) \Rightarrow (\text{uminus}(x) \mapsto \text{Rzero} \in \text{leq})$
138 *uminusInvolutive*:
139 $\forall x \cdot x \in \mathbb{RReal} \Rightarrow (\text{uminus}(\text{uminus}(x)) = x)$

148 *uminusNeutrality*:
149 $\text{uminus}(\text{Rzero}) = \text{Rzero}$
150 *inverseInvolutive*:
151 $\forall x \cdot x \in \text{RReal} \wedge x \neq \text{Rzero} \Rightarrow (\text{inverse}(\text{inverse}(x)) = x)$
152 *sqrtTimesDistrib*:
153 $\forall x, y \cdot x \in \text{RRealPlus} \wedge y \in \text{RRealPlus} \Rightarrow (\text{sqrt}(\text{times}(x \mapsto y)) = \text{times}(\text{sqrt}(x) \mapsto \text{sqrt}(y)))$
154)
155)
156 *sqrtOne*:
157 $\text{sqrt}(\text{Rone}) = \text{Rone}$
158 *sqrtZero*:
159 $\text{sqrt}(\text{Rzero}) = \text{Rzero}$
160 *realNotUpperBounded*:
161 $\forall x \cdot x \in \text{RReal} \Rightarrow (\exists x2 \cdot x2 \in \text{RReal} \wedge x \mapsto x2 \in \text{lt})$
162 *realNotLowerBounded*:
163 $\forall x \cdot x \in \text{RReal} \Rightarrow (\exists x2 \cdot x2 \in \text{RReal} \wedge x2 \mapsto x \in \text{lt})$
164 *realPlusNotBounded*:
165 $\forall x \cdot x \in \text{RRealPlus} \Rightarrow (\exists x2 \cdot x2 \in \text{RRealPlus} \wedge x \mapsto x2 \in \text{lt})$
166 *realMinusNotBounded*:
167 $\forall x \cdot x \in \text{RRealMinus} \Rightarrow (\exists x2 \cdot x2 \in \text{RRealMinus} \wedge x2 \mapsto x \in \text{lt})$
168 *realPlusOrder*:
169 $\forall a, b \cdot a \in \text{RRealPlus} \wedge b \in \text{RReal} \wedge (a \mapsto b \in \text{leq}) \Rightarrow b \in \text{RRealPlus}$
170 *realMinusOrder*:
171 $\forall a, b \cdot a \in \text{RRealMinus} \wedge b \in \text{RReal} \wedge (b \mapsto a \in \text{leq}) \Rightarrow b \in \text{RRealMinus}$
172 *plusCompatibility*:
173 $\forall a, b, c \cdot a \in \text{RReal} \wedge b \in \text{RReal} \wedge c \in \text{RReal} \wedge (a \mapsto b \in \text{leq}) \wedge (a \mapsto c \in \text{leq}) \Rightarrow$
174 $(a \mapsto \text{plus}(b \mapsto c) \in \text{leq})$
175 *plusCompatibility2*:
176 $\forall a, b, c \cdot$
177 $a \in \text{RReal} \wedge b \in \text{RReal} \wedge c \in \text{RReal} \wedge$
178 $\text{plus}(a \mapsto b) \mapsto c \in \text{leq} \wedge \text{Rzero} \mapsto b \in \text{leq} \Rightarrow$
179 $a \mapsto c \in \text{leq}$
180 *extendedTransitivity*:
181 $\forall a, b, c \cdot$
182 $a \in \text{RReal} \wedge b \in \text{RReal} \wedge c \in \text{RReal} \wedge$
183 $a \mapsto b \in \text{leq} \wedge b \mapsto c \in \text{lt} \Rightarrow$
184 $a \mapsto c \in \text{lt}$
185 *extendedTransitivity2*:
186 $\forall a, b, c \cdot$
187 $a \in \text{RReal} \wedge b \in \text{RReal} \wedge c \in \text{RReal} \wedge$
188 $a \mapsto b \in \text{lt} \wedge b \mapsto c \in \text{leq} \Rightarrow$
189 $a \mapsto c \in \text{lt}$
190 *extendedTransitivity3*:
191 $\forall a, b, c \cdot$
192 $a \in \text{RReal} \wedge b \in \text{RReal} \wedge c \in \text{RReal} \wedge$
193 $a \mapsto b \in \text{lt} \wedge b \mapsto c \in \text{lt} \Rightarrow$
194 $a \mapsto c \in \text{lt}$
195 *timesCompatibility*:
196 \top

PROOF RULES

plusAbelianGroup:

Metavariables

a: RReal

b: RReal

c: RReal

Rewrite Rules

plus_commutativity: $\text{plus}(a \mapsto b)$

rhs1: $\top \Rightarrow \text{plus}(b \mapsto a)$

plus_neutralityL: $\text{plus}(a \mapsto \text{Rzero})$

rhs1: $\top \Rightarrow a$

plus_neutralityR: $\text{plus}(\text{Rzero} \mapsto a)$

rhs1: $\top \Rightarrow a$

plus_associativityL: $\text{plus}(a \mapsto \text{plus}(b \mapsto c))$

rhs1: $\top \Rightarrow \text{plus}(\text{plus}(a \mapsto b) \mapsto c)$

plus_associativityR: $\text{plus}(\text{plus}(a \mapsto b) \mapsto c)$

rhs1: $\top \Rightarrow \text{plus}(a \mapsto \text{plus}(b \mapsto c))$

minus_rewrite: $\text{minus}(a \mapsto b)$

rhs1: $\top \Rightarrow \text{plus}(a \mapsto \text{uminus}(b))$

minus_uminus: $\text{uminus}(\text{minus}(a \mapsto b))$

rhs1: $\top \Rightarrow \text{minus}(b \mapsto a)$

minus_uminus_reverse: $\text{minus}(a \mapsto b)$

rhs1: $\top \Rightarrow \text{uminus}(\text{minus}(b \mapsto a))$

uminus_involutive: $\text{uminus}(\text{uminus}(a))$

rhs1: $\top \Rightarrow a$

222 *uminus_reductionL*: plus(a \mapsto uminus(a))
 223 rshl: T \Rightarrow Rzero
 224 *uminus_reductionR*: plus(uminus(a) \mapsto a)
 225 rshl: T \Rightarrow Rzero
 226 *minus_reduction*: minus(a \mapsto a)
 227 rshl: T \Rightarrow Rzero
 228 *zero_minus_left*: minus(Rzero \mapsto a)
 229 rshl: T \Rightarrow uminus(a)
 230 *zero_minus_right*: minus(a \mapsto Rzero)
 231 rshl: T \Rightarrow a
 232 *uminus_neutral*: uminus(Rzero)
 233 rshl: T \Rightarrow Rzero

234 timesCommutativeMonoid:

235 Metavariables

236 a: RReal
 237 b: RReal
 238 c: RReal

239 Rewrite Rules

240 *times_commutativity*: times(a \mapsto b)
 241 rshl: T \Rightarrow times(b \mapsto a)
 242 *times_neutralityL*: times(a \mapsto Rone)
 243 rshl: T \Rightarrow a
 244 *times_neutralityR*: times(Rone \mapsto a)
 245 rshl: T \Rightarrow a
 246 *times_associativityL*: times(a \mapsto times(b \mapsto c))
 247 rshl: T \Rightarrow times(times(a \mapsto b) \mapsto c)
 248 *times_associativityR*: times(times(a \mapsto b) \mapsto c)
 249 rshl: T \Rightarrow times(a \mapsto times(b \mapsto c))
 250 *times_absorbingL*: times(a \mapsto Rzero)
 251 rshl: T \Rightarrow Rzero
 252 *times_absorbingR*: times(Rzero \mapsto a)
 253 rshl: T \Rightarrow Rzero
 254 *division_rewrite*: divide(a \mapsto b)
 255 rshl: T \Rightarrow times(a \mapsto inverse(b))
 256 *divide_inverse*: inverse(divide(a \mapsto b))
 257 rshl: a \neq Rzero \Rightarrow divide(b \mapsto a)
 258 *divide_inverse_reverse*: divide(a \mapsto b)
 259 rshl: a \neq Rzero \Rightarrow inverse(divide(b \mapsto a))
 260 *inverse_involutive*: inverse(inverse(a))
 261 rshl: T \Rightarrow a
 262 *inverse_reductionL*: times(a \mapsto inverse(a))
 263 rshl: T \Rightarrow Rone
 264 *inverse_reductionR*: times(inverse(a) \mapsto a)
 265 rshl: T \Rightarrow Rone
 266 *divide_reduction*: divide(a \mapsto a)
 267 rshl: T \Rightarrow Rone
 268 *one_divide_left*: divide(Rone \mapsto a)
 269 rshl: T \Rightarrow inverse(a)
 270 *one_divide_right*: divide(a \mapsto Rone)
 271 rshl: T \Rightarrow a
 272 *inverse_neutral*: inverse(Rone)
 273 rshl: T \Rightarrow Rone

274 plusTimesDistributive:

275 Metavariables

276 a: RReal
 277 b: RReal
 278 c: RReal

279 Rewrite Rules

280 *left_distribute*: times(a \mapsto plus(b \mapsto c))
 281 rshl: T \Rightarrow plus(times(a \mapsto b) \mapsto times(a \mapsto c))
 282 *right_distribute*: times(plus(b \mapsto c) \mapsto a)
 283 rshl: T \Rightarrow plus(times(b \mapsto a) \mapsto times(c \mapsto a))
 284 *left_factorize*: plus(times(a \mapsto b) \mapsto times(a \mapsto c))
 285 rshl: T \Rightarrow times(a \mapsto plus(b \mapsto c))
 286 *right_factorize*: plus(times(b \mapsto a) \mapsto times(c \mapsto a))
 287 rshl: T \Rightarrow times(plus(b \mapsto c) \mapsto a)
 288 *left_minus_distribute*: times(a \mapsto minus(b \mapsto c))
 289 rshl: T \Rightarrow minus(times(a \mapsto b) \mapsto times(a \mapsto c))
 290 *right_minus_distribute*: times(minus(b \mapsto c) \mapsto a)
 291 rshl: T \Rightarrow minus(times(b \mapsto a) \mapsto times(c \mapsto a))
 292 *left_minus_factorize*: minus(times(a \mapsto b) \mapsto times(a \mapsto c))
 293 rshl: T \Rightarrow times(a \mapsto minus(b \mapsto c))
 294 *right_minus_factorize*: minus(times(b \mapsto a) \mapsto times(c \mapsto a))
 295 rshl: T \Rightarrow times(minus(b \mapsto c) \mapsto a)

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296 minus_distribute: uminus (plus (a ↦ b))
297   rhs1: T ⇒ plus (uminus (a) ↦ uminus (b))
298 minus_factorize: plus (uminus (a) ↦ uminus (b))
299   rhs1: T ⇒ uminus (plus (a ↦ b))
300 divide_distribute: divide (plus (b ↦ c) ↦ a)
301   rhs1: T ⇒ plus (divide (b ↦ a) ↦ divide (c ↦ a))
302 divide_factorize: plus (divide (b ↦ a) ↦ divide (c ↦ a))
303   rhs1: T ⇒ divide (plus (b ↦ c) ↦ a)
304 inverse_uminus: inverse (uminus (a))
305   rhs1: a ≠ Rzero ⇒ uminus (inverse (a))
306 uminus_inverse: uminus (inverse (a))
307   rhs1: a ≠ Rzero ⇒ inverse (uminus (a))
308 divide_left_absorbing: divide (Rzero ↦ a)
309   rhs1: T ⇒ Rzero
310 equations:
311 Metavariables
312   a: RReal
313   b: RReal
314   c: RReal
315 Rewrite Rules
316 plus_left_simplify: plus (a ↦ b) = plus (a ↦ c)
317   rhs1: T ⇒ b = c
318 plus_right_simplify: plus (b ↦ a) = plus (c ↦ a)
319   rhs1: T ⇒ b = c
320 times_left_simplify: times (a ↦ b) = times (a ↦ c)
321   rhs1: a ≠ Rzero ⇒ b = c
322 times_right_simplify: times (b ↦ a) = times (c ↦ a)
323   rhs1: a ≠ Rzero ⇒ b = c
324 minus_left_simplify: minus (a ↦ b) = minus (a ↦ c)
325   rhs1: T ⇒ b = c
326 minus_right_simplify: minus (b ↦ a) = minus (c ↦ a)
327   rhs1: T ⇒ b = c
328 divide_left_simplify: divide (a ↦ b) = divide (a ↦ c)
329   rhs1: a ≠ Rzero ⇒ b = c
330 divide_right_simplify: divide (b ↦ a) = divide (c ↦ a)
331   rhs1: T ⇒ b = c
332 uminus_simplify: uminus (a) = uminus (b)
333   rhs1: T ⇒ a = b
334 inverse_simplify: inverse (a) = inverse (b)
335   rhs1: T ⇒ a = b
336 uminus_unsimplify: a = b
337   rhs1: T ⇒ uminus (a) = uminus (b)
338 inverse_unsimplify: a = b
339   rhs1: a ≠ Rzero ∧ b ≠ Rzero ⇒ inverse (a) = inverse (b)
340 equal_to_minus: a = b
341   rhs1: T ⇒ minus (a ↦ b) = Rzero
342 equal_to_divide: a = b
343   rhs1: b ≠ Rzero ⇒ divide (a ↦ b) = Rone
344 plus_eq_zero: plus (a ↦ b) = Rzero
345   rhs1: T ⇒ a = uminus (b)
346 minus_eq_zero: minus (a ↦ b) = Rzero
347   rhs1: T ⇒ a = b
348 times_eq_one: times (a ↦ b) = Rone
349   rhs1: b ≠ Rzero ⇒ a = inverse (b)
350 divide_eq_one: divide (a ↦ b) = Rone
351   rhs1: T ⇒ a = b
352 times_integral: times (a ↦ b) = Rzero
353   rhs1: a ≠ Rzero ⇒ b = Rzero
354   rhs2: b ≠ Rzero ⇒ a = Rzero
355   rhs3: T ⇒ a = Rzero ∨ b = Rzero
356 divide_integral: divide (a ↦ b) = Rzero
357   rhs1: T ⇒ a = Rzero
358 uminus_eq_rewrite: uminus (a) = b
359   rhs1: T ⇒ a = uminus (b)
360 inverse_eq_rewrite: inverse (a) = b
361   rhs1: b ≠ Rzero ⇒ a = inverse (b)
362 inequationsGeneral:
363 Metavariables
364   a: RReal
365   b: RReal
366   c: RReal
367 Rewrite Rules
368 leq_inversion: a ↦ b ∈ leq
369   rhs1: T ⇒ b ↦ a ∈ geq

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370 $geq_inversion: a \mapsto b \in geq$
371 $rhs1: T \Rightarrow b \mapsto a \in leq$
372 $lt_inversion: a \mapsto b \in lt$
373 $rhs1: T \Rightarrow b \mapsto a \in gt$
374 $gt_inversion: a \mapsto b \in gt$
375 $rhs1: T \Rightarrow b \mapsto a \in lt$
376 $leq_total: \neg(a \mapsto b \in leq)$
377 $rhs1: T \Rightarrow a \mapsto b \in gt$
378 $geq_total: \neg(a \mapsto b \in geq)$
379 $rhs1: T \Rightarrow a \mapsto b \in lt$
380 $geq_total_reverse: \neg(a \mapsto b \in lt)$
381 $rhs1: T \Rightarrow a \mapsto b \in geq$
382 $leq_total_reverse: \neg(a \mapsto b \in gt)$
383 $rhs1: T \Rightarrow a \mapsto b \in leq$
384 $eq_to_leq_weakening: a = b$
385 $rhs1: T \Rightarrow a \mapsto b \in leq$
386 $eq_to_geq_weakening: a = b$
387 $rhs1: T \Rightarrow a \mapsto b \in geq$
388 $noteq_rewrite: a \neq b$
389 $rhs1: T \Rightarrow (a \mapsto b \in lt) \vee (b \mapsto a \in lt)$
390 $lt_to_leq_weakening: a \mapsto b \in lt$
391 $rhs1: T \Rightarrow (a \mapsto b \in leq) \wedge a \neq b$
392 $gt_to_geq_weakening: a \mapsto b \in gt$
393 $rhs1: T \Rightarrow (a \mapsto b \in geq) \wedge a \neq b$
394 $leq_reflexive: a \mapsto a \in leq$
395 $rhs1: T \Rightarrow T$
396 $geq_reflexive: a \mapsto a \in geq$
397 $rhs1: T \Rightarrow T$

Inference Rules

399 $leq_transitivity: a \mapsto b \in leq, b \mapsto c \in leq \vdash a \mapsto c \in leq$
400 $lt_transitivity: a \mapsto b \in lt, b \mapsto c \in lt \vdash a \mapsto c \in lt$
401 $geq_transitivity: a \mapsto b \in geq, b \mapsto c \in geq \vdash a \mapsto c \in geq$
402 $gt_transitivity: a \mapsto b \in gt, b \mapsto c \in gt \vdash a \mapsto c \in gt$
403 $leq_antisymmetry: a \mapsto b \in leq, b \mapsto a \in leq \vdash a = b$
404 $geq_antisymmetry: a \mapsto b \in geq, b \mapsto a \in geq \vdash a = b$
405 $leq_lt_weakening: a \mapsto b \in leq, a \neq b \vdash a \mapsto b \in lt$
406 $geq_gt_weakening: a \mapsto b \in geq, a \neq b \vdash a \mapsto b \in gt$
407 $lt_leq_reduction: (a \mapsto b \in lt) \vee a = b \vdash a \mapsto b \in leq$
408 $gt_geq_reduction: (a \mapsto b \in gt) \vee a = b \vdash a \mapsto b \in geq$
409 inequationsCompatibility:

Metavariables

411 $a: RReal$
412 $b: RReal$
413 $c: RReal$

Rewrite Rules

414 $leq_plus_compatibility: plus(a \mapsto b) \mapsto plus(a \mapsto c) \in leq$
415 $rhs1: T \Rightarrow b \mapsto c \in leq$
416 $lt_plus_compatibility: plus(a \mapsto b) \mapsto plus(a \mapsto c) \in lt$
417 $rhs1: T \Rightarrow b \mapsto c \in lt$
418 $uminus_leq_ineq: a \mapsto b \in leq$
419 $rhs1: T \Rightarrow uminus(b) \mapsto uminus(a) \in leq$
420 $uminus_lt_ineq: a \mapsto b \in lt$
421 $rhs1: T \Rightarrow uminus(b) \mapsto uminus(a) \in lt$
422 $leq_times_compatibility: times(a \mapsto b) \mapsto times(a \mapsto c) \in leq$
423 $rhs1: Rzero \mapsto a \in lt \Rightarrow b \mapsto c \in leq$
424 $rhs2: a \mapsto Rzero \in lt \Rightarrow c \mapsto b \in leq$
425 $lt_times_compatibility: times(a \mapsto b) \mapsto times(a \mapsto c) \in lt$
426 $rhs1: Rzero \mapsto a \in lt \Rightarrow b \mapsto c \in lt$
427 $rhs2: a \mapsto Rzero \in lt \Rightarrow c \mapsto b \in lt$
428 $inverse_leq_ineq: a \mapsto b \in leq$
429 $rhs1: a \neq Rzero \wedge b \neq Rzero \Rightarrow inverse(b) \mapsto inverse(a) \in leq$
430 $inverse_lt_ineq: a \mapsto b \in lt$
431 $rhs1: a \neq Rzero \wedge b \neq Rzero \Rightarrow inverse(b) \mapsto inverse(a) \in lt$
432 $leq_balance_left: a \mapsto b \in leq$
433 $rhs1: T \Rightarrow minus(a \mapsto b) \mapsto Rzero \in leq$
434 $leq_balance_right: a \mapsto b \in leq$
435 $rhs1: T \Rightarrow Rzero \mapsto minus(b \mapsto a) \in leq$
436 $lt_balance_left: a \mapsto b \in lt$
437 $rhs1: T \Rightarrow minus(a \mapsto b) \mapsto Rzero \in lt$
438 $lt_balance_right: a \mapsto b \in lt$
439 $rhs1: T \Rightarrow Rzero \mapsto minus(b \mapsto a) \in lt$
440 $leq_div_balance_left: a \mapsto b \in leq$
441 $rhs1: b \neq Rzero \Rightarrow divide(a \mapsto b) \mapsto Rone \in leq$
442 $leq_div_balance_right: a \mapsto b \in leq$
443

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444   rhs1: a ≠ Rzero ⇒ Rone ⇨ divide(b ⇨ a) ∈ leq
445 lt_div_balance_left: a ⇨ b ∈ lt
446   rhs1: b ≠ Rzero ⇒ divide(a ⇨ b) ⇨ Rone ∈ lt
447 lt_div_balance_right: a ⇨ b ∈ lt
448   rhs1: a ≠ Rzero ⇒ Rone ⇨ divide(b ⇨ a) ∈ lt
449 leq_plus_balance_left: a ⇨ plus(b ⇨ c) ∈ leq
450   rhs1: T ⇒ minus(a ⇨ c) ⇨ b ∈ leq
451 leq_plus_balance_right: plus(a ⇨ b) ⇨ c ∈ leq
452   rhs1: T ⇒ a ⇨ minus(c ⇨ b) ∈ leq
453 leq_times_balance_left: a ⇨ times(b ⇨ c) ∈ leq
454   rhs1: Rzero ⇨ c ∈ lt ⇒ divide(a ⇨ c) ⇨ b ∈ leq
455   rhs2: c ⇨ Rzero ∈ lt ⇒ b ⇨ divide(a ⇨ c) ∈ leq
456 leq_times_balance_right: times(a ⇨ b) ⇨ c ∈ leq
457   rhs1: Rzero ⇨ b ∈ lt ⇒ a ⇨ divide(c ⇨ b) ∈ leq
458   rhs2: b ⇨ Rzero ∈ lt ⇒ divide(c ⇨ b) ⇨ a ∈ leq
459 lt_plus_balance_left: a ⇨ plus(b ⇨ c) ∈ lt
460   rhs1: T ⇒ minus(a ⇨ c) ⇨ b ∈ lt
461 lt_plus_balance_right: plus(a ⇨ b) ⇨ c ∈ lt
462   rhs1: T ⇒ a ⇨ minus(b ⇨ c) ∈ lt
463 lt_times_balance_left: a ⇨ times(b ⇨ c) ∈ lt
464   rhs1: Rzero ⇨ c ∈ lt ⇒ divide(a ⇨ c) ⇨ b ∈ lt
465   rhs2: c ⇨ Rzero ∈ lt ⇒ b ⇨ divide(a ⇨ c) ∈ lt
466 lt_times_balance_right: times(a ⇨ b) ⇨ c ∈ lt
467   rhs1: Rzero ⇨ b ∈ lt ⇒ a ⇨ divide(c ⇨ b) ∈ lt
468   rhs2: b ⇨ Rzero ∈ lt ⇒ divide(c ⇨ b) ⇨ a ∈ lt
469 inverse_positive_leq: Rzero ⇨ a ∈ leq
470   rhs1: a ≠ Rzero ⇒ Rzero ⇨ inverse(a) ∈ lt
471 inverse_negative_leq: a ⇨ Rzero ∈ leq
472   rhs1: a ≠ Rzero ⇒ inverse(a) ⇨ Rzero ∈ lt
473 inverse_positive_lt: Rzero ⇨ a ∈ lt
474   rhs1: T ⇒ Rzero ⇨ inverse(a) ∈ lt
475 inverse_negative_lt: a ⇨ Rzero ∈ lt
476   rhs1: T ⇒ inverse(a) ⇨ Rzero ∈ lt
477 times_compatibility_leq: Rzero ⇨ times(a ⇨ b) ∈ leq
478   rhs5: Rzero ⇨ a ∈ leq ⇒ Rzero ⇨ b ∈ leq
479   rhs6: Rzero ⇨ b ∈ leq ⇒ Rzero ⇨ a ∈ leq
480   rhs7: a ⇨ Rzero ∈ leq ⇒ b ⇨ Rzero ∈ leq
481   rhs8: b ⇨ Rzero ∈ leq ⇒ a ⇨ Rzero ∈ leq
482 times_compatibility_geq: Rzero ⇨ times(a ⇨ b) ∈ geq
483   rhs5: T ⇒ (a ⇨ Rzero ∈ leq ∧ Rzero ⇨ b ∈ leq) ∨ (Rzero ⇨ a ∈ leq ∧ b ⇨ Rzero ∈ leq)
484 absPredicates:
485 Metavariables
486   x: RReal
487   y: RReal
488 Rewrite Rules
489   abs_eqn: abs(x) = y
490     rhs1: T ⇒ x = y ∨ uminus(x) = y
491   abs_contradict: abs(x) = y
492     rhs1: y ⇨ Rzero ∈ lt ⇒ ⊥
493   abs_zero: abs(x) = Rzero
494     rhs1: T ⇒ x = Rzero
495 realSubSetsUtility:
496 Metavariables
497   a: RReal
498 Rewrite Rules
499   realPlus_to_leq: a ∈ RRealPlus
500     rhs1: T ⇒ Rzero ⇨ a ∈ leq
501   realMinus_to_leq: a ∈ RRealMinus
502     rhs1: T ⇒ a ⇨ Rzero ∈ leq
503   realStar_to_neq: a ∈ RRealStar
504     rhs1: T ⇒ a ≠ Rzero
505   realPlusStar_to_leq: a ∈ RRealPlusStar
506     rhs1: T ⇒ Rzero ⇨ a ∈ lt
507   realMinusStar_to_leq: a ∈ RRealMinusStar
508     rhs1: T ⇒ a ⇨ Rzero ∈ lt
509   zero_in_RRealPlus: Rzero ∈ RRealPlus
510     rhs1: T ⇒ T
511 typing:
512 Metavariables
513   a: RReal
514   b: RReal
515 Rewrite Rules
516   plusTypeRew: a ⇨ b ∈ dom(plus)
517     rhs1: T ⇒ a ∈ RReal ∧ b ∈ RReal

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518 *minusTypeRew*: $a \mapsto b \in \text{dom}(\text{minus})$
 519 $\text{rhs1} : T \Rightarrow a \in \text{RReal} \wedge b \in \text{RReal}$
 520 *timesTypeRew*: $a \mapsto b \in \text{dom}(\text{times})$
 521 $\text{rhs1} : T \Rightarrow a \in \text{RReal} \wedge b \in \text{RReal}$
 522 *divideTypeRew*: $a \mapsto b \in \text{dom}(\text{divide})$
 523 $\text{rhs1} : T \Rightarrow a \in \text{RReal} \wedge b \in \text{RRealStar}$
 524 *uminusTypeRew*: $a \in \text{dom}(\text{uminus})$
 525 $\text{rhs1} : T \Rightarrow a \in \text{RReal}$
 526 *inverseTypeRew*: $a \in \text{dom}(\text{inverse})$
 527 $\text{rhs1} : T \Rightarrow a \in \text{RRealStar}$
 528 *absTypeRew*: $a \in \text{dom}(\text{abs})$
 529 $\text{rhs1} : T \Rightarrow a \in \text{RReal}$
 530 *sqrtTypeRew*: $a \in \text{dom}(\text{sqrt})$
 531 $\text{rhs1} : T \Rightarrow a \in \text{RRealPlus}$
 532 *plusBasicTypeRew*: $\text{plus} \in \text{RReal} \times \text{RReal} \rightarrow \text{RReal}$
 533 $\text{rhs1} : T \Rightarrow T$
 534 *minusBasicTypeRew*: $\text{minus} \in \text{RReal} \times \text{RReal} \rightarrow \text{RReal}$
 535 $\text{rhs1} : T \Rightarrow T$
 536 *timesBasicTypeRew*: $\text{times} \in \text{RReal} \times \text{RReal} \rightarrow \text{RReal}$
 537 $\text{rhs1} : T \Rightarrow T$
 538 *divideBasicTypeRew*: $\text{divide} \in \text{RReal} \times \text{RReal} \rightarrow \text{RReal}$
 539 $\text{rhs1} : T \Rightarrow T$
 540 *uminusBasicTypeRew*: $\text{uminus} \in \text{RReal} \rightarrow \text{RReal}$
 541 $\text{rhs1} : T \Rightarrow T$
 542 *inverseBasicTypeRew*: $\text{inverse} \in \text{RReal} \rightarrow \text{RReal}$
 543 $\text{rhs1} : T \Rightarrow T$
 544 *absBasicTypeRew*: $\text{abs} \in \text{RReal} \rightarrow \text{RReal}$
 545 $\text{rhs1} : T \Rightarrow T$
 546 *sqrtBasicTypeRew*: $\text{sqrt} \in \text{RReal} \rightarrow \text{RReal}$
 547 $\text{rhs1} : T \Rightarrow T$

Inference Rules

549 *plusType*: $a \in \text{RReal}, b \in \text{RReal} \vdash \text{plus}(a \mapsto b) \in \text{RReal}$
 550 *minusType*: $a \in \text{RReal}, b \in \text{RReal} \vdash \text{minus}(a \mapsto b) \in \text{RReal}$
 551 *timesType*: $a \in \text{RReal}, b \in \text{RReal} \vdash \text{times}(a \mapsto b) \in \text{RReal}$
 552 *divideType*: $a \in \text{RReal}, b \in \text{RReal}, b \neq \text{Rzero} \vdash \text{divide}(a \mapsto b) \in \text{RRealStar}$
 553 *uminusType*: $a \in \text{RReal} \vdash \text{uminus}(a) \in \text{RReal}$
 554 *inverseType*: $a \in \text{RRealStar} \vdash \text{inverse}(a) \in \text{RRealStar}$
 555 *sqrtType*: $a \in \text{RRealPlus} \vdash \text{sqrt}(a) \in \text{RRealPlus}$
 556 *absType*: $a \in \text{RReal} \vdash \text{abs}(a) \in \text{RRealPlus}$
 557 *plusTypeArg*: $a \in \text{RReal}, b \in \text{RReal} \vdash a \mapsto b \in \text{dom}(\text{plus})$
 558 *minusTypeArg*: $a \in \text{RReal}, b \in \text{RReal} \vdash a \mapsto b \in \text{dom}(\text{minus})$
 559 *timesTypeArg*: $a \in \text{RReal}, b \in \text{RReal} \vdash a \mapsto b \in \text{dom}(\text{times})$
 560 *divideTypeArg*: $a \in \text{RReal}, b \in \text{RReal}, b \neq \text{Rzero} \vdash a \mapsto b \in \text{dom}(\text{divide})$
 561 *uminusTypeArg*: $a \in \text{RReal} \vdash a \in \text{dom}(\text{uminus})$
 562 *inverseTypeArg*: $a \in \text{RRealStar} \vdash a \in \text{dom}(\text{inverse})$
 563 *sqrtTypeArg*: $a \in \text{RRealPlus} \vdash a \in \text{dom}(\text{sqrt})$
 564 *absTypeArg*: $a \in \text{RReal} \vdash a \in \text{dom}(\text{abs})$

565 **END**