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THEORY Piecewise
  IMPORT THEORY PROJECTS
    /SimpleDEq THEORIES /SimpleDEq/Functions.dtf|org.eventb.theory.core.deployedTheoryRoot#Functions
  TYPE PARAMETERS E,F,G
  OPERATORS
    partitionS <predicate> (X: ℙ(E),Xs: ℙ(ℙ(E)))
      direct definition
        (∀ X1,X2 · X1 ∈ Xs ∧ X2 ∈ Xs ∧ X1 ≠ X2 ⇒ X1 ∩ X2 = ∅) ∧
        union(Xs) = X
    piecewiseContinuous <predicate> (Ix: ℙ(ℙ(E)),B: ℙ(F),f: union(Ix) → B)
      well−definedness Ix ≠ ∅,∀ I1,I2 · I1 ∈ Ix ∧ I2 ∈ Ix ∧ I1 ≠ I2 ⇒ I1 ∩ I2 = ∅
      direct definition
        ∀ I0 · I0 ∈ Ix ⇒ (I0 ◁ f) ∈ C0(I0,B)
    partialPiecewiseContinuous <predicate> (Ix: ℙ(ℙ(E)),B: ℙ(F),C: ℙ(G),g: union(Ix)×B → C)
      well−definedness Ix ≠ ∅,∀ I1,I2 · I1 ∈ Ix ∧ I2 ∈ Ix ∧ I1 ≠ I2 ⇒ I1 ∩ I2 = ∅
      direct definition
        ∀ I0 · I0 ∈ Ix ⇒ ((I0×B) ◁ g) ∈ C0(I0×B,C)
    piecewiseLipschitzContinuous <predicate> (Ix: ℙ(ℙ(E)),B: ℙ(F),f: union(Ix) → B)
      well−definedness Ix ≠ ∅,∀ I1,I2 · I1 ∈ Ix ∧ I2 ∈ Ix ∧ I1 ≠ I2 ⇒ I1 ∩ I2 = ∅
      direct definition
        ∀ I0 · I0 ∈ Ix ⇒ lipschitzContinuous(I0,B,I0 ◁ f)
  THEOREMS
    untilPiecewise:
      ∀ s,t0,f,g ·
        s ∈ RReal ∧ t0 ∈ RReal ∧ s ↦ t0 ∈ leq ∧
        f ∈ RReal ⇸ E ∧ g ∈ RReal ⇸ E ∧
        Closed2Open(s,t0) ⊆ dom(f) ∧ Closed2Infinity(t0) ⊆ dom(g) ∧
        f ∈ C0(Closed2Open(s,t0),E) ∧ g ∈ C0(Closed2Infinity(t0),E) ⇒
          piecewiseContinuous({Closed2Open(s,t0),Closed2Infinity(t0)},E,until(s,f,t0,g))
    untilFPartialPiecewise:
      ∀ s,t0,f,g ·
        s ∈ RReal ∧ t0 ∈ RReal ∧ s ↦ t0 ∈ leq ∧
        f ∈ RReal×E ⇸ F ∧ g ∈ RReal×E ⇸ F ∧
        Closed2Open(s,t0)×∅ ⊆ dom(f) ∧ Closed2Infinity(t0)×∅ ⊆ dom(g) ∧
        f ∈ C0(Closed2Open(s,t0)×E,F) ∧ g ∈ C0(Closed2Infinity(t0)×E,F) ⇒
          partialPiecewiseContinuous({Closed2Open(s,t0),Closed2Infinity(t0)},E,F,untilF(s,f,t0,g))
END
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