

1 **THEORY** Monoid
2 **TYPE PARAMETERS** M
3 **OPERATORS**
4 **associative** <predicate> (op: (M × M) → M)
5 **direct definition**
6 $\forall x, y, z \cdot x \in M \wedge y \in M \wedge z \in M \Rightarrow \text{op}(x \mapsto \text{op}(y \mapsto z)) = \text{op}(\text{op}(x \mapsto y) \mapsto z)$
7 **neutral** <predicate> (op: (M × M) → M, e: M)
8 **well-definedness** M ≠ ∅
9 **direct definition**
10 $\forall x \cdot x \in M \Rightarrow (\text{op}(x \mapsto e) = x \wedge \text{op}(e \mapsto x) = x)$
11 **Monoid** <predicate> (op: (M × M) → M, e: M)
12 **well-definedness** M ≠ ∅
13 **direct definition**
14 $\text{associative}(\text{op}) \wedge \text{neutral}(\text{op}, e)$
15 **THEOREMS**
16 *neutralUnicity:*
17 $\forall \text{op}, e \cdot \text{op} \in ((M \times M) \rightarrow M) \wedge e \in M \wedge \text{Monoid}(\text{op}, e) \Rightarrow ($
18 $\forall x \cdot x \in M \wedge \text{neutral}(\text{op}, x) \Rightarrow x = e$
19 $)$
20 **END**