

## THEORY Intervals

### IMPORT THEORY PROJECTS

/SimpleDEq THEORIES /SimpleDEq/Reals.dtf | org.eventb.theory.core.deployedTheoryRoot#Reals

### OPERATORS

**Infinity2Open** <expression> (b: RReal)

**direct definition**

{ t | t  $\mapsto$  b  $\in$  lt }

**Infinity2Closed** <expression> (b: RReal)

**direct definition**

{ t | t  $\mapsto$  b  $\in$  leq }

**Open2Infinity** <expression> (a: RReal)

**direct definition**

{ t | a  $\mapsto$  t  $\in$  lt }

**Closed2Infinity** <expression> (a: RReal)

**direct definition**

{ t | a  $\mapsto$  t  $\in$  leq }

**Open2Open** <expression> (a: RReal, b: RReal)

**direct definition**

{ t | a  $\mapsto$  t  $\in$  lt  $\wedge$  t  $\mapsto$  b  $\in$  lt }

**Open2Closed** <expression> (a: RReal, b: RReal)

**direct definition**

{ t | a  $\mapsto$  t  $\in$  lt  $\wedge$  t  $\mapsto$  b  $\in$  leq }

**Closed2Open** <expression> (a: RReal, b: RReal)

**direct definition**

{ t | a  $\mapsto$  t  $\in$  leq  $\wedge$  t  $\mapsto$  b  $\in$  lt }

**Closed2Closed** <expression> (a: RReal, b: RReal)

**direct definition**

{ t | a  $\mapsto$  t  $\in$  leq  $\wedge$  t  $\mapsto$  b  $\in$  leq }

### THEOREMS

*realPlusIsZero2Infinity:*

RRealPlus = Closed2Infinity(Rzero)

*realMinusIsInfinity2Zero:*

RRealMinus = Infinity2Closed(Rzero)

*c2c\_existence:*

$\forall a, b \cdot a \in \text{RReal} \wedge b \in \text{RReal} \wedge a \mapsto b \in \text{leq} \Rightarrow (\exists x \cdot x \in \text{Closed2Closed}(a, b))$

*c2o\_existence:*

$\forall a, b \cdot a \in \text{RReal} \wedge b \in \text{RReal} \wedge a \mapsto b \in \text{lt} \Rightarrow (\exists x \cdot x \in \text{Closed2Open}(a, b))$

*o2c\_existence:*

$\forall a, b \cdot a \in \text{RReal} \wedge b \in \text{RReal} \wedge a \mapsto b \in \text{lt} \Rightarrow (\exists x \cdot x \in \text{Open2Closed}(a, b))$

*o2o\_existence:*

$\forall a, b \cdot a \in \text{RReal} \wedge b \in \text{RReal} \wedge a \mapsto b \in \text{lt} \Rightarrow (\exists x \cdot x \in \text{Open2Open}(a, b))$

*boundaryInClosed2Closed:*

$\forall a, b \cdot a \in \text{RReal} \wedge b \in \text{RReal} \Rightarrow (a \in \text{Closed2Closed}(a, b) \wedge b \in \text{Closed2Closed}(a, b))$

*boundaryInClosed2Open:*

$\forall a, b \cdot a \in \text{RReal} \wedge b \in \text{RReal} \Rightarrow (a \in \text{Closed2Open}(a, b))$

*boundaryInOpen2Closed:*

$\forall a, b \cdot a \in \text{RReal} \wedge b \in \text{RReal} \Rightarrow (b \in \text{Open2Closed}(a, b))$

*boundaryInInfinity2Closed:*

$\forall b \cdot b \in \text{RReal} \Rightarrow (b \in \text{Infinity2Closed}(b))$

*boundaryInClosed2Infinity:*

$\forall a \cdot a \in \text{RReal} \Rightarrow (a \in \text{Closed2Infinity}(a))$

*closed2ClosedLowerBound:*

$\forall a, b \cdot a \in \text{RReal} \wedge b \in \text{RReal} \wedge a \mapsto b \in \text{leq} \Rightarrow$   
 $\text{lowerBound}(\text{leq}, \text{Closed2Closed}(a, b), a)$

*closed2OpenLowerBound:*

$\forall a, b \cdot a \in \text{RReal} \wedge b \in \text{RReal} \wedge a \mapsto b \in \text{lt} \Rightarrow$   
 $\text{lowerBound}(\text{leq}, \text{Closed2Open}(a, b), a)$

*open2ClosedLowerBound:*

$\forall a, b \cdot a \in \text{RReal} \wedge b \in \text{RReal} \wedge a \mapsto b \in \text{lt} \Rightarrow$   
 $\text{lowerBound}(\text{leq}, \text{Open2Closed}(a, b), a)$

*open2OpenLowerBound:*

$\forall a, b \cdot a \in \text{RReal} \wedge b \in \text{RReal} \wedge a \mapsto b \in \text{lt} \Rightarrow$   
 $\text{lowerBound}(\text{leq}, \text{Open2Open}(a, b), a)$

*closed2InfinityLowerBound:*

$\forall a \cdot a \in \text{RReal} \Rightarrow$   
 $\text{lowerBound}(\text{leq}, \text{Closed2Infinity}(a), a)$

*open2InfinityLowerBound:*

$\forall a \cdot a \in \text{RReal} \Rightarrow$   
 $\text{lowerBound}(\text{leq}, \text{Open2Infinity}(a), a)$

*infinity2ClosedLowerBound:*

$\forall b \cdot b \in \text{RReal} \Rightarrow$   
 $\neg \text{lowerBounded}(\text{leq}, \text{Infinity2Closed}(b))$

*infinity2OpenLowerBound:*

74  $\forall b \cdot b \in \mathbb{RReal} \Rightarrow$   
75  $\neg \text{lowerBounded}(\text{leq}, \text{Infinity2Open}(b))$   
76 *closed2ClosedInfimum:*  
77  $\forall a, b \cdot a \in \mathbb{RReal} \wedge b \in \mathbb{RReal} \wedge a \mapsto b \in \text{leq} \Rightarrow$   
78  $\text{infimum}(\text{leq}, \text{Closed2Closed}(a,b), a)$   
79 *closed2OpenInfimum:*  
80  $\forall a, b \cdot a \in \mathbb{RReal} \wedge b \in \mathbb{RReal} \wedge a \mapsto b \in \text{lt} \Rightarrow$   
81  $\text{infimum}(\text{leq}, \text{Closed2Open}(a,b), a)$   
82 *open2ClosedInfimum:*  
83  $\forall a, b \cdot a \in \mathbb{RReal} \wedge b \in \mathbb{RReal} \wedge a \mapsto b \in \text{lt} \Rightarrow$   
84  $\text{infimum}(\text{leq}, \text{Open2Closed}(a,b), a)$   
85 *open2OpenInfimum:*  
86  $\forall a, b \cdot a \in \mathbb{RReal} \wedge b \in \mathbb{RReal} \wedge a \mapsto b \in \text{lt} \Rightarrow$   
87  $\text{infimum}(\text{leq}, \text{Open2Open}(a,b), a)$   
88 *closed2InfinityInfimum:*  
89  $\forall a \cdot a \in \mathbb{RReal} \Rightarrow$   
90  $\text{infimum}(\text{leq}, \text{Closed2Infinity}(a), a)$   
91 *open2InfinityInfimum:*  
92  $\forall a \cdot a \in \mathbb{RReal} \Rightarrow$   
93  $\text{infimum}(\text{leq}, \text{Open2Infinity}(a), a)$   
94 *closed2ClosedMinimum:*  
95  $\forall a, b \cdot a \in \mathbb{RReal} \wedge b \in \mathbb{RReal} \wedge a \mapsto b \in \text{leq} \Rightarrow$   
96  $\text{minimum}(\text{leq}, \text{Closed2Closed}(a,b), a)$   
97 *closed2OpenMinimum:*  
98  $\forall a, b \cdot a \in \mathbb{RReal} \wedge b \in \mathbb{RReal} \wedge a \mapsto b \in \text{lt} \Rightarrow$   
99  $\text{minimum}(\text{leq}, \text{Closed2Open}(a,b), a)$   
100 *open2ClosedMinimum:*  
101  $\forall a, b \cdot a \in \mathbb{RReal} \wedge b \in \mathbb{RReal} \wedge a \mapsto b \in \text{lt} \Rightarrow$   
102  $\neg \text{hasMinimum}(\text{leq}, \text{Open2Closed}(a,b))$   
103 *open2OpenMinimum:*  
104  $\forall a, b \cdot a \in \mathbb{RReal} \wedge b \in \mathbb{RReal} \wedge a \mapsto b \in \text{lt} \Rightarrow$   
105  $\neg \text{hasMinimum}(\text{leq}, \text{Open2Open}(a,b))$   
106 *closed2InfinityMinimum:*  
107  $\forall a \cdot a \in \mathbb{RReal} \Rightarrow$   
108  $\text{minimum}(\text{leq}, \text{Closed2Infinity}(a), a)$   
109 *open2InfinityMinimum:*  
110  $\forall a \cdot a \in \mathbb{RReal} \Rightarrow$   
111  $\neg \text{hasMinimum}(\text{leq}, \text{Open2Infinity}(a))$   
112 *infinity2ClosedMinimum:*  
113  $\forall b \cdot b \in \mathbb{RReal} \Rightarrow$   
114  $\neg \text{hasMinimum}(\text{leq}, \text{Infinity2Closed}(b))$   
115 *infinity2OpenMinimum:*  
116  $\forall b \cdot b \in \mathbb{RReal} \Rightarrow$   
117  $\neg \text{hasMinimum}(\text{leq}, \text{Infinity2Open}(b))$   
118 *closed2ClosedUpperBound:*  
119  $\forall a, b \cdot a \in \mathbb{RReal} \wedge b \in \mathbb{RReal} \wedge a \mapsto b \in \text{leq} \Rightarrow$   
120  $\text{upperBound}(\text{leq}, \text{Closed2Closed}(a,b), b)$   
121 *closed2OpenUpperBound:*  
122  $\forall a, b \cdot a \in \mathbb{RReal} \wedge b \in \mathbb{RReal} \wedge a \mapsto b \in \text{lt} \Rightarrow$   
123  $\text{upperBound}(\text{leq}, \text{Closed2Open}(a,b), b)$   
124 *open2ClosedUpperBound:*  
125  $\forall a, b \cdot a \in \mathbb{RReal} \wedge b \in \mathbb{RReal} \wedge a \mapsto b \in \text{lt} \Rightarrow$   
126  $\text{upperBound}(\text{leq}, \text{Open2Closed}(a,b), b)$   
127 *open2OpenUpperBound:*  
128  $\forall a, b \cdot a \in \mathbb{RReal} \wedge b \in \mathbb{RReal} \wedge a \mapsto b \in \text{lt} \Rightarrow$   
129  $\text{upperBound}(\text{leq}, \text{Open2Open}(a,b), b)$   
130 *closed2InfinityUpperBound:*  
131  $\forall a \cdot a \in \mathbb{RReal} \Rightarrow$   
132  $\neg \text{upperBounded}(\text{leq}, \text{Closed2Infinity}(a))$   
133 *open2InfinityUpperBound:*  
134  $\forall a \cdot a \in \mathbb{RReal} \Rightarrow$   
135  $\neg \text{upperBounded}(\text{leq}, \text{Open2Infinity}(a))$   
136 *infinity2ClosedUpperBound:*  
137  $\forall b \cdot b \in \mathbb{RReal} \Rightarrow$   
138  $\text{upperBound}(\text{leq}, \text{Infinity2Closed}(b), b)$   
139 *infinity2OpenUpperBound:*  
140  $\forall b \cdot b \in \mathbb{RReal} \Rightarrow$   
141  $\text{upperBound}(\text{leq}, \text{Infinity2Open}(b), b)$   
142 *closed2ClosedSupremum:*  
143  $\forall a, b \cdot a \in \mathbb{RReal} \wedge b \in \mathbb{RReal} \wedge a \mapsto b \in \text{leq} \Rightarrow$   
144  $\text{supremum}(\text{leq}, \text{Closed2Closed}(a,b), b)$   
145 *closed2OpenSupremum:*  
146  $\forall a, b \cdot a \in \mathbb{RReal} \wedge b \in \mathbb{RReal} \wedge a \mapsto b \in \text{lt} \Rightarrow$   
147  $\text{supremum}(\text{leq}, \text{Closed2Open}(a,b), b)$

148 *open2ClosedSupremum* :  
149  $\forall a, b \cdot a \in \mathbb{RReal} \wedge b \in \mathbb{RReal} \wedge a \mapsto b \in \text{lt} \Rightarrow$   
150  $\text{supremum}(\text{leq}, \text{Open2Closed}(a,b), b)$   
151 *open2OpenSupremum* :  
152  $\forall a, b \cdot a \in \mathbb{RReal} \wedge b \in \mathbb{RReal} \wedge a \mapsto b \in \text{lt} \Rightarrow$   
153  $\text{supremum}(\text{leq}, \text{Open2Open}(a,b), b)$   
154 *infinity2ClosedSupremum* :  
155  $\forall b \cdot b \in \mathbb{RReal} \Rightarrow$   
156  $\text{supremum}(\text{leq}, \text{Infinity2Closed}(b), b)$   
157 *infinity2OpenSupremum* :  
158  $\forall b \cdot b \in \mathbb{RReal} \Rightarrow$   
159  $\text{supremum}(\text{leq}, \text{Open2Infinity}(b), b)$   
160 *closed2ClosedMaximum* :  
161  $\forall a, b \cdot a \in \mathbb{RReal} \wedge b \in \mathbb{RReal} \wedge a \mapsto b \in \text{leq} \Rightarrow$   
162  $\text{maximum}(\text{leq}, \text{Closed2Closed}(a,b), b)$   
163 *closed2OpenMaximum* :  
164  $\forall a, b \cdot a \in \mathbb{RReal} \wedge b \in \mathbb{RReal} \wedge a \mapsto b \in \text{lt} \Rightarrow$   
165  $\neg \text{hasMaximum}(\text{leq}, \text{Closed2Open}(a,b))$   
166 *open2ClosedMaximum* :  
167  $\forall a, b \cdot a \in \mathbb{RReal} \wedge b \in \mathbb{RReal} \wedge a \mapsto b \in \text{lt} \Rightarrow$   
168  $\text{maximum}(\text{leq}, \text{Open2Closed}(a,b), b)$   
169 *open2OpenMaximum* :  
170  $\forall a, b \cdot a \in \mathbb{RReal} \wedge b \in \mathbb{RReal} \wedge a \mapsto b \in \text{lt} \Rightarrow$   
171  $\neg \text{hasMaximum}(\text{leq}, \text{Open2Open}(a,b))$   
172 *closed2InfinityMaximum* :  
173  $\forall a \cdot a \in \mathbb{RReal} \Rightarrow$   
174  $\neg \text{hasMaximum}(\text{leq}, \text{Closed2Infinity}(a))$   
175 *open2InfinityMaximum* :  
176  $\forall a \cdot a \in \mathbb{RReal} \Rightarrow$   
177  $\neg \text{hasMaximum}(\text{leq}, \text{Open2Infinity}(a))$   
178 *infinity2ClosedMaximum* :  
179  $\forall b \cdot b \in \mathbb{RReal} \Rightarrow$   
180  $\text{maximum}(\text{leq}, \text{Infinity2Closed}(b), b)$   
181 *infinity2OpenMaximum* :  
182  $\forall b \cdot b \in \mathbb{RReal} \Rightarrow$   
183  $\neg \text{hasMaximum}(\text{leq}, \text{Infinity2Open}(b))$   
184 *c2cUc2c* :  
185  $\forall a, b, c \cdot$   
186  $a \in \mathbb{RReal} \wedge b \in \mathbb{RReal} \wedge c \in \mathbb{RReal} \wedge$   
187  $a \mapsto b \in \text{leq} \wedge b \mapsto c \in \text{leq} \Rightarrow$   
188  $\text{Closed2Closed}(a,b) \cup \text{Closed2Closed}(b,c) = \text{Closed2Closed}(a,c)$   
189 *c2cUc2o* :  
190  $\forall a, b, c \cdot$   
191  $a \in \mathbb{RReal} \wedge b \in \mathbb{RReal} \wedge c \in \mathbb{RReal} \wedge$   
192  $a \mapsto b \in \text{leq} \wedge b \mapsto c \in \text{lt} \Rightarrow$   
193  $\text{Closed2Closed}(a,b) \cup \text{Closed2Open}(b,c) = \text{Closed2Open}(a,c)$   
194 *o2cUc2c* :  
195  $\forall a, b, c \cdot$   
196  $a \in \mathbb{RReal} \wedge b \in \mathbb{RReal} \wedge c \in \mathbb{RReal} \wedge$   
197  $a \mapsto b \in \text{lt} \wedge b \mapsto c \in \text{leq} \Rightarrow$   
198  $\text{Open2Closed}(a,b) \cup \text{Closed2Closed}(b,c) = \text{Open2Closed}(a,c)$   
199 *o2cUc2o* :  
200  $\forall a, b, c \cdot$   
201  $a \in \mathbb{RReal} \wedge b \in \mathbb{RReal} \wedge c \in \mathbb{RReal} \wedge$   
202  $a \mapsto b \in \text{lt} \wedge b \mapsto c \in \text{lt} \Rightarrow$   
203  $\text{Open2Closed}(a,b) \cup \text{Closed2Open}(b,c) = \text{Open2Open}(a,c)$   
204 *inf2cUc2c* :  
205  $\forall b, c \cdot$   
206  $b \in \mathbb{RReal} \wedge c \in \mathbb{RReal} \wedge$   
207  $b \mapsto c \in \text{leq} \Rightarrow$   
208  $\text{Infinity2Closed}(b) \cup \text{Closed2Closed}(b,c) = \text{Infinity2Closed}(c)$   
209 *inf2cUc2o* :  
210  $\forall b, c \cdot$   
211  $b \in \mathbb{RReal} \wedge c \in \mathbb{RReal} \wedge$   
212  $b \mapsto c \in \text{lt} \Rightarrow$   
213  $\text{Infinity2Closed}(b) \cup \text{Closed2Open}(b,c) = \text{Infinity2Open}(c)$   
214 *c2cUc2inf* :  
215  $\forall a, b \cdot$   
216  $a \in \mathbb{RReal} \wedge b \in \mathbb{RReal} \wedge$   
217  $a \mapsto b \in \text{leq} \Rightarrow$   
218  $\text{Closed2Closed}(a,b) \cup \text{Closed2Infinity}(b) = \text{Closed2Infinity}(a)$   
219 *o2cUc2inf* :  
220  $\forall a, b \cdot$   
221  $a \in \mathbb{RReal} \wedge b \in \mathbb{RReal} \wedge$

222  $a \mapsto b \in \text{lt} \Rightarrow$   
223  $\text{Open2Closed}(a,b) \cup \text{Closed2Infinity}(b) = \text{Open2Infinity}(a)$   
224 *inf2cUc2inf*:  
225  $\forall a \cdot$   
226  $a \in \text{RReal} \Rightarrow$   
227  $\text{Infinity2Closed}(a) \cup \text{Closed2Infinity}(a) = \text{RReal}$   
228 *c2cUo2c*:  
229  $\forall a, b, c \cdot$   
230  $a \in \text{RReal} \wedge b \in \text{RReal} \wedge c \in \text{RReal} \wedge$   
231  $a \mapsto b \in \text{leq} \wedge b \mapsto c \in \text{lt} \Rightarrow$   
232  $\text{Closed2Closed}(a,b) \cup \text{Open2Closed}(b,c) = \text{Closed2Closed}(a,c)$   
233 *c2cUo2o*:  
234  $\forall a, b, c \cdot$   
235  $a \in \text{RReal} \wedge b \in \text{RReal} \wedge c \in \text{RReal} \wedge$   
236  $a \mapsto b \in \text{leq} \wedge b \mapsto c \in \text{lt} \Rightarrow$   
237  $\text{Closed2Closed}(a,b) \cup \text{Open2Open}(b,c) = \text{Closed2Open}(a,c)$   
238 *o2cUo2c*:  
239  $\forall a, b, c \cdot$   
240  $a \in \text{RReal} \wedge b \in \text{RReal} \wedge c \in \text{RReal} \wedge$   
241  $a \mapsto b \in \text{lt} \wedge b \mapsto c \in \text{lt} \Rightarrow$   
242  $\text{Open2Closed}(a,b) \cup \text{Open2Closed}(b,c) = \text{Open2Closed}(a,c)$   
243 *o2cUo2o*:  
244  $\forall a, b, c \cdot$   
245  $a \in \text{RReal} \wedge b \in \text{RReal} \wedge c \in \text{RReal} \wedge$   
246  $a \mapsto b \in \text{lt} \wedge b \mapsto c \in \text{lt} \Rightarrow$   
247  $\text{Open2Closed}(a,b) \cup \text{Open2Open}(b,c) = \text{Open2Open}(a,c)$   
248 *inf2cUo2c*:  
249  $\forall b, c \cdot$   
250  $b \in \text{RReal} \wedge c \in \text{RReal} \wedge$   
251  $b \mapsto c \in \text{lt} \Rightarrow$   
252  $\text{Infinity2Closed}(b) \cup \text{Open2Closed}(b,c) = \text{Infinity2Closed}(c)$   
253 *inf2cUo2o*:  
254  $\forall b, c \cdot$   
255  $b \in \text{RReal} \wedge c \in \text{RReal} \wedge$   
256  $b \mapsto c \in \text{lt} \Rightarrow$   
257  $\text{Infinity2Closed}(b) \cup \text{Open2Open}(b,c) = \text{Infinity2Open}(c)$   
258 *c2cUo2inf*:  
259  $\forall a, b \cdot$   
260  $a \in \text{RReal} \wedge b \in \text{RReal} \wedge$   
261  $a \mapsto b \in \text{leq} \Rightarrow$   
262  $\text{Closed2Closed}(a,b) \cup \text{Open2Infinity}(b) = \text{Closed2Infinity}(a)$   
263 *o2cUo2inf*:  
264  $\forall a, b \cdot$   
265  $a \in \text{RReal} \wedge b \in \text{RReal} \wedge$   
266  $a \mapsto b \in \text{lt} \Rightarrow$   
267  $\text{Open2Closed}(a,b) \cup \text{Open2Infinity}(b) = \text{Open2Infinity}(a)$   
268 *inf2cUo2inf*:  
269  $\forall a \cdot$   
270  $a \in \text{RReal} \Rightarrow$   
271  $\text{Infinity2Closed}(a) \cup \text{Open2Infinity}(a) = \text{RReal}$   
272 *c2oUc2c*:  
273  $\forall a, b, c \cdot$   
274  $a \in \text{RReal} \wedge b \in \text{RReal} \wedge c \in \text{RReal} \wedge$   
275  $a \mapsto b \in \text{lt} \wedge b \mapsto c \in \text{leq} \Rightarrow$   
276  $\text{Closed2Open}(a,b) \cup \text{Closed2Closed}(b,c) = \text{Closed2Closed}(a,c)$   
277 *c2oUc2o*:  
278  $\forall a, b, c \cdot$   
279  $a \in \text{RReal} \wedge b \in \text{RReal} \wedge c \in \text{RReal} \wedge$   
280  $a \mapsto b \in \text{lt} \wedge b \mapsto c \in \text{lt} \Rightarrow$   
281  $\text{Closed2Open}(a,b) \cup \text{Closed2Open}(b,c) = \text{Closed2Open}(a,c)$   
282 *o2oUc2c*:  
283  $\forall a, b, c \cdot$   
284  $a \in \text{RReal} \wedge b \in \text{RReal} \wedge c \in \text{RReal} \wedge$   
285  $a \mapsto b \in \text{lt} \wedge b \mapsto c \in \text{leq} \Rightarrow$   
286  $\text{Open2Open}(a,b) \cup \text{Closed2Closed}(b,c) = \text{Open2Closed}(a,c)$   
287 *o2oUc2o*:  
288  $\forall a, b, c \cdot$   
289  $a \in \text{RReal} \wedge b \in \text{RReal} \wedge c \in \text{RReal} \wedge$   
290  $a \mapsto b \in \text{lt} \wedge b \mapsto c \in \text{lt} \Rightarrow$   
291  $\text{Open2Open}(a,b) \cup \text{Closed2Open}(b,c) = \text{Open2Open}(a,c)$   
292 *inf2oUc2c*:  
293  $\forall b, c \cdot$   
294  $b \in \text{RReal} \wedge c \in \text{RReal} \wedge$   
295  $b \mapsto c \in \text{leq} \Rightarrow$

296  $\text{Infinity2Open}(b) \cup \text{Closed2Closed}(b,c) = \text{Infinity2Closed}(c)$   
297 *inf2oUc2o*:  
298  $\forall b, c \cdot$   
299  $b \in \text{RReal} \wedge c \in \text{RReal} \wedge$   
300  $b \mapsto c \in \text{lt} \Rightarrow$   
301  $\text{Infinity2Open}(b) \cup \text{Closed2Open}(b,c) = \text{Infinity2Open}(c)$   
302 *c2oUc2inf*:  
303  $\forall a, b \cdot$   
304  $a \in \text{RReal} \wedge b \in \text{RReal} \wedge$   
305  $a \mapsto b \in \text{lt} \Rightarrow$   
306  $\text{Closed2Open}(a,b) \cup \text{Closed2Infinity}(b) = \text{Closed2Infinity}(a)$   
307 *o2oUc2inf*:  
308  $\forall a, b \cdot$   
309  $a \in \text{RReal} \wedge b \in \text{RReal} \wedge$   
310  $a \mapsto b \in \text{lt} \Rightarrow$   
311  $\text{Open2Open}(a,b) \cup \text{Closed2Infinity}(b) = \text{Open2Infinity}(a)$   
312 *inf2oUc2inf*:  
313  $\forall a \cdot$   
314  $a \in \text{RReal} \Rightarrow$   
315  $\text{Infinity2Open}(a) \cup \text{Closed2Infinity}(a) = \text{RReal}$   
316 *c2cCc2c*:  
317  $\forall a, b, c \cdot$   
318  $a \in \text{RReal} \wedge b \in \text{RReal} \wedge c \in \text{RReal} \wedge$   
319  $a \mapsto b \in \text{leq} \wedge b \mapsto c \in \text{leq} \Rightarrow$   
320  $\text{Closed2Closed}(a,b) \cap \text{Closed2Closed}(b,c) = \{b\}$   
321 *o2cCc2c*:  
322  $\forall a, b, c \cdot$   
323  $a \in \text{RReal} \wedge b \in \text{RReal} \wedge c \in \text{RReal} \wedge$   
324  $a \mapsto b \in \text{lt} \wedge b \mapsto c \in \text{leq} \Rightarrow$   
325  $\text{Open2Closed}(a,b) \cap \text{Closed2Closed}(b,c) = \{b\}$   
326 *c2cCc2o*:  
327  $\forall a, b, c \cdot$   
328  $a \in \text{RReal} \wedge b \in \text{RReal} \wedge c \in \text{RReal} \wedge$   
329  $a \mapsto b \in \text{leq} \wedge b \mapsto c \in \text{lt} \Rightarrow$   
330  $\text{Closed2Closed}(a,b) \cap \text{Closed2Open}(b,c) = \{b\}$   
331 *o2cCc2o*:  
332  $\forall a, b, c \cdot$   
333  $a \in \text{RReal} \wedge b \in \text{RReal} \wedge c \in \text{RReal} \wedge$   
334  $a \mapsto b \in \text{lt} \wedge b \mapsto c \in \text{lt} \Rightarrow$   
335  $\text{Open2Closed}(a,b) \cap \text{Closed2Open}(b,c) = \{b\}$   
336 *inf2cCc2c*:  
337  $\forall b, c \cdot$   
338  $b \in \text{RReal} \wedge c \in \text{RReal} \wedge$   
339  $b \mapsto c \in \text{leq} \Rightarrow$   
340  $\text{Infinity2Closed}(b) \cap \text{Closed2Closed}(b,c) = \{b\}$   
341 *inf2cCc2o*:  
342  $\forall b, c \cdot$   
343  $b \in \text{RReal} \wedge c \in \text{RReal} \wedge$   
344  $b \mapsto c \in \text{lt} \Rightarrow$   
345  $\text{Infinity2Closed}(b) \cap \text{Closed2Open}(b,c) = \{b\}$   
346 *c2cCc2inf*:  
347  $\forall a, b \cdot$   
348  $a \in \text{RReal} \wedge b \in \text{RReal} \wedge$   
349  $a \mapsto b \in \text{leq} \Rightarrow$   
350  $\text{Closed2Closed}(a,b) \cap \text{Closed2Infinity}(b) = \{b\}$   
351 *o2cCc2inf*:  
352  $\forall a, b \cdot$   
353  $a \in \text{RReal} \wedge b \in \text{RReal} \wedge$   
354  $a \mapsto b \in \text{lt} \Rightarrow$   
355  $\text{Open2Closed}(a,b) \cap \text{Closed2Infinity}(b) = \{b\}$   
356 *inf2cCc2inf*:  
357  $\forall a \cdot$   
358  $a \in \text{RReal} \Rightarrow$   
359  $\text{Infinity2Closed}(a) \cap \text{Closed2Infinity}(a) = \{a\}$   
360 *c2oCc2c*:  
361  $\forall a, b, c \cdot$   
362  $a \in \text{RReal} \wedge b \in \text{RReal} \wedge c \in \text{RReal} \wedge$   
363  $a \mapsto b \in \text{lt} \wedge b \mapsto c \in \text{leq} \Rightarrow$   
364  $\text{Closed2Open}(a,b) \cap \text{Closed2Closed}(b,c) = \emptyset$   
365 *o2oCc2c*:  
366  $\forall a, b, c \cdot$   
367  $a \in \text{RReal} \wedge b \in \text{RReal} \wedge c \in \text{RReal} \wedge$   
368  $a \mapsto b \in \text{lt} \wedge b \mapsto c \in \text{leq} \Rightarrow$   
369  $\text{Open2Open}(a,b) \cap \text{Closed2Closed}(b,c) = \emptyset$

370 *c2oCc2o*:  
371  $\forall a, b, c \cdot$   
372  $a \in \text{RReal} \wedge b \in \text{RReal} \wedge c \in \text{RReal} \wedge$   
373  $a \mapsto b \in \text{lt} \wedge b \mapsto c \in \text{lt} \Rightarrow$   
374  $\text{Closed2Open}(a,b) \cap \text{Closed2Open}(b,c) = \emptyset$

375 *o2oCc2o*:  
376  $\forall a, b, c \cdot$   
377  $a \in \text{RReal} \wedge b \in \text{RReal} \wedge c \in \text{RReal} \wedge$   
378  $a \mapsto b \in \text{lt} \wedge b \mapsto c \in \text{lt} \Rightarrow$   
379  $\text{Open2Open}(a,b) \cap \text{Closed2Open}(b,c) = \emptyset$

380 *inf2oCc2c*:  
381  $\forall b, c \cdot$   
382  $b \in \text{RReal} \wedge c \in \text{RReal} \wedge$   
383  $b \mapsto c \in \text{leq} \Rightarrow$   
384  $\text{Infinity2Open}(b) \cap \text{Closed2Closed}(b,c) = \emptyset$

385 *inf2oCc2o*:  
386  $\forall b, c \cdot$   
387  $b \in \text{RReal} \wedge c \in \text{RReal} \wedge$   
388  $b \mapsto c \in \text{lt} \Rightarrow$   
389  $\text{Infinity2Open}(b) \cap \text{Closed2Open}(b,c) = \emptyset$

390 *c2oCc2inf*:  
391  $\forall a, b \cdot$   
392  $a \in \text{RReal} \wedge b \in \text{RReal} \wedge$   
393  $a \mapsto b \in \text{lt} \Rightarrow$   
394  $\text{Closed2Open}(a,b) \cap \text{Closed2Infinity}(b) = \emptyset$

395 *o2oCc2inf*:  
396  $\forall a, b \cdot$   
397  $a \in \text{RReal} \wedge b \in \text{RReal} \wedge$   
398  $a \mapsto b \in \text{lt} \Rightarrow$   
399  $\text{Open2Open}(a,b) \cap \text{Closed2Infinity}(b) = \emptyset$

400 *inf2oCc2inf*:  
401  $\forall a \cdot$   
402  $a \in \text{RReal} \Rightarrow$   
403  $\text{Infinity2Closed}(a) \cap \text{Closed2Infinity}(a) = \emptyset$

404 *c2cCo2c*:  
405  $\forall a, b, c \cdot$   
406  $a \in \text{RReal} \wedge b \in \text{RReal} \wedge c \in \text{RReal} \wedge$   
407  $a \mapsto b \in \text{leq} \wedge b \mapsto c \in \text{lt} \Rightarrow$   
408  $\text{Closed2Closed}(a,b) \cap \text{Open2Closed}(b,c) = \emptyset$

409 *o2cCo2c*:  
410  $\forall a, b, c \cdot$   
411  $a \in \text{RReal} \wedge b \in \text{RReal} \wedge c \in \text{RReal} \wedge$   
412  $a \mapsto b \in \text{lt} \wedge b \mapsto c \in \text{lt} \Rightarrow$   
413  $\text{Open2Closed}(a,b) \cap \text{Open2Closed}(b,c) = \emptyset$

414 *c2cCo2o*:  
415  $\forall a, b, c \cdot$   
416  $a \in \text{RReal} \wedge b \in \text{RReal} \wedge c \in \text{RReal} \wedge$   
417  $a \mapsto b \in \text{leq} \wedge b \mapsto c \in \text{lt} \Rightarrow$   
418  $\text{Closed2Closed}(a,b) \cap \text{Open2Open}(b,c) = \emptyset$

419 *o2cCo2o*:  
420  $\forall a, b, c \cdot$   
421  $a \in \text{RReal} \wedge b \in \text{RReal} \wedge c \in \text{RReal} \wedge$   
422  $a \mapsto b \in \text{lt} \wedge b \mapsto c \in \text{lt} \Rightarrow$   
423  $\text{Open2Closed}(a,b) \cap \text{Open2Open}(b,c) = \emptyset$

424 *inf2cCo2c*:  
425  $\forall b, c \cdot$   
426  $b \in \text{RReal} \wedge c \in \text{RReal} \wedge$   
427  $b \mapsto c \in \text{lt} \Rightarrow$   
428  $\text{Infinity2Closed}(b) \cap \text{Open2Closed}(b,c) = \emptyset$

429 *inf2cCo2o*:  
430  $\forall b, c \cdot$   
431  $b \in \text{RReal} \wedge c \in \text{RReal} \wedge$   
432  $b \mapsto c \in \text{lt} \Rightarrow$   
433  $\text{Infinity2Closed}(b) \cap \text{Open2Open}(b,c) = \emptyset$

434 *c2cCo2inf*:  
435  $\forall a, b \cdot$   
436  $a \in \text{RReal} \wedge b \in \text{RReal} \wedge$   
437  $a \mapsto b \in \text{leq} \Rightarrow$   
438  $\text{Closed2Closed}(a,b) \cap \text{Open2Infinity}(b) = \emptyset$

439 *o2cCo2inf*:  
440  $\forall a, b \cdot$   
441  $a \in \text{RReal} \wedge b \in \text{RReal} \wedge$   
442  $a \mapsto b \in \text{lt} \Rightarrow$   
443  $\text{Open2Closed}(a,b) \cap \text{Open2Infinity}(b) = \emptyset$

444 *inf2cCo2inf* :  
445  $\forall a \cdot$   
446  $a \in \text{RReal} \Rightarrow$   
447  $\text{Infinity2Closed}(a) \cap \text{Open2Infinity}(a) = \emptyset$   
448 *c2oCo2c* :  
449  $\forall a, b, c \cdot$   
450  $a \in \text{RReal} \wedge b \in \text{RReal} \wedge c \in \text{RReal} \wedge$   
451  $a \mapsto b \in \text{lt} \wedge b \mapsto c \in \text{lt} \Rightarrow$   
452  $\text{Closed2Open}(a, b) \cap \text{Open2Closed}(b, c) = \emptyset$   
453 *o2oCo2c* :  
454  $\forall a, b, c \cdot$   
455  $a \in \text{RReal} \wedge b \in \text{RReal} \wedge c \in \text{RReal} \wedge$   
456  $a \mapsto b \in \text{lt} \wedge b \mapsto c \in \text{lt} \Rightarrow$   
457  $\text{Open2Open}(a, b) \cap \text{Open2Closed}(b, c) = \emptyset$   
458 *c2oCo2o* :  
459  $\forall a, b, c \cdot$   
460  $a \in \text{RReal} \wedge b \in \text{RReal} \wedge c \in \text{RReal} \wedge$   
461  $a \mapsto b \in \text{lt} \wedge b \mapsto c \in \text{lt} \Rightarrow$   
462  $\text{Closed2Open}(a, b) \cap \text{Open2Open}(b, c) = \emptyset$   
463 *o2oCo2o* :  
464  $\forall a, b, c \cdot$   
465  $a \in \text{RReal} \wedge b \in \text{RReal} \wedge c \in \text{RReal} \wedge$   
466  $a \mapsto b \in \text{lt} \wedge b \mapsto c \in \text{lt} \Rightarrow$   
467  $\text{Open2Open}(a, b) \cap \text{Open2Open}(b, c) = \emptyset$   
468 *inf2oCo2c* :  
469  $\forall b, c \cdot$   
470  $b \in \text{RReal} \wedge c \in \text{RReal} \wedge$   
471  $b \mapsto c \in \text{lt} \Rightarrow$   
472  $\text{Infinity2Open}(b) \cap \text{Open2Closed}(b, c) = \emptyset$   
473 *inf2oCo2o* :  
474  $\forall b, c \cdot$   
475  $b \in \text{RReal} \wedge c \in \text{RReal} \wedge$   
476  $b \mapsto c \in \text{lt} \Rightarrow$   
477  $\text{Infinity2Open}(b) \cap \text{Open2Open}(b, c) = \emptyset$   
478 *c2oCo2inf* :  
479  $\forall a, b \cdot$   
480  $a \in \text{RReal} \wedge b \in \text{RReal} \wedge$   
481  $a \mapsto b \in \text{lt} \Rightarrow$   
482  $\text{Closed2Open}(a, b) \cap \text{Open2Infinity}(b) = \emptyset$   
483 *o2oCo2inf* :  
484  $\forall a, b \cdot$   
485  $a \in \text{RReal} \wedge b \in \text{RReal} \wedge$   
486  $a \mapsto b \in \text{lt} \Rightarrow$   
487  $\text{Open2Open}(a, b) \cap \text{Open2Infinity}(b) = \emptyset$   
488 *inf2oCo2inf* :  
489  $\forall a \cdot$   
490  $a \in \text{RReal} \Rightarrow$   
491  $\text{Infinity2Open}(a) \cap \text{Open2Infinity}(a) = \emptyset$

## PROOF RULES

493 *minimumRew* :

### Metavariables

495  $a : \text{RReal}$

496  $b : \text{RReal}$

### Rewrite Rules

498 *minClosed2Closed* :  $\text{Rmin}(\text{Closed2Closed}(a, b))$

499  $\text{rhs1} : a \mapsto b \in \text{leq} \Rightarrow a$

500 *minClosed2Open* :  $\text{Rmin}(\text{Closed2Open}(a, b))$

501  $\text{rhs1} : a \mapsto b \in \text{lt} \Rightarrow a$

502 *minClosed2Infinity* :  $\text{Rmin}(\text{Closed2Infinity}(a))$

503  $\text{rhs1} : \top \Rightarrow a$

504 *maximumRew* :

### Metavariables

506  $a : \text{RReal}$

507  $b : \text{RReal}$

### Rewrite Rules

509 *maxClosed2Closed* :  $\text{Rmax}(\text{Closed2Closed}(a, b))$

510  $\text{rhs1} : a \mapsto b \in \text{leq} \Rightarrow b$

511 *maxOpen2Closed* :  $\text{Rmax}(\text{Open2Closed}(a, b))$

512  $\text{rhs1} : a \mapsto b \in \text{lt} \Rightarrow b$

513 *maxInfinity2Closed* :  $\text{Rmax}(\text{Infinity2Closed}(b))$

514  $\text{rhs1} : \top \Rightarrow b$

515 *infimumRew* :

### Metavariables

517  $a : \text{RReal}$

```

518   b: RReal
519 Rewrite Rules
520   infClosed2Closed: Rinf( Closed2Closed(a, b))
521     rhs1: a ↦ b ∈ leq ⇒ a
522   infClosed2Open: Rinf( Closed2Open(a, b))
523     rhs1: a ↦ b ∈ lt ⇒ a
524   infOpen2Closed: Rinf( Open2Closed(a, b))
525     rhs1: a ↦ b ∈ lt ⇒ a
526   infOpen2Open: Rinf( Open2Open(a, b))
527     rhs1: a ↦ b ∈ lt ⇒ a
528   infClosed2Infinity: Rinf( Closed2Infinity(a))
529     rhs1: T ⇒ a
530   infOpen2Infinity: Rinf( Open2Infinity(a))
531     rhs1: T ⇒ a
532 supremumRew:
533 Metavariables
534   a: RReal
535   b: RReal
536 Rewrite Rules
537   supClosed2Closed: Rsup( Closed2Closed(a, b))
538     rhs1: a ↦ b ∈ leq ⇒ b
539   supClosed2Open: Rsup( Closed2Open(a, b))
540     rhs1: a ↦ b ∈ lt ⇒ b
541   supOpen2Closed: Rsup( Open2Closed(a, b))
542     rhs1: a ↦ b ∈ lt ⇒ b
543   supOpen2Open: Rsup( Open2Open(a, b))
544     rhs1: a ↦ b ∈ lt ⇒ b
545   supInfinity2Closed: Rsup( Infinity2Closed(b))
546     rhs1: T ⇒ b
547   supInfinity2Open: Rsup( Infinity2Open(b))
548     rhs1: T ⇒ b
549 intervalURew:
550 Metavariables
551   a: RReal
552   b: RReal
553   c: RReal
554 Rewrite Rules
555   c2cUc2c_rew: Closed2Closed(a, b) ∪ Closed2Closed(b, c)
556     rhs1: T ⇒ Closed2Closed(a, c)
557   o2cUc2c_rew: Open2Closed(a, b) ∪ Closed2Closed(b, c)
558     rhs1: T ⇒ Open2Closed(a, c)
559   c2cUc2o_rew: Closed2Closed(a, b) ∪ Closed2Open(b, c)
560     rhs1: T ⇒ Closed2Open(a, c)
561   o2cUc2o_rew: Open2Closed(a, b) ∪ Closed2Open(b, c)
562     rhs1: T ⇒ Open2Open(a, c)
563   inf2cUc2c_rew: Infinity2Closed(b) ∪ Closed2Closed(b, c)
564     rhs1: T ⇒ Infinity2Closed(c)
565   inf2cUc2o_rew: Infinity2Closed(b) ∪ Closed2Open(b, c)
566     rhs1: T ⇒ Infinity2Open(c)
567   c2cUc2inf_rew: Closed2Closed(a, b) ∪ Closed2Infinity(b)
568     rhs1: T ⇒ Closed2Infinity(a)
569   o2cUc2inf_rew: Open2Closed(a, b) ∪ Closed2Infinity(b)
570     rhs1: T ⇒ Open2Infinity(a)
571   c2oUc2c_rew: Closed2Open(a, b) ∪ Closed2Closed(b, c)
572     rhs1: T ⇒ Closed2Closed(a, c)
573   o2oUc2c_rew: Open2Open(a, b) ∪ Closed2Closed(b, c)
574     rhs1: T ⇒ Open2Closed(a, c)
575   c2oUc2o_rew: Closed2Open(a, b) ∪ Closed2Open(b, c)
576     rhs1: T ⇒ Closed2Open(a, c)
577   o2oUc2o_rew: Open2Open(a, b) ∪ Closed2Open(b, c)
578     rhs1: T ⇒ Open2Open(a, c)
579   inf2oUc2c_rew: Infinity2Open(b) ∪ Closed2Closed(b, c)
580     rhs1: T ⇒ Infinity2Closed(c)
581   inf2oUc2o_rew: Infinity2Open(b) ∪ Closed2Open(b, c)
582     rhs1: T ⇒ Infinity2Open(c)
583   c2oUc2inf_rew: Closed2Open(a, b) ∪ Closed2Infinity(b)
584     rhs1: T ⇒ Closed2Infinity(a)
585   o2oUc2inf_rew: Open2Open(a, b) ∪ Closed2Infinity(b)
586     rhs1: T ⇒ Open2Infinity(a)
587   c2cUo2c_rew: Closed2Closed(a, b) ∪ Open2Closed(b, c)
588     rhs1: T ⇒ Closed2Closed(a, c)
589   o2cUo2c_rew: Open2Closed(a, b) ∪ Open2Closed(b, c)
590     rhs1: T ⇒ Open2Closed(a, c)
591   c2cUo2o_rew: Closed2Closed(a, b) ∪ Open2Open(b, c)

```



592         $\text{rhs1} : T \Rightarrow \text{Closed2Open}(a, c)$   
 593     $\text{o2cUo2o\_rew} : \text{Open2Closed}(a, b) \cup \text{Open2Open}(b, c)$   
 594         $\text{rhs1} : T \Rightarrow \text{Open2Open}(a, c)$   
 595     $\text{inf2cUo2c\_rew} : \text{Infinity2Closed}(b) \cup \text{Open2Closed}(b, c)$   
 596         $\text{rhs1} : T \Rightarrow \text{Infinity2Closed}(c)$   
 597     $\text{inf2cUo2o\_rew} : \text{Infinity2Closed}(b) \cup \text{Open2Open}(b, c)$   
 598         $\text{rhs1} : T \Rightarrow \text{Infinity2Open}(c)$   
 599     $\text{c2cUo2inf\_rew} : \text{Closed2Closed}(a, b) \cup \text{Open2Infinity}(b)$   
 600         $\text{rhs1} : T \Rightarrow \text{Closed2Infinity}(a)$   
 601     $\text{o2cUo2inf\_rew} : \text{Open2Closed}(a, b) \cup \text{Open2Infinity}(b)$   
 602         $\text{rhs1} : T \Rightarrow \text{Open2Infinity}(a)$

603 **intervalCRew :**

604 **Metavariables**

605     $a : \text{RReal}$   
 606     $b : \text{RReal}$   
 607     $c : \text{RReal}$

608 **Rewrite Rules**

609     $\text{c2cCc2c\_rew} : \text{Closed2Closed}(a, b) \cap \text{Closed2Closed}(b, c)$   
 610         $\text{rhs1} : T \Rightarrow \{b\}$   
 611     $\text{o2cCc2c\_rew} : \text{Open2Closed}(a, b) \cap \text{Closed2Closed}(b, c)$   
 612         $\text{rhs1} : T \Rightarrow \{b\}$   
 613     $\text{c2cCc2o\_rew} : \text{Closed2Closed}(a, b) \cap \text{Closed2Open}(b, c)$   
 614         $\text{rhs1} : T \Rightarrow \{b\}$   
 615     $\text{o2cCc2o\_rew} : \text{Open2Closed}(a, b) \cap \text{Closed2Open}(b, c)$   
 616         $\text{rhs1} : T \Rightarrow \{b\}$   
 617     $\text{inf2cCc2c\_rew} : \text{Infinity2Closed}(b) \cap \text{Closed2Closed}(b, c)$   
 618         $\text{rhs1} : T \Rightarrow \{b\}$   
 619     $\text{inf2cCc2o\_rew} : \text{Infinity2Closed}(b) \cap \text{Closed2Open}(b, c)$   
 620         $\text{rhs1} : T \Rightarrow \{b\}$   
 621     $\text{c2cCc2inf\_rew} : \text{Closed2Closed}(a, b) \cap \text{Closed2Infinity}(b)$   
 622         $\text{rhs1} : T \Rightarrow \{b\}$   
 623     $\text{o2cCc2inf\_rew} : \text{Open2Closed}(a, b) \cap \text{Closed2Infinity}(b)$   
 624         $\text{rhs1} : T \Rightarrow \{b\}$

625 **intervalRealParts :**

626 **Rewrite Rules**

627     $\text{RPlus2Int} : \text{RRealPlus}$   
 628         $\text{rhs1} : T \Rightarrow \text{Closed2Infinity}(\text{Rzero})$   
 629     $\text{Int2RPlus} : \text{Closed2Infinity}(\text{Rzero})$   
 630         $\text{rhs1} : T \Rightarrow \text{RRealPlus}$   
 631     $\text{RMinus2Int} : \text{RRealMinus}$   
 632         $\text{rhs1} : T \Rightarrow \text{Infinity2Closed}(\text{Rzero})$   
 633     $\text{Int2RMinus} : \text{Infinity2Closed}(\text{Rzero})$   
 634         $\text{rhs1} : T \Rightarrow \text{RRealMinus}$   
 635     $\text{RPlusStar2Int} : \text{RRealPlusStar}$   
 636         $\text{rhs1} : T \Rightarrow \text{Open2Infinity}(\text{Rzero})$   
 637     $\text{Int2RPlusStar} : \text{Open2Infinity}(\text{Rzero})$   
 638         $\text{rhs1} : T \Rightarrow \text{RRealPlusStar}$   
 639     $\text{RMinusStar2Int} : \text{RRealMinusStar}$   
 640         $\text{rhs1} : T \Rightarrow \text{Infinity2Open}(\text{Rzero})$   
 641     $\text{Int2RMinusStar} : \text{Infinity2Open}(\text{Rzero})$   
 642         $\text{rhs1} : T \Rightarrow \text{RRealMinusStar}$

643 **intervalInclusion :**

644 **Metavariables**

645     $a1 : \text{RReal}$   
 646     $b1 : \text{RReal}$   
 647     $a2 : \text{RReal}$   
 648     $b2 : \text{RReal}$

649 **Rewrite Rules**

650     $\text{c2InfIncc2Inf} : \text{Closed2Infinity}(a1) \subseteq \text{Closed2Infinity}(a2)$   
 651         $\text{rhs1} : T \Rightarrow a2 \mapsto a1 \in \text{leq}$   
 652     $\text{o2InfIncc2Inf} : \text{Open2Infinity}(a1) \subseteq \text{Closed2Infinity}(a2)$   
 653         $\text{rhs1} : T \Rightarrow a2 \mapsto a1 \in \text{leq}$   
 654     $\text{c2InfInco2Inf} : \text{Closed2Infinity}(a1) \subseteq \text{Open2Infinity}(a2)$   
 655         $\text{rhs1} : T \Rightarrow a2 \mapsto a1 \in \text{lt}$   
 656     $\text{o2InfInco2Inf} : \text{Open2Infinity}(a1) \subseteq \text{Open2Infinity}(a2)$   
 657         $\text{rhs1} : T \Rightarrow a2 \mapsto a1 \in \text{leq}$   
 658     $\text{inf2cIncinf2c} : \text{Infinity2Closed}(b1) \subseteq \text{Infinity2Closed}(b2)$   
 659         $\text{rhs1} : T \Rightarrow b1 \mapsto b2 \in \text{leq}$   
 660     $\text{inf2oIncinf2c} : \text{Infinity2Open}(b1) \subseteq \text{Infinity2Closed}(b2)$   
 661         $\text{rhs1} : T \Rightarrow b1 \mapsto b2 \in \text{leq}$   
 662     $\text{inf2cIncinf2o} : \text{Infinity2Closed}(b1) \subseteq \text{Infinity2Open}(b2)$   
 663         $\text{rhs1} : T \Rightarrow b1 \mapsto b2 \in \text{lt}$   
 664     $\text{inf2oIncinf2o} : \text{Infinity2Open}(b1) \subseteq \text{Infinity2Open}(b2)$   
 665         $\text{rhs1} : T \Rightarrow b1 \mapsto b2 \in \text{leq}$

666  $o2oIno2o$ :  $\text{Open2Open}(a1, b1) \subseteq \text{Open2Open}(a2, b2)$   
667  $\text{rhs1}$ :  $T \Rightarrow a1 \mapsto a2 \in \text{lt} \wedge b1 \mapsto b2 \in \text{gt}$   
668  $c2oInc2c$ :  $\text{Closed2Open}(a1, b1) \subseteq \text{Closed2Closed}(a2, b2)$   
669  $\text{rhs1}$ :  $T \Rightarrow a1 \mapsto a2 \in \text{leq} \wedge b1 \mapsto b2 \in \text{geq}$   
670  $\text{intervalInclusion2}$ :  
671 **Metavariables**  
672  $a$ :  $\mathbb{R}\text{Real}$   
673  $b$ :  $\mathbb{R}\text{Real}$   
674 **Rewrite Rules**  
675  $s\_o2oIno2Inf$ :  $\text{Open2Open}(a, b) \subseteq \text{Open2Infinity}(a)$   
676  $\text{rhs1}$ :  $T \Rightarrow T$   
677  $s\_o2cIno2Inf$ :  $\text{Open2Closed}(a, b) \subseteq \text{Open2Infinity}(a)$   
678  $\text{rhs1}$ :  $T \Rightarrow T$   
679  $s\_o2oInc2Inf$ :  $\text{Open2Open}(a, b) \subseteq \text{Closed2Infinity}(a)$   
680  $\text{rhs1}$ :  $T \Rightarrow T$   
681  $s\_o2cInc2Inf$ :  $\text{Open2Closed}(a, b) \subseteq \text{Closed2Infinity}(a)$   
682  $\text{rhs1}$ :  $T \Rightarrow T$   
683  $s\_c2oInc2Inf$ :  $\text{Closed2Open}(a, b) \subseteq \text{Closed2Infinity}(a)$   
684  $\text{rhs1}$ :  $T \Rightarrow T$   
685  $s\_c2cInc2Inf$ :  $\text{Closed2Closed}(a, b) \subseteq \text{Closed2Infinity}(a)$   
686  $\text{rhs1}$ :  $T \Rightarrow T$   
687  $s\_o2oIno2c$ :  $\text{Open2Open}(a, b) \subseteq \text{Open2Closed}(a, b)$   
688  $\text{rhs1}$ :  $T \Rightarrow T$   
689  $s\_o2oInc2o$ :  $\text{Open2Open}(a, b) \subseteq \text{Closed2Open}(a, b)$   
690  $\text{rhs1}$ :  $T \Rightarrow T$   
691  $s\_o2oInc2c$ :  $\text{Open2Open}(a, b) \subseteq \text{Closed2Closed}(a, b)$   
692  $\text{rhs1}$ :  $T \Rightarrow T$   
693  $s\_c2oInc2c$ :  $\text{Closed2Open}(a, b) \subseteq \text{Closed2Closed}(a, b)$   
694  $\text{rhs1}$ :  $T \Rightarrow T$   
695  $s\_o2cInc2c$ :  $\text{Open2Closed}(a, b) \subseteq \text{Closed2Closed}(a, b)$   
696  $\text{rhs1}$ :  $T \Rightarrow T$   
697  $\text{degenerated\_intervals}$ :  
698 **Metavariables**  
699  $a$ :  $\mathbb{R}\text{Real}$   
700 **Rewrite Rules**  
701  $c2c\_single$ :  $\text{Closed2Closed}(a, a)$   
702  $\text{rhs1}$ :  $T \Rightarrow \{a\}$   
703  $o2c\_empty$ :  $\text{Open2Closed}(a, a)$   
704  $\text{rhs1}$ :  $T \Rightarrow \emptyset : \mathbb{P}(\mathbb{R}\text{Real})$   
705  $c2o\_empty$ :  $\text{Closed2Open}(a, a)$   
706  $\text{rhs1}$ :  $T \Rightarrow \emptyset : \mathbb{P}(\mathbb{R}\text{Real})$   
707  $o2o\_empty$ :  $\text{Open2Open}(a, a)$   
708  $\text{rhs1}$ :  $T \Rightarrow \emptyset : \mathbb{P}(\mathbb{R}\text{Real})$   
709  $\text{intervalElements}$ :  
710 **Metavariables**  
711  $a$ :  $\mathbb{R}\text{Real}$   
712  $b$ :  $\mathbb{R}\text{Real}$   
713 **Rewrite Rules**  
714  $a\_in\_c2c$ :  $a \in \text{Closed2Closed}(a, b)$   
715  $\text{rhs1}$ :  $T \Rightarrow a \mapsto b \in \text{leq}$   
716  $a\_in\_c2o$ :  $a \in \text{Closed2Open}(a, b)$   
717  $\text{rhs1}$ :  $T \Rightarrow a \mapsto b \in \text{lt}$   
718  $b\_in\_c2c$ :  $b \in \text{Closed2Closed}(a, b)$   
719  $\text{rhs1}$ :  $T \Rightarrow a \mapsto b \in \text{leq}$   
720  $b\_in\_o2c$ :  $b \in \text{Open2Closed}(a, b)$   
721  $\text{rhs1}$ :  $T \Rightarrow a \mapsto b \in \text{lt}$   
722  $a\_in\_c2Inf$ :  $a \in \text{Closed2Infinity}(a)$   
723  $\text{rhs1}$ :  $T \Rightarrow T$   
724  $b\_in\_Inf2c$ :  $b \in \text{Infinity2Closed}(b)$   
725  $\text{rhs1}$ :  $T \Rightarrow T$   
726 **END**