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1 THEORY Approximation
2 IMPORT THEORY PROJECTS
3 /SimpleDEq THEORIES /SimpleDEq/ApproximationBase.dtf | org.eventb.theory.core.deployedTheoryRoot#
4 ApproximationBase
5 TYPE PARAMETERS E, F1, F2, F3, UF1, UF2, F
6 OPERATORS
7 FDeltaNeighborhood <predicate> (PE:  $\mathbb{P}(E)$ , delta:  $\mathbb{RRealPlus}$ , a: F, f2:  $E \rightarrow F$ )
8 well-definedness  $PE \subseteq \text{dom}(f2)$ 
9 direct definition
10  $\forall x \cdot x \in PE \Rightarrow \text{DeltaNeighborhood}(\text{delta}, a, f2(x))$ 
11 DeltaApproximation <predicate> (PE:  $\mathbb{P}(E)$ , delta:  $\mathbb{RRealPlus}$ , f1:  $E \rightarrow F$ , f2:  $E \rightarrow F$ )
12 well-definedness  $PE \subseteq \text{dom}(f1), PE \subseteq \text{dom}(f2)$ 
13 direct definition
14  $\forall x \cdot x \in PE \Rightarrow \text{DeltaNeighborhood}(\text{delta}, f1(x), f2(x))$ 
15 DeltaApproximationEq <predicate> (DR:  $\mathbb{P}(\mathbb{RReal})$ , delta:  $\mathbb{RRealPlus}$ , e1:  $DE(F)$ , e2:  $DE(F)$ )
16 well-definedness  $\text{Solvable}(\text{DR}, e1), \text{Solvable}(\text{DR}, e2)$ 
17 direct definition
18  $\forall \text{eta1}, \text{eta2} \cdot$ 
19  $\text{eta1} \in \mathbb{RReal} \rightarrow F \wedge \text{eta2} \in \mathbb{RReal} \rightarrow F \wedge$ 
20  $\text{DR} \subseteq \text{dom}(\text{eta1}) \wedge \text{DR} \subseteq \text{dom}(\text{eta2}) \wedge$ 
21  $\text{solutionOf}(\text{DR}, \text{eta1}, e1) \wedge \text{solutionOf}(\text{DR}, \text{eta2}, e2)$ 
22  $\Rightarrow \text{DeltaApproximation}(\text{DR}, \text{delta}, \text{eta1}, \text{eta2})$ 
23 DeltaApproximationEqObs <predicate> (DR:  $\mathbb{P}(\mathbb{RReal})$ , delta:  $\mathbb{RRealPlus}$ , ee1:  $DE(F1)$ , g1:  $F1 \rightarrow F$ , ee2:  $DE(F2)$ ,
24 g2:  $F2 \rightarrow F$ )
25 well-definedness  $\text{Solvable}(\text{DR}, ee1), \text{Solvable}(\text{DR}, ee2)$ 
26 direct definition
27  $\forall \text{eta1}, \text{eta2} \cdot$ 
28  $\text{eta1} \in \mathbb{RReal} \rightarrow F1 \wedge \text{eta2} \in \mathbb{RReal} \rightarrow F2 \wedge$ 
29  $\text{DR} \subseteq \text{dom}(\text{eta1}) \wedge \text{DR} \subseteq \text{dom}(\text{eta2}) \wedge$ 
30  $\text{solutionOf}(\text{DR}, \text{eta1}, ee1) \wedge \text{solutionOf}(\text{DR}, \text{eta2}, ee2)$ 
31  $\Rightarrow \text{DeltaApproximation}(\text{DR}, \text{delta}, g1 \circ \text{eta1}, g2 \circ \text{eta2})$ 
32 DeltaApproximationEqF <predicate> (DR:  $\mathbb{P}(\mathbb{RReal})$ , delta:  $\mathbb{RRealPlus}$ , e:  $DE(F)$ , g:  $\mathbb{RReal} \rightarrow F$ )
33 well-definedness  $\text{Solvable}(\text{DR}, e), \text{DR} \subseteq \text{dom}(g)$ 
34 direct definition
35  $\forall \text{eta} \cdot$ 
36  $\text{eta} \in \mathbb{RReal} \rightarrow F \wedge \text{DR} \subseteq \text{dom}(\text{eta}) \wedge \text{solutionOf}(\text{DR}, \text{eta}, e)$ 
37  $\Rightarrow \text{DeltaApproximation}(\text{DR}, \text{delta}, \text{eta}, g)$ 
38 ProjectiveHull <expression> (delta:  $\mathbb{RRealPlus}$ , SF:  $\mathbb{P}(F)$ )
39 direct definition
40  $\{ f2 \mid f2 \in F \wedge (\exists f1 \cdot f1 \in \text{SF} \wedge \text{DeltaNeighborhood}(\text{delta}, f1, f2)) \}$ 
41 DeltaNeighborhoodSet <expression> (delta:  $\mathbb{RRealPlus}$ , x: E)
42 direct definition
43  $\{ y \mid y \in E \wedge \text{DeltaNeighborhood}(\text{delta}, x, y) \}$ 
44 AXIOMATIC DEFINITIONS simulation_functions:
45 OPERATORS
46 SimulationFunctions <expression> (DF1:  $\mathbb{P}(F1)$ , DF2:  $\mathbb{P}(F2)$ , DUF1:  $\mathbb{P}(UF1)$ , DUF2:  $\mathbb{P}(UF2)$ , f1:  $F1 \times UF1 \rightarrow F1$ ,
47 f2:  $F2 \times UF2 \rightarrow F2$ , g1:  $F1 \rightarrow F$ , g2:  $F2 \rightarrow F$ ) :  $\mathbb{P}(\mathbb{P}((F1 \times F2) \times \mathbb{RReal}))$ 
48 well-definedness  $\text{DF1} \times \text{DUF1} \subseteq \text{dom}(f1), \text{DF2} \times \text{DUF2} \subseteq \text{dom}(f2)$ 
49 AXIOMS
50 SimulationFunctions_char:
51  $\forall \text{DF1}, \text{DF2}, \text{DUF1}, \text{DUF2}, f1, f2, g1, g2, V \cdot$ 
52  $\text{DF1} \subseteq F1 \wedge \text{DF2} \subseteq F2 \wedge \text{DUF1} \subseteq UF1 \wedge \text{DUF2} \subseteq UF2 \wedge$ 
53  $f1 \in F1 \times UF1 \rightarrow F1 \wedge f2 \in F2 \times UF2 \rightarrow F2 \wedge$ 
54  $g1 \in F1 \rightarrow F \wedge g2 \in F2 \rightarrow F \wedge$ 
55  $\text{DF1} \times \text{DUF1} \subseteq \text{dom}(f1) \wedge \text{DF2} \times \text{DUF2} \subseteq \text{dom}(f2) \wedge$ 
56  $V \in \text{SimulationFunctions}(\text{DF1}, \text{DF2}, \text{DUF1}, \text{DUF2}, f1, f2, g1, g2) \wedge$ 
57  $V \in F1 \times F2 \rightarrow \mathbb{RReal}$ 
58  $\Rightarrow ($ 
59  $\text{DF1} \times \text{DF2} \subseteq \text{dom}(V)$ 
60  $\wedge (\forall x1, x2 \cdot x1 \in \text{DF1} \wedge x2 \in \text{DF2} \Rightarrow V(x1 \mapsto x2) \in \mathbb{RRealPlus})$ 
61  $)$ 
62 SimulationFunction_control_ODE:
63  $\forall \text{DR}, \text{eta01}, \text{eta02}, t0, \text{DF1}, \text{DF2}, \text{DUF1}, \text{DUF2}, f1, f2, g1, g2, u1, u2, V \cdot$ 
64  $\text{DR} \subseteq \mathbb{RReal} \wedge t0 \in \text{DR} \wedge \text{eta01} \in F1 \wedge \text{eta02} \in F2 \wedge$ 
65  $\text{DF1} \subseteq F1 \wedge \text{DF2} \subseteq F2 \wedge \text{DUF1} \subseteq UF1 \wedge \text{DUF2} \subseteq UF2 \wedge$ 
66  $f1 \in F1 \times UF1 \rightarrow F1 \wedge \text{DF1} \times \text{DUF1} \subseteq \text{dom}(f1) \wedge$ 
67  $f2 \in F2 \times UF2 \rightarrow F2 \wedge \text{DF2} \times \text{DUF2} \subseteq \text{dom}(f2) \wedge$ 
68  $g1 \in F1 \rightarrow F \wedge g2 \in F2 \rightarrow F \wedge$ 
69  $u1 \in \mathbb{RReal} \rightarrow UF1 \wedge \text{DR} \subseteq \text{dom}(u1) \wedge \text{ran}(u1) \subseteq UF1 \wedge$ 
70  $u2 \in \mathbb{RReal} \rightarrow UF2 \wedge \text{DR} \subseteq \text{dom}(u2) \wedge \text{ran}(u2) \subseteq UF2 \wedge$ 
71  $V \in \text{SimulationFunctions}(\text{DF1}, \text{DF2}, \text{DUF1}, \text{DUF2}, f1, f2, g1, g2) \wedge$ 
72  $\text{SolvableWith}(\text{DR}, \text{caode}(f1, \text{eta01}, t0), u1) \wedge \text{SolvableWith}(\text{DR}, \text{caode}(f2, \text{eta02}, t0), u2) \Rightarrow$ 
73  $(\exists \text{delta} \cdot$ 

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71 $\text{delta} \in \text{RRealPlus} \wedge$
72 $\text{DeltaApproximationEqObs}(\text{DR}, \text{delta},$
73 $\quad \text{withControl}(\text{DR}, \text{caode}(f1, \text{eta01}, t0), u1), g1,$
74 $\quad \text{withControl}(\text{DR}, \text{caode}(f2, \text{eta02}, t0), u2), g2$
75 $\quad)$
76 $)$

77 *SimulationFunction_control_ODE_delta:*

78 $\forall \text{DR}, \text{eta01}, \text{eta02}, t0, \text{DF1}, \text{DF2}, \text{DUF1}, \text{DUF2}, f1, f2, g1, g2, u1, u2, V, \text{delta} \cdot$
79 $\text{DR} \subseteq \text{RReal} \wedge t0 \in \text{DR} \wedge \text{eta01} \in F1 \wedge \text{eta02} \in F2 \wedge$
80 $\text{DF1} \subseteq F1 \wedge \text{DF2} \subseteq F2 \wedge \text{DUF1} \subseteq \text{UF1} \wedge \text{DUF2} \subseteq \text{UF2} \wedge$
81 $f1 \in F1 \times \text{UF1} \rightarrow F1 \wedge \text{DF1} \times \text{DUF1} \subseteq \text{dom}(f1) \wedge$
82 $f2 \in F2 \times \text{UF2} \rightarrow F2 \wedge \text{DF2} \times \text{DUF2} \subseteq \text{dom}(f2) \wedge$
83 $g1 \in F1 \rightarrow F \wedge g2 \in F2 \rightarrow F \wedge$
84 $u1 \in \text{RReal} \rightarrow \text{UF1} \wedge \text{DR} \subseteq \text{dom}(u1) \wedge \text{ran}(u1) \subseteq \text{UF1} \wedge$
85 $u2 \in \text{RReal} \rightarrow \text{UF2} \wedge \text{DR} \subseteq \text{dom}(u2) \wedge \text{ran}(u2) \subseteq \text{UF2} \wedge$
86 $V \in \text{SimulationFunctions}(\text{DF1}, \text{DF2}, \text{DUF1}, \text{DUF2}, f1, f2, g1, g2) \wedge$
87 $\text{delta} \in \text{RReal} \wedge \text{Rzero} \mapsto \text{delta} \in \text{lt} \wedge \text{boundedBy}(\text{DF1} \times \text{DF2}, V, \text{Rzero}, \text{delta}) \wedge$
88 $\text{SolvableWith}(\text{DR}, \text{caode}(f1, \text{eta01}, t0), u1) \wedge \text{SolvableWith}(\text{DR}, \text{caode}(f2, \text{eta02}, t0), u2) \Rightarrow$
89 $\quad \text{DeltaApproximationEqObs}(\text{DR}, \text{delta},$
90 $\quad \quad \text{withControl}(\text{DR}, \text{caode}(f1, \text{eta01}, t0), u1), g1,$
91 $\quad \quad \text{withControl}(\text{DR}, \text{caode}(f2, \text{eta02}, t0), u2), g2$
92 $\quad)$ neighborhoods:

AXIOMS

94 *deltaNSet_open:*

95 $\forall \text{delta}, x \cdot \text{delta} \in \text{RRealPlus} \wedge \text{Rzero} \mapsto \text{delta} \in \text{lt} \wedge x \in E \Rightarrow$
96 $\quad \text{IsOpen}(\text{DeltaNeighborhoodSet}(\text{delta}, x))$

THEOREMS

98 *deltaA_restriction:*

99 $\forall \text{PE}, \text{PE2}, \text{delta}, f1, f2 \cdot$
100 $\text{PE} \subseteq E \wedge \text{PE2} \subseteq \text{PE} \wedge \text{delta} \in \text{RRealPlus} \wedge$
101 $f1 \in E \rightarrow F \wedge f2 \in E \rightarrow F \wedge$
102 $\text{PE} \subseteq \text{dom}(f1) \wedge \text{PE} \subseteq \text{dom}(f2) \wedge$
103 $\text{DeltaApproximation}(\text{PE}, \text{delta}, f1, f2)$
104 \Rightarrow
105 $\quad \text{DeltaApproximation}(\text{PE2}, \text{delta}, f1, f2)$

106 *deltaAeq_restriction:*

107 $\forall \text{DR}, \text{DR2}, \text{delta}, e1, e2 \cdot$
108 $\text{DR} \subseteq \text{RReal} \wedge \text{DR2} \subseteq \text{DR} \wedge \text{delta} \in \text{RRealPlus} \wedge$
109 $e1 \in \text{DE}(F) \wedge e2 \in \text{DE}(F) \wedge$
110 $\text{Solvable}(\text{DR}, e1) \wedge \text{Solvable}(\text{DR2}, e2) \wedge$
111 $\text{DeltaApproximationEq}(\text{DR}, \text{delta}, e1, e2)$
112 \Rightarrow
113 $\quad \text{DeltaApproximationEq}(\text{DR2}, \text{delta}, e1, e2)$

114 *deltaAeqobs_restriction:*

115 $\forall \text{DR}, \text{DR2}, \text{delta}, e1, e2, g1, g2 \cdot$
116 $\text{DR} \subseteq \text{RReal} \wedge \text{DR2} \subseteq \text{DR} \wedge \text{delta} \in \text{RRealPlus} \wedge$
117 $e1 \in \text{DE}(F1) \wedge e2 \in \text{DE}(F2) \wedge$
118 $g1 \in F1 \rightarrow F \wedge g2 \in F2 \rightarrow F \wedge$
119 $\text{Solvable}(\text{DR}, e1) \wedge \text{Solvable}(\text{DR2}, e2) \wedge$
120 $\text{DeltaApproximationEqObs}(\text{DR}, \text{delta}, e1, g1, e2, g2)$
121 \Rightarrow
122 $\quad \text{DeltaApproximationEqObs}(\text{DR2}, \text{delta}, e1, g1, e2, g2)$

123 *deltaA_comp:*

124 $\forall \text{DF1}, \text{PE}, f, g, h, \text{delta} \cdot$
125 $\text{DF1} \subseteq F1 \wedge \text{PE} \subseteq E \wedge$
126 $g \in F1 \rightarrow E \wedge f \in E \rightarrow F \wedge h \in E \rightarrow F \wedge$
127 $\text{DF1} \subseteq \text{dom}(g) \wedge \text{PE} \subseteq \text{dom}(f) \wedge \text{PE} \subseteq \text{dom}(h) \wedge$
128 $\text{DeltaApproximation}(\text{PE}, \text{delta}, f, h) \wedge$
129 $(\forall x \cdot x \in \text{DF1} \Rightarrow g(x) \in \text{PE}) \Rightarrow$
130 $\quad \text{DeltaApproximation}(\text{DF1}, \text{delta}, f \circ g, h \circ g)$

131 *deltaA_commutative:*

132 $\forall \text{PE}, \text{delta}, f1, f2 \cdot$
133 $\text{PE} \subseteq E \wedge \text{delta} \in \text{RRealPlus} \wedge f1 \in E \rightarrow F \wedge f2 \in E \rightarrow F \wedge$
134 $\text{PE} \subseteq \text{dom}(f1) \wedge \text{PE} \subseteq \text{dom}(f2) \Rightarrow$
135 $\quad (\text{DeltaApproximation}(\text{PE}, \text{delta}, f1, f2) \Leftrightarrow \text{DeltaApproximation}(\text{PE}, \text{delta}, f2, f1))$

136 *deltaAeq_commutative:*

137 $\forall \text{DR}, \text{delta}, e1, e2 \cdot$
138 $\text{DR} \subseteq \text{RReal} \wedge \text{delta} \in \text{RRealPlus} \wedge e1 \in \text{DE}(F) \wedge e2 \in \text{DE}(F) \wedge \text{Solvable}(\text{DR}, e1) \wedge \text{Solvable}(\text{DR}, e2)$
139 \Rightarrow
140 $\quad (\text{DeltaApproximationEq}(\text{DR}, \text{delta}, e1, e2) \Leftrightarrow \text{DeltaApproximationEq}(\text{DR}, \text{delta}, e2, e1))$

141 *deltaAeqobs_commutative:*

142 $\forall \text{DR}, \text{delta}, e1, e2, g1, g2 \cdot$
143 $\text{DR} \subseteq \text{RReal} \wedge \text{delta} \in \text{RRealPlus} \wedge$
 $e1 \in \text{DE}(F1) \wedge e2 \in \text{DE}(F2) \wedge$

144 Solvable(DR, e1) \wedge Solvable(DR, e2) \wedge

145 $g1 \in F1 \rightarrow F \wedge g2 \in F2 \rightarrow F$

146 \Rightarrow

147 (DeltaApproximationEqObs(DR, delta, e1, g1, e2, g2)

148 \Leftrightarrow

149 DeltaApproximationEqObs(DR, delta, e2, g2, e1, g1))

150 *deltaApp_induction:*

151 \forall delta, eqA, eqC, etaA, etaC, etaAp, etaCp, t, tp, InvA, InvC .

152 delta \in RRealPlus \wedge

153 t \in RRealPlus \wedge

154 eqA \in DE(F) \wedge eqC \in DE(F) \wedge

155 etaA \in RReal \Rightarrow F \wedge Closed2Closed(Rzero, t) \subseteq dom(etaA) \wedge

156 etaC \in RReal \Rightarrow F \wedge Closed2Closed(Rzero, t) \subseteq dom(etaC) \wedge

157 DeltaApproximationEq(Closed2Closed(t, tp), delta, eqA, eqC) \wedge

158 DeltaApproximation(Closed2Closed(Rzero, t), delta, etaA, etaC) \wedge

159 CBAPsolutionOf(t, tp, etaA, etaAp, eqA, InvA) \wedge CBAPsolutionOf(t, tp, etaC, etaCp, eqC, InvC)

160 \Rightarrow DeltaApproximation(Closed2Closed(Rzero, tp), delta, etaAp, etaCp)

161 *deltaApp_obs_induction:*

162 \forall delta, eqA, eqC, gA, gC, etaA, etaC, etaAp, etaCp, t, tp, InvA, InvC .

163 delta \in RRealPlus \wedge

164 t \in RRealPlus \wedge

165 eqA \in DE(F1) \wedge eqC \in DE(F2) \wedge gA \in F1 \rightarrow F \wedge gC \in F2 \rightarrow F \wedge

166 etaA \in RReal \Rightarrow F1 \wedge Closed2Closed(Rzero, t) \subseteq dom(etaA) \wedge

167 etaC \in RReal \Rightarrow F2 \wedge Closed2Closed(Rzero, t) \subseteq dom(etaC) \wedge

168 DeltaApproximationEqObs(Closed2Closed(t, tp), delta, eqA, gA, eqC, gC) \wedge

169 DeltaApproximation(Closed2Closed(Rzero, t), delta, gA \circ etaA, gC \circ etaC) \wedge

170 CBAPsolutionOf(t, tp, etaA, etaAp, eqA, InvA) \wedge CBAPsolutionOf(t, tp, etaC, etaCp, eqC, InvC)

171 \Rightarrow DeltaApproximation(Closed2Closed(Rzero, tp), delta, gA \circ etaAp, gC \circ etaCp)

172 *ProjectiveHull_INV:*

173 \forall delta, xA, xC, IA, IC .

174 delta \in RRealPlus \wedge

175 IA \subseteq F \wedge xA \in F \wedge

176 IC \subseteq F \wedge xC \in F \wedge xC \in IC \wedge

177 ProjectiveHull(delta, IC) \subseteq IA \Rightarrow

178 xA \in IA

179 **END**