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1 THEORY PlannarControlTheory
2 IMPORT THEORY PROJECTS
3 /SimpleDEq THEORIES /SimpleDEq/Approximation.dtf|org.eventb.theory.core.deployedTheoryRoot#
4 Approximation
5 OPERATORS
6 SecondOrder2DimensionSystemFunction <expression> (correction_coeff: RReal, controlArea4:  $\mathbb{P}((\text{RReal} \times \text{RReal}) \times (\text{RReal} \times \text{RReal}))$ )
7 direct definition
8  $(\lambda ((x1 \mapsto x2) \mapsto (x3 \mapsto x4)) \mapsto ((vx \mapsto vy) \mapsto (wx \mapsto wy))) \cdot$ 
9  $((x1 \mapsto x2) \mapsto (x3 \mapsto x4)) \in (\text{RReal} \times \text{RReal}) \times (\text{RReal} \times \text{RReal}) \wedge ((vx \mapsto vy) \mapsto (wx \mapsto wy)) \in (\text{RReal} \times \text{RReal}) \times (\text{RReal} \times \text{RReal}) \wedge ((vx \mapsto vy) \mapsto (wx \mapsto wy)) \in \text{controlArea4}$ 
10  $| (\text{minus}(\text{divide}(vx \mapsto \text{Rtwo}) \mapsto \text{minus}(\text{times}(\text{correction\_coeff} \mapsto \text{minus}(x3 \mapsto wx)) \mapsto x1))$ 
11  $\mapsto \text{minus}(\text{divide}(vy \mapsto \text{Rtwo}) \mapsto \text{minus}(\text{times}(\text{correction\_coeff} \mapsto \text{minus}(x4 \mapsto wy)) \mapsto x2)))$ 
12  $\mapsto (x1$ 
13  $\mapsto x2)$ 
14 FirstOrder2DimensionSystemFunction <expression> (controlArea2:  $\mathbb{P}(\text{RReal} \times \text{RReal})$ )
15 direct definition
16  $(\lambda (x1 \mapsto x2) \mapsto (vx \mapsto vy)) \cdot$ 
17  $(x1 \mapsto x2) \in \text{RReal} \times \text{RReal} \wedge (vx \mapsto vy) \in \text{RReal} \times \text{RReal} \wedge (vx \mapsto vy) \in \text{controlArea2}$ 
18  $| (vx$ 
19  $\mapsto vy$ 
20  $)$ 
21  $)$ 
22 SecondOrder2DimensionSystem <expression> (correction_coeff: RReal, controlArea4:  $\mathbb{P}((\text{RReal} \times \text{RReal}) \times (\text{RReal} \times \text{RReal}))$ , t0: RRealPlus, x0:  $(\text{RReal} \times \text{RReal}) \times (\text{RReal} \times \text{RReal})$ )
23 direct definition
24 caode(
25 SecondOrder2DimensionSystemFunction(correction_coeff, controlArea4),
26 x0,
27 t0
28 )
29 FirstOrder2DimensionSystem <expression> (controlArea2:  $\mathbb{P}(\text{RReal} \times \text{RReal})$ , t0: RRealPlus, y0: RReal × RReal)
30 direct definition
31 caode(
32 FirstOrder2DimensionSystemFunction(controlArea2),
33 y0,
34 t0
35 )
36 PointwiseSlopedControl <expression> (DR:  $\mathbb{P}(\text{RReal})$ , vx: RReal, vy: RReal, t0: RReal)
37 well-definedness t0 ∈ DR
38 direct definition
39  $(\lambda t \cdot t \in \text{DR} \wedge t0 \mapsto t \in \text{leq} |$ 
40  $(vx \mapsto vy)$ 
41  $\mapsto$ 
42  $(\text{times}(vx \mapsto \text{minus}(t \mapsto t0)) \mapsto \text{times}(vy \mapsto \text{minus}(t \mapsto t0))))$ 
43  $)$ 
44 PointwiseControl <expression> (DR:  $\mathbb{P}(\text{RReal})$ , vx: RReal, vy: RReal, t0: RReal)
45 well-definedness t0 ∈ DR
46 direct definition
47  $(\lambda t \cdot t \in \text{DR} \wedge t0 \mapsto t \in \text{leq} | (vx \mapsto vy))$ 
48 FirstOrderSystemObserver <expression> ()
49 direct definition
50  $(\lambda x \cdot x \in \text{RReal} \times \text{RReal} | x)$ 
51 SecondOrderSystemObserver <expression> ()
52 direct definition
53  $(\lambda (x1 \mapsto x2) \mapsto (x3 \mapsto x4)) \cdot (x1 \mapsto x2 \mapsto x3 \mapsto x4) \in \text{RReal} \times \text{RReal} \times \text{RReal} \times \text{RReal} | x3 \mapsto x4)$ 
54 AXIOMATIC DEFINITIONS pc_control:
55 AXIOMS
56 second_order_control:
57  $\forall \text{DR}, \text{UF}, vx, vy, cc, t0, x0 \cdot$ 
58  $cc \in \text{RReal} \wedge$ 
59  $\text{DR} \subseteq \text{RReal} \wedge \text{UF} \subseteq \text{RReal} \times \text{RReal} \wedge$ 
60  $(vx \mapsto vy) \in \text{UF} \wedge$ 
61  $t0 \in \text{RReal} \wedge t0 \in \text{DR} \wedge$ 
62  $x0 \in (\text{RReal} \times \text{RReal}) \times (\text{RReal} \times \text{RReal}) \Rightarrow$ 
63  $\text{SolvableWith}(\text{DR},$ 
64  $\text{SecondOrder2DimensionSystem}(cc, \text{UF} \times (\text{RReal} \times \text{RReal}), t0, x0),$ 
65  $\text{PointwiseSlopedControl}(\text{DR}, vx, vy, t0)$ 
66  $)$ 
67 first_order_control:
68  $\forall \text{DR}, \text{UF}, vx, vy, t0, x0 \cdot$ 
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70 DR ⊆ RReal ∧ UF ⊆ RReal×RReal ∧
71 (vx⇒vy) ∈ UF ∧
72 t0 ∈ RReal ∧ t0 ∈ DR ∧
73 x0 ∈ RReal×RReal ⇒
74 SolvableWith(
75   DR,
76   FirstOrder2DimensionSystem(UF, t0 , x0),
77   PointwiseControl(DR, vx , vy , t0)
78 ) pc_sim:

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OPERATORS

NeighborhoodSimulationCondition <predicate> (mu: RRealPlus , nu: RRealPlus , cc: RReal) :
well-definedness Rzero ⇒ cc ∈ It

AXIOMS

nsc_def:

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84 ∀ mu, nu, cc · mu ∈ RRealPlus ∧ nu ∈ RRealPlus ∧ cc ∈ RReal ∧ Rzero ⇒ cc ∈ It ⇒ (
85   NeighborhoodSimulationCondition(mu, nu, cc) ⇔
86   (times(nu ⇒ plus(divide(Rone ⇒ Rtwo) ⇒ plus(times(Rtwo ⇒ cc) ⇒ sqrt(plus(Rone ⇒ times(
      plus(Rtwo ⇒ Rtwo) ⇒ cc)))))) ⇒ mu ∈ leq)
87 )

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first_second_order_simulation_function:

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89 ∀ UF1, UF2, cc , mu, nu ·
90   cc ∈ RReal ∧
91   UF1 ⊆ RReal×RReal ∧ UF2 ⊆ RReal×RReal ∧
92   mu ∈ RRealPlus ∧ nu ∈ RRealPlus ∧
93   UF1 ⊆ DeltaNeighborhoodSet(mu, Rzero⇒Rzero) ∧ UF2 ⊆ DeltaNeighborhoodSet(nu, Rzero⇒Rzero) ∧
94   NeighborhoodSimulationCondition(mu, nu, cc)
95   ⇒ (
96     ∃ V · V ∈ SimulationFunctions(
97       (RReal×RReal)×(RReal×RReal), RReal×RReal,
98       UF1×(RReal×RReal), UF2,
99       SecondOrder2DimensionSystemFunction(cc, UF1×(RReal×RReal)), SecondOrderSystemObserver ,
100      FirstOrder2DimensionSystemFunction(UF2), FirstOrderSystemObserver
101     )
102   )

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first_second_order_simulation_delta:

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104 ∀ UF1, UF2, cc , mu, nu, V ·
105   cc ∈ RReal ∧
106   UF1 ⊆ RReal×RReal ∧ UF2 ⊆ RReal×RReal ∧
107   mu ∈ RRealPlus ∧ nu ∈ RRealPlus ∧
108   NeighborhoodSimulationCondition(mu, nu, cc) ∧
109   V ∈ SimulationFunctions(
110     (RReal×RReal)×(RReal×RReal), RReal×RReal,
111     UF1×(RReal×RReal), UF2,
112     SecondOrder2DimensionSystemFunction(cc, UF1×(RReal×RReal)), SecondOrderSystemObserver ,
113     FirstOrder2DimensionSystemFunction(UF2), FirstOrderSystemObserver
114   ) ⇒
115   boundedBy(((RReal×RReal)×(RReal×RReal))×(RReal×RReal), V, Rzero , times(Rtwo⇒nu)) distance:

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OPERATORS

plannar_distance <expression> (r1: RReal×RReal, r2: RReal×RReal) : RReal

AXIOMS

dist_sym:

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120 ∀ x, y · x ∈ RReal×RReal ∧ y ∈ RReal×RReal ⇒
121   plannar_distance(x, y) = plannar_distance(y, x)

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dist_sep:

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123 ∀ x, y · x ∈ RReal×RReal ∧ y ∈ RReal×RReal ⇒
124   (plannar_distance(x, y) = Rzero ⇔ x = y)

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dist_tri:

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126 ∀ x, y, z · x ∈ RReal×RReal ∧ y ∈ RReal×RReal ∧ z ∈ RReal×RReal ⇒
127   plannar_distance(x, z) ⇒ plus(plannar_distance(x, y)⇒plannar_distance(y, z)) ∈ leq

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open_ball_is_open:

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129 ∀ x, delta · x ∈ RReal×RReal ∧ delta ∈ RRealPlus ∧ Rzero ⇒ delta ∈ It ⇒
130   IsOpen({ y | plannar_distance(x, y) ⇒ delta ∈ gt })

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dist_choice:

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132 ∀ P, d · P ∈ RReal×RReal ∧ d ∈ RReal ∧ Rzero ⇒ d ∈ It ⇒ (
133   ∃ Q · Q ∈ RReal×RReal ∧ d ⇒ plannar_distance(P, Q) ∈ It
134 )

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neighborhood_plannar_distance:

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136 ∀ a, b, delta ·
137   a ∈ RReal×RReal ∧ b ∈ RReal×RReal ∧ delta ∈ RRealPlus ∧ Rzero ⇒ delta ∈ It ∧
138   DeltaNeighborhood(delta, a, b)
139   ⇒ plannar_distance(a, b) ⇒ delta ∈ It

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THEOREMS

first_order_system_solvable:

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141 ∀ DR, UF, t0 , y0 , vx , vy ·
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143 DR ⊆ ℝReal ∧
144 UF ⊆ ℝReal×ℝReal ∧
145 t0 ∈ DR ∧ y0 ∈ ℝReal×ℝReal ∧
146 vx↦vy ∈ UF ⇒
147 SolvableWith(DR, FirstOrder2DimensionSystem(UF, t0 , y0) , PointwiseControl(DR, vx , vy , t0))
148 second_order_system_solvable:
149 ∀ DR, UF, t0 , x0 , vx , vy , cc .
150 DR ⊆ ℝReal ∧
151 UF ⊆ ℝReal×ℝReal ∧
152 t0 ∈ DR ∧ x0 ∈ (ℝReal×ℝReal)×(ℝReal×ℝReal) ∧
153 vx↦vy ∈ UF ∧
154 cc ∈ ℝReal ⇒
155 SolvableWith(DR, SecondOrder2DimensionSystem(cc , UF×(ℝReal×ℝReal) , t0 , x0) ,
156 PointwiseSlopedControl(DR, vx , vy , t0))

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END