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THEORY PlannarControlTheory
  IMPORT THEORY PROJECTS
    /SimpleDEq THEORIES /SimpleDEq/Approximation.dtf|org.eventb.theory.core.deployedTheoryRoot#
        Approximation
  OPERATORS
    SecondOrder2DimensionSystemFunction <expression> (correction_coeff: RReal,controlArea4: ℙ((RReal×RReal)×
        (RReal×RReal)))
      direct definition
        (λ ((x1↦x2)↦(x3↦x4))↦((vx↦vy)↦(wx↦wy)) ·
          ((x1↦x2)↦(x3↦x4)) ∈ (RReal×RReal)×(RReal×RReal) ∧ ((vx↦vy)↦(wx↦wy)) ∈ (RReal×RReal)×(
              RReal×RReal) ∧ ((vx↦vy)↦(wx↦wy)) ∈ controlArea4
        | (minus(divide(vx↦Rtwo)↦minus(times(correction_coeff↦minus(x3↦wx))↦x1))
          ↦ minus(divide(vy↦Rtwo)↦minus(times(correction_coeff↦minus(x4↦wy))↦x2)))
          ↦(x1
          ↦ x2)
        )
    FirstOrder2DimensionSystemFunction <expression> (controlArea2: ℙ(RReal×RReal))
      direct definition
        (λ (x1↦x2)↦(vx↦vy) ·
          (x1↦x2) ∈ RReal×RReal ∧ (vx↦vy) ∈ RReal×RReal ∧ (vx↦vy) ∈ controlArea2
        | ( vx
          ↦ vy
          )
        )
    SecondOrder2DimensionSystem <expression> (correction_coeff: RReal,controlArea4: ℙ((RReal×RReal)×(RReal
        ×RReal)),t0: RRealPlus,x0: (RReal×RReal)×(RReal×RReal))
      direct definition
        caode(
          SecondOrder2DimensionSystemFunction(correction_coeff,controlArea4),
          x0,
          t0
        )
    FirstOrder2DimensionSystem <expression> (controlArea2: ℙ(RReal×RReal),t0: RRealPlus,y0: RReal×RReal)
      direct definition
        caode(
          FirstOrder2DimensionSystemFunction(controlArea2),
          y0,
          t0
        )
    PointwiseSlopedControl <expression> (DR: ℙ(RReal),vx: RReal,vy: RReal,t0: RReal)
      well-definedness t0 ∈ DR
      direct definition
        (λ t · t ∈ DR ∧ t0 ↦ t ∈ leq |
          (vx ↦ vy)
          ↦
          (times(vx ↦ minus(t ↦ t0)) ↦ times(vy ↦ minus(t ↦ t0)))
        )
    PointwiseControl <expression> (DR: ℙ(RReal),vx: RReal,vy: RReal,t0: RReal)
      well-definedness t0 ∈ DR
      direct definition
        (λ t · t ∈ DR ∧ t0 ↦ t ∈ leq | (vx ↦ vy))
    FirstOrderSystemObserver <expression> ()
      direct definition
        (λ x · x ∈ RReal×RReal | x )
    SecondOrderSystemObserver <expression> ()
      direct definition
        (λ (x1↦x2)↦(x3↦x4) · (x1↦x2↦x3↦x4) ∈ RReal×RReal×RReal×RReal | x3↦x4)
  AXIOMATIC DEFINITIONS  pc_control:
    AXIOMS
      second_order_control:
        ∀ DR,UF,vx,vy,cc,t0,x0 ·
          cc ∈ RReal ∧
          DR ⊆ RReal ∧ UF ⊆ RReal×RReal ∧
          (vx↦vy) ∈ UF ∧
          t0 ∈ RReal ∧ t0 ∈ DR ∧
          x0 ∈ (RReal×RReal)×(RReal×RReal) ⇒
            SolvableWith(
              DR,
              SecondOrder2DimensionSystem(cc,UF×(RReal×RReal),t0,x0),
              PointwiseSlopedControl(DR,vx,vy,t0)
            )
      first_order_control:
        ∀ DR,UF,vx,vy,t0,x0 ·
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        DR ⊆ RReal ∧ UF ⊆ RReal×RReal ∧
        (vx↦vy) ∈ UF ∧
        t0 ∈ RReal ∧ t0 ∈ DR ∧
        x0 ∈ RReal×RReal ⇒
          SolvableWith(
            DR,
            FirstOrder2DimensionSystem(UF,t0,x0),
            PointwiseControl(DR,vx,vy,t0)
          )  pc_sim:
  OPERATORS
    NeighborhoodSimulationCondition <predicate> (mu: RRealPlus,nu: RRealPlus,cc: RReal) :
      well-definedness Rzero ↦ cc ∈ lt
  AXIOMS
    nsc_def:
      ∀ mu,nu,cc · mu ∈ RRealPlus ∧ nu ∈ RRealPlus ∧ cc ∈ RReal ∧ Rzero ↦ cc ∈ lt ⇒ (
        NeighborhoodSimulationCondition(mu,nu,cc) ⇔
        (times(nu ↦ plus(divide(Rone ↦ Rtwo) ↦ plus(times(Rtwo ↦ cc) ↦ sqrt(plus(Rone ↦ times(
            plus(Rtwo ↦ Rtwo) ↦ cc)))))) ↦ mu ∈ leq)
      )
    first_second_order_simulation_function:
      ∀ UF1,UF2,cc,mu,nu ·
        cc ∈ RReal ∧
        UF1 ⊆ RReal×RReal ∧ UF2 ⊆ RReal×RReal ∧
        mu ∈ RRealPlus ∧ nu ∈ RRealPlus ∧
        UF1 ⊆ DeltaNeighborhoodSet(mu,Rzero↦Rzero) ∧ UF2 ⊆ DeltaNeighborhoodSet(nu,Rzero↦Rzero) ∧
        NeighborhoodSimulationCondition(mu,nu,cc)
         ⇒ (
          ∃ V · V ∈ SimulationFunctions(
            (RReal×RReal)×(RReal×RReal), RReal×RReal,
            UF1×(RReal×RReal),      UF2,
            SecondOrder2DimensionSystemFunction(cc,UF1×(RReal×RReal)), SecondOrderSystemObserver,
            FirstOrder2DimensionSystemFunction(UF2),          FirstOrderSystemObserver
          )
        )
    first_second_order_simulation_delta:
      ∀ UF1,UF2,cc,mu,nu,V ·
        cc ∈ RReal ∧
        UF1 ⊆ RReal×RReal ∧ UF2 ⊆ RReal×RReal ∧
        mu ∈ RRealPlus ∧ nu ∈ RRealPlus ∧
        NeighborhoodSimulationCondition(mu,nu,cc) ∧
        V ∈ SimulationFunctions(
          (RReal×RReal)×(RReal×RReal), RReal×RReal,
          UF1×(RReal×RReal),      UF2,
          SecondOrder2DimensionSystemFunction(cc,UF1×(RReal×RReal)), SecondOrderSystemObserver,
          FirstOrder2DimensionSystemFunction(UF2),          FirstOrderSystemObserver
        ) ⇒
          boundedBy(((RReal×RReal)×(RReal×RReal))×(RReal×RReal),V,Rzero,times(Rtwo↦nu)) distance:
  OPERATORS
    plannar_distance <expression> (r1: RReal×RReal,r2: RReal×RReal) : RReal
  AXIOMS
    dist_sym:
      ∀ x,y · x ∈ RReal×RReal ∧ y ∈ RReal×RReal ⇒
        plannar_distance(x,y) = plannar_distance(y,x)
    dist_sep:
      ∀ x,y · x ∈ RReal×RReal ∧ y ∈ RReal×RReal ⇒
        (plannar_distance(x,y) = Rzero ⇔ x = y)
    dist_tri:
      ∀ x,y,z · x ∈ RReal×RReal ∧ y ∈ RReal×RReal ∧ z ∈ RReal×RReal ⇒
        plannar_distance(x,z) ↦ plus(plannar_distance(x,y)↦plannar_distance(y,z)) ∈ leq
    open_ball_is_open:
      ∀ x,delta · x ∈ RReal×RReal ∧ delta ∈ RRealPlus ∧ Rzero ↦ delta ∈ lt ⇒
        IsOpen({ y | plannar_distance(x,y) ↦ delta ∈ gt })
    dist_choice:
      ∀ P,d · P ∈ RReal×RReal ∧ d ∈ RReal ∧ Rzero ↦ d ∈ lt ⇒ (
          ∃ Q · Q ∈ RReal×RReal ∧ d ↦ plannar_distance(P,Q) ∈ lt
        )
    neighborhood_plannar_distance:
      ∀ a,b,delta ·
        a ∈ RReal×RReal ∧ b ∈ RReal×RReal ∧ delta ∈ RRealPlus ∧ Rzero ↦ delta ∈ lt ∧
        DeltaNeighborhood(delta,a,b)
          ⇒ plannar_distance(a,b) ↦ delta ∈ lt
THEOREMS
  first_order_system_solvable:
    ∀ DR,UF,t0,y0,vx,vy ·
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143         DR ⊆ RReal ∧
144         UF ⊆ RReal×RReal ∧
145         t0 ∈ DR ∧ y0 ∈ RReal×RReal ∧
146         vx↦vy ∈ UF ⇒
147           SolvableWith(DR, FirstOrder2DimensionSystem(UF, t0, y0), PointwiseControl(DR, vx, vy, t0))
148     second_order_system_solvable:
149       ∀ DR, UF, t0, x0, vx, vy, cc ·
150         DR ⊆ RReal ∧
151         UF ⊆ RReal×RReal ∧
152         t0 ∈ DR ∧ x0 ∈ (RReal×RReal)×(RReal×RReal) ∧
153         vx↦vy ∈ UF ∧
154         cc ∈ RReal ⇒
155           SolvableWith(DR, SecondOrder2DimensionSystem(cc, UF×(RReal×RReal), t0, x0),
                PointwiseSlopedControl(DR, vx, vy, t0))
156 END
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