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1 MACHINE
2   PendulumLin
3   REFINES
4     Pendulum
5   SEES
6     PendulumLinCtx
7   VARIABLES t, theta, thetap, t_sense, control_fun, theta_sense, thetap_sense, control_fun_lin, theta_lin, thetap_lin,
8     theta_lin_sense, thetap_lin_sense
9   INVARIANTS
10    inv1: theta_lin ∈ RReal ↔ RReal
11    inv2: thetap_lin ∈ RReal ↔ RReal
12    inv3: Closed2Closed(Rzero,t) ⊆ dom(theta_lin)
13    inv4: Closed2Closed(Rzero,t) ⊆ dom(thetap_lin)
14    inv5: DeltaApproximation(Closed2Closed(Rzero,t),delta,bind(theta,thetap),bind(theta_lin,thetap_lin))
15    inv6:
16      ∀t_ · t_ ∈ Closed2Closed(Rzero,t) ⇒
17        abs(theta(t_)) ↪ thetabound ∈ lt ∧
18        abs(theta_lin(t_)) ↪ minus(thetabound ↪ delta) ∈ lt
19    inv7: control_fun_lin ∈ RReal ↔ RReal
20    inv8: DeltaApproximation(Closed2Infinity(t),control_delta,control_fun,control_fun_lin)
21    inv9: theta_lin_sense ∈ RReal
22    inv10: thetap_lin_sense ∈ RReal
23    inv11: abs(theta_lin_sense) ↪ minus(thetabound ↪ delta) ∈ leq
24    inv12: DeltaNeighborhood(delta,theta_sense ↪ thetap_sense,theta_lin_sense ↪ thetap_lin_sense)
25   EVENTS
26   INITIALISATION
27   THEN
28     act1: t := Rzero
29     act2: theta := {Rzero ↪ theta0}
30     act3: thetap := {Rzero ↪ Rzero}
31     act4: t_sense := Rzero
32     act5: theta_sense,thetap_sense := theta0,Rzero
33     act6: control_fun := PendulumRawControl(omega0,theta0,Rzero,Rzero)
34     act7: control_fun_lin := PendulumLinControl(omega0,theta0,Rzero,Rzero)
35     act8: theta_lin := {Rzero ↪ theta0}
36     act9: thetap_lin := {Rzero ↪ Rzero}
37     act10: theta_lin_sense,thetap_lin_sense := theta0,Rzero
38   END
39
40   Behave
41   REFINES Behave
42   ANY e, tp
43   WHERE
44     grd1: e ∈ DE(S)
45     grd2: Solvable(Closed2Closed(t,tp),e)
46     grd4: theta(t) ↪ thetabound ∈ lt
47     grd5: tp ∈ RRealPlus
48     grd6: t ↪ tp ∈ lt
49     grd7:
50       CBAPsolutionOfFIS(t,tp,bind(theta,thetap),e,
51         {theta_,thetap_ · theta_ ∈ RReal ∧ thetap_ ∈ RReal ∧
52           abs(theta_) ↪ thetabound ∈ lt
53           | theta_ ↪ thetap_})
54     grd8:
55       CBAPsolutionOfFIS(t,tp,bind(theta_lin,thetap_lin),e,
56         {theta_,thetap_ · theta_ ∈ RReal ∧ thetap_ ∈ RReal ∧
57           abs(theta_) ↪ thetabound ∈ lt
58           | theta_ ↪ thetap_})
59   THEN
60     act1:
61       t,theta,thetap,theta_lin,thetap_lin :|
62         t' = tp ∧
63         theta' ∈ RReal ↔ RReal ∧ Closed2Closed(Rzero,t') ⊆ dom(theta') ∧
64         thetap' ∈ RReal ↔ RReal ∧ Closed2Closed(Rzero,t') ⊆ dom(thetap') ∧
65         CBAPsolutionOf(t,t',bind(theta,thetap),bind(theta',thetap'),e,
66           {theta_,thetap_ · theta_ ∈ RReal ∧ thetap_ ∈ RReal ∧
67             abs(theta_) ↪ thetabound ∈ lt
68             | theta_ ↪ thetap_}) ∧
69         theta_lin' ∈ RReal ↔ RReal ∧ Closed2Closed(Rzero,t') ⊆ dom(theta_lin') ∧
70         thetap_lin' ∈ RReal ↔ RReal ∧ Closed2Closed(Rzero,t') ⊆ dom(thetap_lin') ∧
71         CBAPsolutionOf(t,t',bind(theta_lin,thetap_lin),bind(theta_lin',thetap_lin'),e,
72           {theta_,thetap_ · theta_ ∈ RReal ∧ thetap_ ∈ RReal ∧
73             abs(theta_) ↪ thetabound ∈ lt
74             | theta_ ↪ thetap_})

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73 | theta_  $\mapsto$  thetap_})
74 END
75
76 sense_angle
77 REFINES sense_angle
78 WHERE
79   grd1 : Rzero  $\mapsto$  abs(theta_lin(t))  $\in$  lt
80 THEN
81   act1 : t_sense,theta_sense,thetap_sense := t,theta(t),thetap(t)
82   act2 : theta_lin_sense,thetap_lin_sense := theta_lin(t),thetap_lin(t)
83 END
84
85 transition_calculate_control
86 REFINES transition_calculate_control
87 THEN
88   act1 : control_fun := PendulumRawControl(omega0,theta_sense,thetap_sense,t_sense)
89   act2 : control_fun_lin := PendulumLinControl(omega0,theta_lin_sense,thetap_lin_sense,t_sense)
90 END
91
92 actuate_balance
93 REFINES actuate_balance
94 ANY tp
95 WHERE
96   grd0 : tp  $\in$  RRealPlus  $\wedge$  t  $\mapsto$  tp  $\in$  lt
97   grd2 :
98     SolvableWith(
99       Closed2Closed(t,tp),
100      PendulumRaw(omega0,(theta(t)  $\mapsto$  thetap(t)),t),
101      control_fun
102    )
103   grd3 :
104     SolvableWith(
105       Closed2Closed(t,tp),
106       PendulumLin(omega0,(theta_lin(t)  $\mapsto$  thetap_lin(t)),t),
107       control_fun_lin
108     )
109   grd4 : abs(theta(t))  $\mapsto$  thetabound  $\in$  lt
110   grd5 : abs(theta_lin(t))  $\mapsto$  minus(thetabound  $\mapsto$  delta)  $\in$  lt
111 THEN
112   act1 :
113     t,theta,thetap,theta_lin,thetap_lin :|
114       t' = tp  $\wedge$ 
115       theta'  $\in$  RReal  $\Rightarrow$  RReal  $\wedge$  Closed2Closed(Rzero,t')  $\subseteq$  dom(theta')  $\wedge$ 
116       thetap'  $\in$  RReal  $\Rightarrow$  RReal  $\wedge$  Closed2Closed(Rzero,t')  $\subseteq$  dom(thetap')  $\wedge$ 
117       CBAPsolutionOf(
118         t,t',
119         bind(theta,thetap),
120         bind(theta',thetap'),
121         withControl(
122           Closed2Closed(t,t'),
123           PendulumRaw(omega0,(theta(t)  $\mapsto$  thetap(t)),t),
124           control_fun
125         ),
126         {theta_,thetap_ · theta_  $\in$  RReal  $\wedge$  thetap_  $\in$  RReal  $\wedge$ 
127           abs(theta_)  $\mapsto$  thetabound  $\in$  lt
128           | theta_  $\mapsto$  thetap_}
129       )  $\wedge$ 
130       theta_lin'  $\in$  RReal  $\Rightarrow$  RReal  $\wedge$  Closed2Closed(Rzero,t')  $\subseteq$  dom(theta_lin')  $\wedge$ 
131       thetap_lin'  $\in$  RReal  $\Rightarrow$  RReal  $\wedge$  Closed2Closed(Rzero,t')  $\subseteq$  dom(thetap_lin')  $\wedge$ 
132       CBAPsolutionOf(
133         t,t',
134         bind(theta_lin,thetap_lin),
135         bind(theta_lin',thetap_lin'),
136         withControl(
137           Closed2Closed(t,t'),
138           PendulumLin(omega0,(theta_lin(t)  $\mapsto$  thetap_lin(t)),t),
139           control_fun_lin
140         ),
141         {theta_,thetap_ · theta_  $\in$  RReal  $\wedge$  thetap_  $\in$  RReal  $\wedge$ 
142           abs(theta_)  $\mapsto$  minus(thetabound  $\mapsto$  delta)  $\in$  lt
143           | theta_  $\mapsto$  thetap_}
144       )
145 END
146

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