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1 MACHINE
2   Pendulum
3 REFINES
4   Generic
5 SEES
6   PendulumCtx
7 VARIABLES  $t, \theta, \dot{\theta}, t_{sense}, \theta_{sense}, \dot{\theta}_{sense}, control\_fun$ 
8 INVARIANTS
9   inv1:  $\theta \in \mathbb{R} \leftrightarrow \mathbb{R}$ 
10  inv2:  $\dot{\theta} \in \mathbb{R} \leftrightarrow \mathbb{R}$ 
11  inv3:  $Closed2Closed(Rzero, t) \subseteq dom(\theta)$ 
12  inv4:  $Closed2Closed(Rzero, t) \subseteq dom(\dot{\theta})$ 
13  inv5:  $x_p = bind(\theta, \dot{\theta})$ 
14  inv6:  $\forall t \cdot t \in Closed2Closed(Rzero, t) \Rightarrow abs(\theta(t)) \mapsto \theta_{max} \in lt$ 
15  inv7:  $x_s = control$ 
16  inv8:  $t_{sense} \in \mathbb{R}^+$ 
17  inv9:  $\theta_{sense} \in \mathbb{R}$ 
18  inv10:  $\dot{\theta}_{sense} \in \mathbb{R}$ 
19  inv11:  $control\_fun \in \mathbb{R} \leftrightarrow \mathbb{R}$ 
20  inv12:  $Closed2Infinity(t_{sense}) \subseteq dom(control\_fun)$ 
21  inv13:  $abs(\theta_{sense}) \mapsto \theta_{max} \in leq$ 
22 EVENTS
23 INITIALISATION
24 WITH
25    $x_p' : x_p' = \{Rzero \mapsto (\theta_0 \mapsto Rzero)\}$ 
26    $x_s' : x_s' = control$ 
27 THEN
28   act1:  $t := Rzero$ 
29   act2:  $\theta := \{Rzero \mapsto \theta_0\}$ 
30   act3:  $\dot{\theta} := \{Rzero \mapsto Rzero\}$ 
31   act4:  $t_{sense} := Rzero$ 
32   act5:  $\theta_{sense}, \dot{\theta}_{sense} := \theta_0, Rzero$ 
33   act6:  $control\_fun := PendulumRawControl(\omega_0, \theta_0, Rzero, Rzero)$ 
34 END
35
36 Behave
37 REFINES Behave
38 ANY  $e, tp$ 
39 WHERE
40   grd1:  $e \in DE(S)$ 
41   grd2:  $Solvable(Closed2Closed(t, tp), e)$ 
42   grd4:  $\theta(t) \mapsto \theta_{max} \in lt$ 
43   grd5:  $tp \in \mathbb{R}^+$ 
44   grd6:  $t \mapsto tp \in lt$ 
45   grd7:
46      $CBAPsolutionOfFIS(t, tp, bind(\theta, \dot{\theta}), e,$ 
47        $\{ \theta_-, \dot{\theta}_- \cdot \theta_- \in \mathbb{R} \wedge \dot{\theta}_- \in \mathbb{R} \wedge$ 
48          $\theta_- \mapsto \theta_{max} \in lt$ 
49          $\mid \theta_- \mapsto \dot{\theta}_- \})$ 
50 WITH
51   Inv:
52      $Inv = \{ \theta_-, \dot{\theta}_- \cdot \theta_- \in \mathbb{R} \wedge \dot{\theta}_- \in \mathbb{R} \wedge$ 
53        $\theta_- \mapsto \theta_{max} \in lt$ 
54        $\mid \theta_- \mapsto \dot{\theta}_- \}$ 
55    $x_p' : x_p' = bind(\theta', \dot{\theta}')$ 
56 THEN
57   act1:
58      $t, \theta, \dot{\theta} : |$ 
59      $t' = tp \wedge$ 
60      $\theta' \in \mathbb{R} \leftrightarrow \mathbb{R} \wedge Closed2Closed(Rzero, t') \subseteq dom(\theta') \wedge$ 
61      $\dot{\theta}' \in \mathbb{R} \leftrightarrow \mathbb{R} \wedge Closed2Closed(Rzero, t') \subseteq dom(\dot{\theta}')$ 
62      $CBAPsolutionOf(t, t', bind(\theta, \dot{\theta}), bind(\theta', \dot{\theta}'), e,$ 
63        $\{ \theta_-, \dot{\theta}_- \cdot \theta_- \in \mathbb{R} \wedge \dot{\theta}_- \in \mathbb{R} \wedge$ 
64          $\theta_- \mapsto \theta_{max} \in lt$ 
65          $\mid \theta_- \mapsto \dot{\theta}_- \})$ 
66 END
67
68 sense_angle
69 REFINES Sense
70 WHERE
71   grd1:  $Rzero \mapsto abs(\theta(t)) \in lt$ 
72 WITH
73    $x_s' : x_s' = control$ 

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74   s: s = {control}
75   p:
76     p = {control} × RReal × {theta_, thetap_ · theta_ ∈ RReal ∧ thetap_ ∈ RReal ∧
77       abs(theta_) ↦ thetamax ∈ geq
78       | theta_ ↦ thetap_}
79   THEN
80     act1 : t_sense, theta_sense, thetap_sense := t, theta(t), thetap(t)
81   END
82
83   transition_calculate_control
84   REFINES Transition
85   WITH
86     x_s': x_s' = control
87     s: s = {control}
88   THEN
89     act1 : control_fun := PendulumRawControl(omega0, theta_sense, thetap_sense, t_sense)
90   END
91
92   actuate_balance
93   REFINES Actuate
94   ANY tp
95   WHERE
96     grd0 : tp ∈ RRealPlus ∧ t ↦ tp ∈ lt
97     grd2 :
98       SolvableWith(
99         Closed2Closed(t, tp),
100        PendulumRaw(omega0, (theta(t) ↦ thetap(t)), t),
101        control_fun
102      )
103     grd4 : theta(t) ↦ thetamax ∈ lt
104   WITH
105     e:
106       e = withControl(
107         Closed2Closed(t, t'),
108         PendulumRaw(omega0, (theta(t) ↦ thetap(t)), t),
109         control_fun
110       )
111     Inv:
112       Inv = {theta_, thetap_ · theta_ ∈ RReal ∧ thetap_ ∈ RReal ∧
113         abs(theta_) ↦ thetamax ∈ lt
114         | theta_ ↦ thetap_}
115     x_p': x_p' = bind(theta', thetap')
116     s: s = {control}
117   THEN
118     act1 :
119       t, theta, thetap :|
120         t' = tp ∧
121         theta' ∈ RReal → RReal ∧ Closed2Closed(Rzero, t') ⊆ dom(theta') ∧
122         thetap' ∈ RReal → RReal ∧ Closed2Closed(Rzero, t') ⊆ dom(thetap') ∧
123         CBAPsolutionOf(
124           t, t',
125           bind(theta, thetap),
126           bind(theta', thetap'),
127           withControl(
128             Closed2Closed(t, t'),
129             PendulumRaw(omega0, (theta(t) ↦ thetap(t)), t),
130             control_fun
131           ),
132           {theta_, thetap_ · theta_ ∈ RReal ∧ thetap_ ∈ RReal ∧
133             abs(theta_) ↦ thetamax ∈ lt
134             | theta_ ↦ thetap_}
135         )
136   END
137
138   END

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