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MACHINE
   Pendulum
REFINES
   Generic
SEES
   PendulumCtx
VARIABLES   t , theta , thetap , t_sense , theta_sense , thetap_sense , control_fun
INVARIANTS
   inv1 :  theta ∈ RReal ⇸ RReal
   inv2 :  thetap ∈ RReal ⇸ RReal
   inv3 :  Closed2Closed(Rzero, t) ⊆ dom(theta)
   inv4 :  Closed2Closed(Rzero, t) ⊆ dom(thetap)
   inv5 :  x_p = bind(theta, thetap)
   inv6 :  ∀t_ · t_ ∈ Closed2Closed(Rzero, t) ⇒ abs(theta(t_)) ↦ thetamax ∈ lt
   inv7 :  x_s = control
   inv8 :  t_sense ∈ RRealPlus
   inv9 :  theta_sense ∈ RReal
   inv10 :  thetap_sense ∈ RReal
   inv11 :  control_fun ∈ RReal ⇸ RReal
   inv12 :  Closed2Infinity(t_sense) ⊆ dom(control_fun)
   inv13 :  abs(theta_sense) ↦ thetamax ∈ leq
EVENTS
   INITIALISATION
   WITH
      x_p' :  x_p' = {Rzero ↦ (theta0 ↦ Rzero)}
      x_s' :  x_s' = control
   THEN
      act1 :  t := Rzero
      act2 :  theta := {Rzero ↦ theta0}
      act3 :  thetap := {Rzero ↦ Rzero}
      act4 :  t_sense := Rzero
      act5 :  theta_sense, thetap_sense := theta0, Rzero
      act6 :  control_fun := PendulumRawControl(omega0, theta0, Rzero, Rzero)
   END

   Behave
   REFINES  Behave
   ANY   e , tp
   WHERE
      grd1 :  e ∈ DE(S)
      grd2 :  Solvable(Closed2Closed(t, tp), e)
      grd4 :  theta(t) ↦ thetamax ∈ lt
      grd5 :  tp ∈ RRealPlus
      grd6 :  t ↦ tp ∈ lt
      grd7 :
         CBAPsolutionOfFIS(t, tp, bind(theta, thetap), e,
            {theta_, thetap_ · theta_ ∈ RReal ∧ thetap_ ∈ RReal∧
                  theta_ ↦ thetamax ∈ lt
               | theta_ ↦ thetap_})
   WITH
      Inv :
         Inv = {theta_, thetap_ · theta_ ∈ RReal ∧ thetap_ ∈ RReal∧
                  theta_ ↦ thetamax ∈ lt
               | theta_ ↦ thetap_}
      x_p' :  x_p' = bind(theta', thetap')
   THEN
      act1 :
         t, theta, thetap :|
            t' = tp∧
            theta' ∈ RReal ⇸ RReal ∧ Closed2Closed(Rzero, t') ⊆ dom(theta')∧
            thetap' ∈ RReal ⇸ RReal ∧ Closed2Closed(Rzero, t') ⊆ dom(thetap')∧
            CBAPsolutionOf(t, t', bind(theta, thetap), bind(theta', thetap'), e,
               {theta_, thetap_ · theta_ ∈ RReal ∧ thetap_ ∈ RReal∧
                  theta_ ↦ thetamax ∈ lt
               | theta_ ↦ thetap_})
   END

   sense_angle
   REFINES  Sense
   WHERE
      grd1 :  Rzero ↦ abs(theta(t)) ∈ lt
   WITH
      x_s' :  x_s' = control
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        s:  s = {control}
        p:
            p = {control} × RReal × {theta_, thetap_ · theta_ ∈ RReal ∧ thetap_ ∈ RReal∧
                                abs(theta_) ↦ thetamax ∈ geq
                            | theta_ ↦ thetap_}
    THEN
        act1:  t_sense, theta_sense, thetap_sense := t, theta(t), thetap(t)
    END

    transition_calculate_control
    REFINES  Transition
    WITH
        x_s':  x_s' = control
        s:  s = {control}
    THEN
        act1:  control_fun := PendulumRawControl(omega0, theta_sense, thetap_sense, t_sense)
    END

    actuate_balance
    REFINES  Actuate
    ANY  tp
    WHERE
        grd0:  tp ∈ RRealPlus ∧ t ↦ tp ∈ lt
        grd2:
            SolvableWith(
                Closed2Closed(t, tp),
                PendulumRaw(omega0, (theta(t) ↦ thetap(t)), t),
                    control_fun
            )
        grd4:  theta(t) ↦ thetamax ∈ lt
    WITH
        e:
            e = withControl(
                Closed2Closed(t, t'),
                PendulumRaw(omega0, (theta(t) ↦ thetap(t)), t),
                control_fun
            )
        Inv:
            Inv = {theta_, thetap_ · theta_ ∈ RReal ∧ thetap_ ∈ RReal∧
                        abs(theta_) ↦ thetamax ∈ lt
                    | theta_ ↦ thetap_}
        x_p':  x_p' = bind(theta', thetap')
        s:  s = {control}
    THEN
        act1:
            t, theta, thetap :|
                t' = tp∧
                theta' ∈ RReal ⇸ RReal ∧ Closed2Closed(Rzero, t') ⊆ dom(theta')∧
                thetap' ∈ RReal ⇸ RReal ∧ Closed2Closed(Rzero, t') ⊆ dom(thetap')∧
                CBAPsolutionOf(
                    t, t',
                    bind(theta, thetap),
                    bind(theta', thetap'),
                    withControl(
                        Closed2Closed(t, t'),
                        PendulumRaw(omega0, (theta(t) ↦ thetap(t)), t),
                        control_fun
                    ),
                    {theta_, thetap_ · theta_ ∈ RReal ∧ thetap_ ∈ RReal∧
                        abs(theta_) ↦ thetamax ∈ lt
                    | theta_ ↦ thetap_}
                )
    END

END
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