

THEORY

IMPORT THEORY PROJECTS

/SimpleDEq THEORIES /SimpleDEq/Approximation.dtf|org.eventb.theory.core.deployedTheoryRoot#
Approximation

OPERATORS

PendulumRawFun <expression> (omega0: RReal)

direct definition

$(\lambda t_ \mapsto (x1_ \mapsto x2_) \mapsto u_ \cdot$
 $t_ \in \text{RRealPlus} \wedge x1_ \in \text{RReal} \wedge x2_ \in \text{RReal} \wedge u_ \in \text{RReal}$
 $| x2_ \mapsto (\text{plus}(\text{times}(u_ \mapsto \cos(x1_)) \mapsto \text{times}(\text{times}(\text{omega0} \mapsto \text{omega0}) \mapsto \sin(x1_))))$
)

PendulumRaw <expression> (omega0: RReal, x0: RReal×RReal, t0: RRealPlus)

direct definition

code(PendulumRawFun(omega0), x0, t0)

PendulumLinFun <expression> (omega0: RReal)

direct definition

$(\lambda t_ \mapsto (x1_ \mapsto x2_) \mapsto u_ \cdot$
 $t_ \in \text{RRealPlus} \wedge x1_ \in \text{RReal} \wedge x2_ \in \text{RReal} \wedge u_ \in \text{RReal}$
 $| x2_ \mapsto (\text{plus}(u_ \mapsto \text{times}(\text{times}(\text{omega0} \mapsto \text{omega0}) \mapsto x1_)))$
)

PendulumLin <expression> (omega0: RReal, x0: RReal×RReal, t0: RRealPlus)

direct definition

code(PendulumLinFun(omega0), x0, t0)

AXIOMATIC DEFINITIONS pendulum_solvability:

OPERATORS

theta_max <expression> (omega0: RReal) : RReal

AXIOMS

theta_max_bounds:

$\forall \text{omega0} \cdot \text{omega0} \in \text{RReal} \Rightarrow \text{Rzero} \mapsto \text{theta_max}(\text{omega0}) \in \text{leq} \wedge \text{theta_max}(\text{omega0}) \mapsto \text{divide}(\text{pi} \mapsto$
 $\text{Rtwo}) \in \text{leq}$

pendulum_raw_controllability:

$\forall \text{omega0}, \text{theta0}, \text{thetap0}, t0 \cdot$
 $\text{omega0} \in \text{RReal} \wedge$
 $\text{theta0} \in \text{RReal} \wedge \text{abs}(\text{theta0}) \mapsto \text{theta_max}(\text{omega0}) \in \text{lt} \wedge$
 $\text{thetap0} \in \text{RReal} \wedge$
 $t0 \in \text{RRealPlus}$
 \Rightarrow (
 $\exists t1 \cdot t1 \in \text{RRealPlus} \wedge t0 \mapsto t1 \in \text{lt} \wedge$
 $\text{Controllable}(\text{Closed2Closed}(t0, t1), \text{PendulumRaw}(\text{omega0}, (\text{theta0} \mapsto \text{thetap0}), t0))$
)

pendulum_lin_controllability:

$\forall \text{omega0}, \text{theta0}, \text{thetap0}, t0 \cdot$
 $\text{omega0} \in \text{RReal} \wedge$
 $\text{theta0} \in \text{RReal} \wedge \text{abs}(\text{theta0}) \mapsto \text{theta_max}(\text{omega0}) \in \text{lt} \wedge$
 $\text{thetap0} \in \text{RReal} \wedge$
 $t0 \in \text{RRealPlus}$
 \Rightarrow (
 $\exists t1 \cdot t1 \in \text{RRealPlus} \wedge t0 \mapsto t1 \in \text{lt} \wedge$
 $\text{Controllable}(\text{Closed2Closed}(t0, t1), \text{PendulumLin}(\text{omega0}, (\text{theta0} \mapsto \text{thetap0}), t0))$
) pendulum_approx:

OPERATORS

PendulumApproxWD <predicate> (delta: RReal, omega0: RReal, theta_bound: RReal, ctrl_bound: RReal,
ctrl_bound_lin: RReal, ctrl_delta: RReal, t0: RRealPlus, t1: RRealPlus) :

well-definedness $\text{Rzero} \mapsto \text{delta} \in \text{lt}, \text{Rzero} \mapsto \text{theta_bound} \in \text{lt}, \text{theta_bound} \mapsto \text{theta_max}(\text{omega0}) \in$
 $\text{lt}, \text{Rzero} \mapsto \text{ctrl_bound} \in \text{lt}, \text{Rzero} \mapsto \text{ctrl_bound_lin} \in \text{lt}, \text{Rzero} \mapsto \text{ctrl_delta} \in \text{lt}, t0 \mapsto t1 \in \text{lt}$

AXIOMS

PAWD_Approximation:

$\forall \text{delta}, \text{omega0}, \text{theta_bound}, \text{ctrl_bound}, \text{ctrl_bound_lin}, \text{ctrl_delta}, t0, t1, x0_raw, x0_lin, u_raw,$
 $u_lin \cdot$
 $\text{delta} \in \text{RReal} \wedge \text{Rzero} \mapsto \text{delta} \in \text{lt} \wedge$
 $\text{omega0} \in \text{RReal} \wedge$
 $\text{theta_bound} \in \text{RReal} \wedge \text{Rzero} \mapsto \text{theta_bound} \in \text{lt} \wedge \text{theta_bound} \mapsto \text{theta_max}(\text{omega0}) \in \text{lt} \wedge$
 $\text{ctrl_bound} \in \text{RReal} \wedge \text{Rzero} \mapsto \text{ctrl_bound} \in \text{lt} \wedge$
 $\text{ctrl_bound_lin} \in \text{RReal} \wedge \text{Rzero} \mapsto \text{ctrl_bound_lin} \in \text{lt} \wedge$
 $\text{ctrl_delta} \in \text{RReal} \wedge \text{Rzero} \mapsto \text{ctrl_delta} \in \text{lt} \wedge$
 $t0 \in \text{RRealPlus} \wedge t1 \in \text{RRealPlus} \wedge t0 \mapsto t1 \in \text{lt} \wedge$
 $\text{PendulumApproxWD}(\text{delta}, \text{omega0}, \text{theta_bound}, \text{ctrl_bound}, \text{ctrl_bound_lin}, \text{ctrl_delta}, t0, t1) \wedge$
 $u_raw \in \text{RReal} \mapsto \text{RReal} \wedge \text{Closed2Closed}(t0, t1) \subseteq \text{dom}(u_raw) \wedge (\forall t_ \cdot t_ \in \text{Closed2Closed}(t0, t1)$
 $) \Rightarrow \text{abs}(u_raw(t_)) \mapsto \text{ctrl_bound} \in \text{lt}) \wedge$
 $u_lin \in \text{RReal} \mapsto \text{RReal} \wedge \text{Closed2Closed}(t0, t1) \subseteq \text{dom}(u_lin) \wedge (\forall t_ \cdot t_ \in \text{Closed2Closed}(t0, t1)$
 $) \Rightarrow \text{abs}(u_lin(t_)) \mapsto \text{ctrl_bound_lin} \in \text{lt}) \wedge$
 $\text{DeltaApproximation}(\text{Closed2Closed}(t0, t1), \text{ctrl_delta}, u_raw, u_lin) \wedge$
 $x0_raw \in \text{RReal} \times \text{RReal} \wedge x0_lin \in \text{RReal} \times \text{RReal} \wedge \text{DeltaNeighborhood}(\text{delta}, x0_raw, x0_lin)$

67 \Rightarrow
68 $\Delta\text{ApproximationEq}(\text{Closed2Closed}(t_0, t_1), \text{delta},$
69 $\quad \text{withControl}(\text{Closed2Closed}(t_0, t_1), \text{PendulumRaw}(\omega_0, x_{0_raw}, t_0), u_raw),$
70 $\quad \text{withControl}(\text{Closed2Closed}(t_0, t_1), \text{PendulumLin}(\omega_0, x_{0_lin}, t_0), u_lin)$
71 $) \text{ pendulum_control}:$

OPERATORS

72 **PendulumRawControl** $\langle \text{expression} \rangle (\omega_0: \mathbb{R}\text{Real}, \theta_0: \mathbb{R}\text{Real}, \theta_{\text{tap}0}: \mathbb{R}\text{Real}, t_0: \mathbb{R}\text{RealPlus}) : \mathbb{P}(\mathbb{R}\text{Real} \times \mathbb{R}\text{Real})$

73 **well-definedness** $\text{abs}(\theta_0) \mapsto \theta_{\text{max}}(\omega_0) \in \text{lt}$

74 **PendulumLinControl** $\langle \text{expression} \rangle (\omega_0: \mathbb{R}\text{Real}, \theta_0: \mathbb{R}\text{Real}, \theta_{\text{tap}0}: \mathbb{R}\text{Real}, t_0: \mathbb{R}\text{RealPlus}) : \mathbb{P}(\mathbb{R}\text{Real} \times \mathbb{R}\text{Real})$

75 **well-definedness** $\text{abs}(\theta_0) \mapsto \theta_{\text{max}}(\omega_0) \in \text{lt}$

76 **PC_raw_bound** $\langle \text{expression} \rangle () : \mathbb{R}\text{Real}$

77 **PC_lin_bound** $\langle \text{expression} \rangle () : \mathbb{R}\text{Real}$

78 **PendulumControlDelta** $\langle \text{expression} \rangle (\omega_0: \mathbb{R}\text{Real}, \text{delta}: \mathbb{R}\text{Real}) : \mathbb{R}\text{Real}$

79 **well-definedness** $\text{Rzero} \mapsto \text{delta} \in \text{lt}$

AXIOMS

80 *pendulum_raw_control_type:*

81 $\forall \omega_0, \theta_0, \theta_{\text{tap}0}, t_0 \cdot$

82 $\omega_0 \in \mathbb{R}\text{Real} \wedge$

83 $\theta_0 \in \mathbb{R}\text{Real} \wedge \text{abs}(\theta_0) \mapsto \theta_{\text{max}}(\omega_0) \in \text{lt} \wedge$

84 $\theta_{\text{tap}0} \in \mathbb{R}\text{Real} \wedge$

85 $t_0 \in \mathbb{R}\text{RealPlus}$

86 $\Rightarrow ($

87 $\text{PendulumRawControl}(\omega_0, \theta_0, \theta_{\text{tap}0}, t_0) \in \mathbb{R}\text{Real} \leftrightarrow \mathbb{R}\text{Real} \wedge$

88 $\text{Closed2Infinity}(t_0) \subseteq \text{dom}(\text{PendulumRawControl}(\omega_0, \theta_0, \theta_{\text{tap}0}, t_0))$

89 $)$

90 *pendulum_lin_control_type:*

91 $\forall \omega_0, \theta_0, \theta_{\text{tap}0}, t_0 \cdot$

92 $\omega_0 \in \mathbb{R}\text{Real} \wedge$

93 $\theta_0 \in \mathbb{R}\text{Real} \wedge \text{abs}(\theta_0) \mapsto \theta_{\text{max}}(\omega_0) \in \text{lt} \wedge$

94 $\theta_{\text{tap}0} \in \mathbb{R}\text{Real} \wedge$

95 $t_0 \in \mathbb{R}\text{RealPlus}$

96 $\Rightarrow ($

97 $\text{PendulumLinControl}(\omega_0, \theta_0, \theta_{\text{tap}0}, t_0) \in \mathbb{R}\text{Real} \leftrightarrow \mathbb{R}\text{Real} \wedge$

98 $\text{Closed2Infinity}(t_0) \subseteq \text{dom}(\text{PendulumLinControl}(\omega_0, \theta_0, \theta_{\text{tap}0}, t_0))$

99 $)$

100 *pendulum_raw_control_bound_def:*

101 $\text{Rzero} \mapsto \text{PC_raw_bound} \in \text{lt}$

102 *pendulum_raw_control_bound:*

103 $\forall \omega_0, \theta_0, \theta_{\text{tap}0}, t_0 \cdot$

104 $\omega_0 \in \mathbb{R}\text{Real} \wedge$

105 $\theta_0 \in \mathbb{R}\text{Real} \wedge \text{abs}(\theta_0) \mapsto \theta_{\text{max}}(\omega_0) \in \text{lt} \wedge$

106 $\theta_{\text{tap}0} \in \mathbb{R}\text{Real} \wedge$

107 $t_0 \in \mathbb{R}\text{RealPlus}$

108 $\Rightarrow ($

109 $\forall t_- \cdot t_- \in \mathbb{R}\text{RealPlus} \wedge t_0 \mapsto t_- \in \text{leq} \Rightarrow$

110 $\text{abs}(\text{PendulumRawControl}(\omega_0, \theta_0, \theta_{\text{tap}0}, t_0)(t_-)) \mapsto \text{PC_raw_bound} \in \text{lt}$

111 $)$

112 *pendulum_raw_control_acceptable:*

113 $\forall \omega_0, \theta_0, \theta_{\text{tap}0}, t_0 \cdot$

114 $\omega_0 \in \mathbb{R}\text{Real} \wedge$

115 $\theta_0 \in \mathbb{R}\text{Real} \wedge \text{abs}(\theta_0) \mapsto \theta_{\text{max}}(\omega_0) \in \text{lt} \wedge$

116 $\theta_{\text{tap}0} \in \mathbb{R}\text{Real} \wedge$

117 $t_0 \in \mathbb{R}\text{RealPlus}$

118 $\Rightarrow ($

119 $\exists t_1 \cdot t_1 \in \mathbb{R}\text{RealPlus} \wedge t_0 \mapsto t_1 \in \text{lt} \wedge$

120 $\text{SolvableWith}(\$

121 $\text{Closed2Closed}(t_0, t_1),$

122 $\text{PendulumRaw}(\omega_0, (\theta_0 \mapsto \theta_{\text{tap}0}), t_0),$

123 $\text{PendulumRawControl}(\omega_0, \theta_0, \theta_{\text{tap}0}, t_0)$

124 $)$

125 $)$

126 *pendulum_raw_control_solution_bounded:*

127 $\forall \omega_0, \theta_0, \theta_{\text{tap}0}, t_0, t_1 \cdot$

128 $\omega_0 \in \mathbb{R}\text{Real} \wedge$

129 $\theta_0 \in \mathbb{R}\text{Real} \wedge \text{abs}(\theta_0) \mapsto \theta_{\text{max}}(\omega_0) \in \text{lt} \wedge$

130 $\theta_{\text{tap}0} \in \mathbb{R}\text{Real} \wedge$

131 $t_0 \in \mathbb{R}\text{RealPlus} \wedge t_1 \in \mathbb{R}\text{RealPlus} \wedge t_0 \mapsto t_1 \in \text{lt} \wedge$

132 $\text{SolvableWith}(\text{Closed2Closed}(t_0, t_1), \text{PendulumRaw}(\omega_0, (\theta_0 \mapsto \theta_{\text{tap}0}), t_0), \text{PendulumRawControl}(\omega_0, \theta_0, \theta_{\text{tap}0}, t_0))$

133 $\Rightarrow ($

134 $\forall \theta_-, \theta_{\text{tap}-} \cdot$

135 $\theta_- \in \mathbb{R}\text{Real} \leftrightarrow \mathbb{R}\text{Real} \wedge \text{Closed2Closed}(t_0, t_1) \subseteq \text{dom}(\theta_-) \wedge$

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138   thetap_ ∈ RReal ⇒ RReal ∧ Closed2Closed(t0, t1) ⊆ dom(thetap_) ∧
139   solutionOf(
140     Closed2Closed(t0, t1),
141     bind(theta_, thetap_),
142     withControl(
143       Closed2Closed(t0, t1),
144       PendulumRaw(omega0, (theta0 ⇒ thetap0), t0),
145       PendulumRawControl(omega0, theta0, thetap0, t0)
146     )
147   )
148   ⇒ (
149     ∀ t_ · t_ ∈ Closed2Closed(t0, t1) ⇒ abs(theta_(t_)) ⇒ theta0 ∈ lt
150   )
151 )
152 pendulum_lin_control_bound_def:
153   Rzero ⇒ PC_lin_bound ∈ lt
154 pendulum_lin_control_bound:
155   ∀ omega0, theta0, thetap0, t0 ·
156     omega0 ∈ RReal ∧
157     theta0 ∈ RReal ∧ abs(theta0) ⇒ theta_max(omega0) ∈ lt ∧
158     thetap0 ∈ RReal ∧
159     t0 ∈ RRealPlus
160   ⇒ (
161     ∀ t_ · t_ ∈ RRealPlus ∧ t0 ⇒ t_ ∈ leq ⇒
162       abs(PendulumLinControl(omega0, theta0, thetap0, t0)(t_)) ⇒ PC_lin_bound ∈ lt
163   )
164 pendulum_lin_control_acceptable:
165   ∀ omega0, theta0, thetap0, t0 ·
166     omega0 ∈ RReal ∧
167     theta0 ∈ RReal ∧ abs(theta0) ⇒ theta_max(omega0) ∈ lt ∧
168     thetap0 ∈ RReal ∧
169     t0 ∈ RRealPlus
170   ⇒ (
171     ∃ t1 · t1 ∈ RRealPlus ∧ t0 ⇒ t1 ∈ lt ∧
172     SolvableWith(
173       Closed2Closed(t0, t1),
174       PendulumLin(omega0, (theta0 ⇒ thetap0), t0),
175       PendulumLinControl(omega0, theta0, thetap0, t0)
176     )
177   )
178 pendulum_lin_control_solution_bounded:
179   ∀ omega0, theta0, thetap0, t0, t1 ·
180     omega0 ∈ RReal ∧
181     theta0 ∈ RReal ∧ abs(theta0) ⇒ theta_max(omega0) ∈ lt ∧
182     thetap0 ∈ RReal ∧
183     t0 ∈ RRealPlus ∧ t1 ∈ RRealPlus ∧ t0 ⇒ t1 ∈ lt ∧
184     SolvableWith(Closed2Closed(t0, t1), PendulumLin(omega0, (theta0 ⇒ thetap0), t0), PendulumLinControl
185       (omega0, theta0, thetap0, t0))
186   ⇒ (
187     ∀ theta_, thetap_ ·
188       theta_ ∈ RReal ⇒ RReal ∧ Closed2Closed(t0, t1) ⊆ dom(theta_) ∧
189       thetap_ ∈ RReal ⇒ RReal ∧ Closed2Closed(t0, t1) ⊆ dom(thetap_) ∧
190       solutionOf(
191         Closed2Closed(t0, t1),
192         bind(theta_, thetap_),
193         withControl(
194           Closed2Closed(t0, t1),
195           PendulumLin(omega0, (theta0 ⇒ thetap0), t0),
196           PendulumLinControl(omega0, theta0, thetap0, t0)
197         )
198       )
199     ⇒ (
200       ∀ t_ · t_ ∈ Closed2Closed(t0, t1) ⇒ abs(theta_(t_)) ⇒ theta0 ∈ lt
201     )
202   )
203 pendulum_control_delta_def:
204   ∀ omega0, delta ·
205     omega0 ∈ RReal ∧
206     delta ∈ RReal ∧ Rzero ⇒ delta ∈ lt
207   ⇒
208     Rzero ⇒ PendulumControlDelta(omega0, delta) ∈ lt
209 pendulum_control_approx:
210   ∀ delta, omega0, theta0_raw, thetap0_raw, theta0_lin, thetap0_lin, t0 ·
211     delta ∈ RReal ∧ Rzero ⇒ delta ∈ lt ∧

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211 omega0 ∈ RReal ∧
212 theta0_raw ∈ RReal ∧ abs(theta0_raw) ↦ theta_max(omega0) ∈ It ∧
213 thetap0_raw ∈ RReal ∧
214 theta0_lin ∈ RReal ∧ abs(theta0_lin) ↦ theta_max(omega0) ∈ It ∧
215 thetap0_lin ∈ RReal ∧
216 t0 ∈ RRealPlus ∧
217 DeltaNeighborhood(delta ,( theta0_raw↦thetap0_raw ) ,( theta0_lin↦thetap0_lin))
218 ⇒
219   DeltaApproximation(
220     Closed2Infinity(t0),
221     PendulumControlDelta(omega0,delta),
222     PendulumRawControl(omega0,theta0_raw,thetap0_raw,t0),
223     PendulumLinControl(omega0,theta0_lin,thetap0_lin,t0)
224   )
225 END
```