A Lightweight Epistemic Logic and its Application to Planning

Elise Perrotin
Joint work with Martin Cooper, Andreas Herzig, Faustine Maffre, Frédéric Maris and Pierre Régnier

IRIT, Toulouse

April 2nd, 2019
Introductory example: learning a message

Two agents are outside a room, in which there is a message $m$. Agents can:

- enter and leave the room;
- display the message;
- ask one another about the message.

Possible goals:
- for both agents to know the message;
- for them to have common knowledge of the message.
Introductory example: learning a message

Two agents are outside a room, in which there is a message $m$. Agents can:
- enter and leave the room;
- display the message;
- ask one another about the message.

Possible goals:
- for both agents to know the message;
- for them to have common knowledge of the message.

- Typical epistemic planning problem

Can we build a lightweight framework in which to model this?
A lightweight epistemic planning framework

Standard DEL planning is undecidable.
Other approaches to simplifying epistemic planning:

- no common knowledge;
- public actions;
- restrict the scope of knowledge operators (e.g., allow $K_i \ldots K_j p$ but not $K_i(p \lor q)$).
  - In particular, $K_1 K_2(m \lor \neg m)$ is not allowed.
A lightweight epistemic planning framework

Standard DEL planning is undecidable.
Other approaches to simplifying epistemic planning:

- no common knowledge;
- public actions;
- restrict the scope of knowledge operators (e.g., allow $K_i \ldots K_j p$ but not $K_i(p \lor q)$).
  
  ▶ In particular, $K_1 K_2(m \lor \neg m)$ is not allowed.

Our approach: use a **visibility-based logic** (inspired by DEL-PAO) and go from *knowing that* to *knowing whether.*
Visibility

We have a set of *observability operators*

\[ OBS = \{ S_i : i \in Agt \} \cup \{ JS \} \]

and a set of *visibility atoms*

\[ ATM = \{ \sigma p : \sigma \in OBS^*, p \in Prop \}. \]

We can now express “\( K_1 K_2 (m \lor \neg m) \)” as \( S_2 m \land S_1 S_2 m \).
Introspection

Agents should be aware of what they (individually and jointly) see.

The set of all introspective atoms is

\[ I-ATM = \{ \sigma S_i S_i \alpha : \sigma \in OBS^* \text{ and } \alpha \in ATM \} \cup \{ \sigma JS \alpha : \sigma \in OBS^+ \text{ and } \alpha \in ATM \} . \]

Atomic consequence:

\[ \alpha \Rightarrow \beta \ \text{iff} \begin{cases} \text{either } \alpha = \beta, \\ \text{or } \alpha = JS \alpha' \text{ and } \beta = \sigma \alpha' \text{ for some } \sigma \in OBS^+ \end{cases} \]
Language

The language of EL-O is defined by the following grammar:

\[ \varphi ::= \alpha \mid \neg \varphi \mid (\varphi \land \varphi) \]

where \( \alpha \) ranges over \( ATM \).

- \( s \models \alpha \) iff \( \alpha \in I-ATM \) or \( \beta \Rightarrow \alpha \) for some \( \beta \in s \)
- \( s \models \neg \varphi \) iff not \( s \models \varphi \)
- \( s \models \varphi \land \varphi' \) iff \( s \models \varphi \) and \( s \models \varphi' \)
Relation with Classical Propositional Calculus (CPC)

\[ s \models_{\text{CPC}} \alpha \text{ iff } \alpha \in s \]

Proposition (Expansion of states)

For every state \( s \subseteq \text{ATM} \) and formula \( \varphi \in \text{Fml}_{\text{EL-O}} \), \( s \models \varphi \) if and only if \( s \Rightarrow \bigcup I\text{-ATM} \models_{\text{CPC}} \varphi \).
Relation with Classical Propositional Calculus (CPC)

\[ s \models_{\text{CPC}} \alpha \iff \alpha \in s \]

Proposition (Expansion of formulas)

Define the expansion of formulas homomorphically from

\[ \text{Exp}(\alpha) = \begin{cases} 
\top & \text{if } \alpha \in I\text{-ATM} \\
(\bigvee \alpha^{\Leftarrow}) & \text{otherwise}
\end{cases} \]

Then for every state \( s \subseteq \text{ATM} \) and formula \( \varphi \in Fml_{\text{EL-O}} \), \( s \models \varphi \iff s \models_{\text{CPC}} \text{Exp}(\varphi) \).

▶ Using expansion, EL-O model checking problems can be polynomially reduced to classical model checking problems.
Proposition (Axiomatization)

For every formula $\varphi \in \text{Fml}_{\text{EL-O}}$, $\varphi$ is EL-O valid iff $\varphi$ is provable in CPC from the following five axiom schemas:

\begin{align*}
S_i S_i \alpha & \quad (Vis_1) \\
JS JS \alpha & \quad (Vis_2) \\
JS S_i S_i \alpha & \quad (Vis_3) \\
JS \alpha \rightarrow S_i \alpha & \quad (Vis_4) \\
JS \alpha \rightarrow JS S_i \alpha & \quad (Vis_5)
\end{align*}

Proposition (Finite model property)

Let $\varphi \in \text{Fml}_{\text{EL-O}}$ be a formula and $s \subseteq \text{ATM}$ a state. Let $s_{\varphi} = (s \Rightarrow \cup I-\text{ATM}) \cap \text{ATM}(\varphi)$. Then $s \models \varphi$ iff $s_{\varphi} \models \varphi$. 
Adding ‘knowing-that’ operators

**Definition (Accessibility relations)**

We associate accessibility relations to agents as follows:

\[ s \sim_i s' \iff s \text{ and } s' \text{ agree on every } \alpha \text{ such that } s \models S_i \alpha; \]

\[ s \sim_{\text{Agt}} s' \iff s \text{ and } s' \text{ agree on every } \alpha \text{ such that } s \models JS \alpha. \]

We can extend the language of EL-O by the standard operators \( K_i \varphi \) and \( CK \varphi \), interpreted as:

\[ s \models K_i \varphi \iff s' \models \varphi \text{ for every } s' \text{ such that } s \sim_i s'; \]

\[ s \models CK \varphi \iff s' \models \varphi \text{ for every } s' \text{ such that } s \sim_{\text{Agt}} s'. \]
Relation with standard epistemic logic

- $\models K_i \alpha \leftrightarrow \alpha \land S_i \alpha$
- $\models CK \alpha \leftrightarrow \alpha \land JS \alpha$
- Distributivity over disjunctions: $\models K_i (p \lor q) \rightarrow K_i p \lor K_i q$
- The fixed point axiom

$$CK p \rightarrow p \land \left( \bigwedge_{i \in \text{Agt}} K_i CK p \right)$$

is valid...

...but not the induction axiom

$$\left( \varphi \land CK (\varphi \rightarrow \bigwedge_{i \in \text{Agt}} K_i \varphi) \right) \rightarrow CK \varphi.$$
Definition (Consistent action descriptions)

An action description is a pair \( a = \langle \text{pre}(a), \text{eff}(a) \rangle \) where \( \text{pre}(a) \) is the precondition of \( a \) and \( \text{eff}(a) \) are the conditional effects of \( a \).

For each conditional effect

\[
\text{ce} = \langle \text{cnd}(\text{ce}), \text{ceff}^+(\text{ce}), \text{ceff}^-(\text{ce}) \rangle,
\]

in \( \text{eff}(a) \), \( \text{cnd}(\text{ce}) \) is the condition of \( \text{ce} \), \( \text{ceff}^+(\text{ce}) \) are the added atoms, and \( \text{ceff}^-(\text{ce}) \) are the deleted atoms.

An action description \( a \) is consistent if and only if

1. for every \( \text{ce} \in \text{eff}(a) \), \( \text{ceff}^-(\text{ce}) \) contains no introspective atoms;

2. for every \( \text{ce}_1, \text{ce}_2 \in \text{eff}(a) \), if \( \text{ceff}^+(\text{ce}_1) \cap (\text{ceff}^-(\text{ce}_2)) \not\subseteq \emptyset \)
   then \( \text{pre}(a) \land \text{cnd}(\text{ce}_1) \land \text{cnd}(\text{ce}_2) \) is unsatisfiable in EL-O.
Example (Learning a message)

- $\text{enter}_i = \langle \neg \text{in}_i, \{ \langle \top, \{ \text{in}_i \}, \emptyset \} \rangle \rangle$;
- $\text{leave}_i = \langle \text{in}_i, \{ \langle \top, \emptyset, \{ \text{in}_i \} \rangle \} \rangle$;
- $\text{reveal}_i =$
  $\langle \text{in}_i, \{ \langle \top, \{ \text{S}_i \text{m} \}, \emptyset \rangle, \langle \text{in}_j, \{ \text{JS m} \}, \emptyset \rangle, \langle \neg \text{in}_j, \{ \text{S}_j \text{S}_i \text{m} \}, \emptyset \rangle \} \rangle$;
- $\text{ask}_{i,j} =$
  $\langle (\text{in}_i \leftrightarrow \text{in}_j) \land \neg \text{S}_i \text{m} \land \text{S}_j \text{m} \land \text{S}_i \text{S}_j \text{m}, \{ \langle \top, \{ \text{JS m} \}, \emptyset \} \rangle \rangle$

for $i, j \in \{1, 2\}$ and $j \neq i$. 
Example (Calls in the original gossip problem)

\[
\text{Call}_j^i = \langle \text{pre}(\text{Call}_j^i), \text{eff}(\text{Call}_j^i) \rangle \quad \text{with} \quad \text{pre}(\text{Call}_j^i) = \top \quad \text{and:}
\]

\[
\text{eff}(\text{Call}_j^i) = \{ \langle S_i s_1 \lor S_j s_1, \{ S_i s_1, S_j s_1 \}, \emptyset \rangle, \ldots, \langle S_i s_n \lor S_j s_n, \{ S_i s_n, S_j s_n \}, \emptyset \rangle \}.
\]
Definition (Semantics)

We define the relation $R_a$ by:

$s R_a s' \iff s \models pre(a)$ and

$$s' = \left( s \setminus \bigcup_{ce \in eff(a),\ s \models cnd(ce)} (ceff^- (ce)) \right) \bigcup \bigcup_{ce \in eff(a),\ s \models cnd(ce)} ceff^+(ce).$$
Definition (Simple planning tasks)

A simple epistemic planning task is a triple \( \mathcal{P} = \langle \text{Act}, s_0, \text{Goal} \rangle \) where \( \text{Act} \) is a finite set of consistent action descriptions, \( s_0 \in 2^{\text{ATM}} \) is a finite state (the initial state) and \( \text{Goal} \in \text{Fml}_{\text{EL-O}} \) is a boolean formula. It is solvable if at least one state \( s \) such that \( s \models \text{Goal} \) is reachable from \( s_0 \) via some sequence of actions from \( \text{Act} \).
Example (Learning a message)

\[ \mathcal{P} = \langle \text{Act}, s_0, \text{Goal} \rangle \text{ with:} \]

- \text{Act} = \{ \text{enter}_i, \text{leave}_i, \text{reveal}_i, \text{ask}_{i,j} : i, j \in \{1, 2\}, i \neq j \} \\
- s_0 = \{ m \} \\
- \text{Goal} = \neg \text{in}_1 \land \neg \text{in}_2 \land \text{JS} m \\

Possible solutions:

- \text{enter}_1, \text{enter}_2, \text{reveal}_1, \text{leave}_1, \text{leave}_2 \\
- \text{enter}_2, \text{reveal}_2, \text{leave}_2, \text{ask}_{1,2}
Example (Generalized gossip problem of depth $k$)

$G_k = \langle \text{Act}^{G_k}, s_0^{G_k}, \text{Goal}^{G_k} \rangle$ where:

- $s_0^{G_k} = \{ S_i s_i : i \in \text{Agt} \} \cup \{ s_i : i \in \text{Agt} \}$
- $\text{Goal}^{G_k} = \bigwedge_{\sigma \in \{ S_i : i \in \text{Agt} \}^{\leq k}} \bigwedge_{j \in \text{Agt}} \sigma s_j$
- $\text{Act}^{G_k} = \{ \text{Call}^i_j : i, j \in \text{Agt}, i \neq j \}$, where

  \[
  \text{Call}^i_j = \langle \top, \{ \langle S_i \sigma_m s_r \lor S_j \sigma_m s_r, \langle \sigma \sigma_m s_r : \sigma \in \{ S_i, S_j \}^{\leq k-m} \}, \emptyset : m < k, \sigma_m \in \{ S_i : i \in \text{Agt} \}^m, r \in \text{Agt} \rangle \]
**Translation into classical planning**

**Definition**

We define the relations $R_a^{\text{class}}$ by:

\[
sR_a^{\text{class}} s' \text{ iff } s \models_{\text{CPC}} \text{pre}(a) \text{ and } s' = \left( s \setminus \bigcup_{ce \in \text{eff}(a), \ s \models_{\text{CPC}} \text{cnd}(ce)} \text{ceff}^-(ce) \right) \cup \bigcup_{ce \in \text{eff}(a), \ s \models_{\text{CPC}} \text{cnd}(ce)} \text{ceff}^+(ce).
\]
**Definition (Expansion of simple epistemic planning tasks)**

Consider the simple epistemic planning task \( P = \langle \text{Act}, s_0, \text{Goal} \rangle \). Its expansion is defined as

\[
\text{Exp}(P) = \langle \{\langle \text{Exp}(\text{pre}(a)), \text{Exp}(\text{eff}(a)) \rangle : \langle \text{pre}(a), \text{eff}(a) \rangle \in \text{Act} \}, \\
\quad s_0, \text{Exp}(\text{Goal}) \rangle,
\]

where

\[
\text{Exp}(\text{eff}(a)) = \{\langle \text{Exp}(\text{cnd}(ce)), \text{eff}^+(ce), (\text{eff}^-(ce)) \rangle^{\,\leftrightarrow} : ce \in \text{eff}(a) \}.
\]

**Proposition**

*Let a be an action description. Then \( R_a = R_{\text{Exp}(a)}^{\text{class}} \).*
Proposition

Let $\mathcal{P}$ be a simple planning task. Then $\mathcal{P}$ is solvable iff its expansion $\text{Exp}(\mathcal{P})$ is classically solvable.

Proposition

The problem of deciding solvability of a simple epistemic planning task is $\text{PSPACE}$ complete.
Using a visibility logic, we’ve designed a method for epistemic planning in which solvability of planning tasks is decidable (as opposed to DEL), and deciding is $\text{PSpace}$ complete.

Our method is less restrictive than other approaches (joint vision, private announcements, knowing-whether).

Limitation: we can’t model problems such as the muddy children problem, where agents know disjunctions.

Future work: distributed planning; joint vision restricted to groups.