Multi-agent Abduction Using Doxastic Temporal Models

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Pepper passing false-belief tasks

http://www2.compute.dtu.dk/~tobo/forskerzonen_trimmed.mp4

Forskerens favorit: Robotten Pepper lærer at sætte sig i andres sted.
https://videnskab.dk/teknologi-innovation/
forskerens-favorit-robotten-pepper-laerer-at-saette-sig-i-andres-sted
Research in social artificial intelligence at DTU: A Pepper robot with social perspective-taking abilities.

The robot solves cognitive tasks: **false-belief tasks** of arbitrary order. Humans can solve first-order at age 4, second-order at age 10 and third-order at age 20.
Plausibility models

Agents: $A$.
Propositions: $P$.

(Multi-pointed) plausibility model: $\mathcal{M} = \langle W, (\preceq_i)_{i \in A}, V, W_d \rangle$, where

- $W$ is a set of possible worlds, marked $\bullet$.
- Each $\preceq_i$ is a plausibility relation: a set of mutually disjoint well-preorders covering $W$.
- $V$ is a valuation.
- $W_d \subseteq W$ is a set of designated worlds (one of which is the actual), marked $\circ$.

$w_1 \preceq S w_2$ iff $S$ finds $w_1$ more plausible than $w_2$.

$\sim_i := \preceq_i \cup \succeq_i$. $w_1 \sim_i w_2$ means $w_1$ and $w_2$ are (epistemically) indistinguishable to agent $i$. 
Language and semantics

Language:

\[ \phi ::= \neg \phi \mid \phi \land \phi \mid B_i \phi \mid K_i \phi \mid C \phi. \]

Semantics:

- \( M, w \models B_i \phi \) iff \( \phi \) holds in the most plausible worlds \( i \) cannot epistemically distinguish from \( w \).
- \( M, w \models K_i \) iff \( \phi \) holds in all worlds \( i \) cannot epistemically distinguish from \( w \).
- \( M \models \phi \) iff \( M, w \models \phi \) for all \( w \in W_d \).

\[ M = \begin{array}{c}
  w_1 : t \\
  S \\
  w_2 : x
\end{array} \]

\[ M \models B_S t \land \neg K_S t \]
(Multi-pointed) event model: $\mathcal{E} = \langle E, (\preceq i)_{i \in A}, \text{pre}, \text{post}, E_d \rangle$, where

- $E$ is a set of possible events, marked ●.
- Each $\preceq_i$ is a plausibility relation (as before).
- For each $e \in E$, $\text{pre}(e)$ is a precondition: a formula.
- For each $e \in E$, $\text{post}(e)$ is a simple postcondition: a conjunction of literals.
- $E_d \subseteq E$ is a set of designated events (one of which is the actual), marked ○.

Events $e$ are labelled by $(\text{pre}(e), \text{post}(e))$.  

\[ e_1 : (T, T) \quad S \quad e_2 : (T, x \land \lnot t) \]
Product update

The **product update** of a plausibility model \( \mathcal{M} = \langle W, (\preceq_i)_{i \in A}, V, W_d \rangle \) with an event model \( \mathcal{E} = \langle E, (\preceq_i)_{i \in A}, \text{pre}, \text{post}, E_d \rangle \) is the plausibility model

\[
\mathcal{M} \otimes \mathcal{E} = \langle W', (\preceq'_i)_{i \in A}, V', W'_d \rangle
\]

where

- \( W' = \{(w, e) \in W \times E \mid \mathcal{M}, w \models \text{pre}(e)\} \)
- \( \preceq'_i = \{((w, e), (v, f)) \in W' \times W' \mid (w \sim_i v \text{ and } e \preceq_i f \text{ and } f \not\preceq_i e) \text{ or } (e \preceq_i f \text{ and } f \preceq_i e \text{ and } w \preceq_i v)\} \) (action-priority update).
- \( V'(p) = \{(w, e) \in W' \mid \text{post}(e) \models p \text{ or } (w \in V(p) \text{ and } \text{post}(e) \not\models \neg p)\} \).
- \( W'_d = \{(w, e) \in W' \mid w \in W_d \text{ and } e \in E_d\} \).
DoTL models

A DoTL model is a plausibility model where the worlds are structured into histories over an alphabet of events. Formally: A DoTL model (model of Doxastic Temporal Logic) over a set of events $\Sigma$ is a plausibility model $D = \langle H, (\preceq_i)_{i \in A}, V, H_d \rangle$, where $H \subseteq \Sigma^*$ is closed under non-empty prefixes. Elements of $H$ are called histories.

The frontier of a DoTL model is the plausibility model consisting of the maximal histories. Formally: The frontier of a DoTL model $D = \langle H, (\preceq_i)_{i \in A}, V, H_d \rangle$ is the plausibility model given by $\text{frontier}(D) = \langle H_{\max}, (\preceq_i \upharpoonright H_{\max}^2)_{i \in A}, V \upharpoonright H_{\max}, H_d \cap H_{\max} \rangle$, where $H_{\max} = \{ h \in H \mid \text{there is no } h' \in H \text{ with } |h'| > |h| \}$.

The inclusive (product) update of a DoTL model $D$ with an event model $E$ extends $D$ by updating its frontier with $E$. Formally: The inclusive (product) update of a DoTL model $D = \langle H, (\preceq_i)_{i \in A}, V, H_d \rangle$ over $\Sigma$ and an event model $E = \langle E, (\preceq_i)_{i \in A}, \text{pre, post}, E_d \rangle$ is the DoTL model $D \otimes \cup E$ over $\Sigma \cup E$ given by the union of $D$ and $\text{frontier}(D) \otimes E$. 
Example: The Sally-Anne task

initial state: \( w_0 : S, t \)

Leave: \( \text{leave:} (\top, \neg S) \)

Swap: \( \text{skip:} (\top, \top) \quad \text{move:} (\top, x \land \neg t) \)

Enter: \( \text{enter:} (\top, S) \)

PeekBasket: \( \neg t! : (\neg t, \top) \)
Sally-Anne DoTL model: inclusive updates

\[ w_0 : S, t \]

\[ \text{leave}: (\top, \neg S) \quad \text{Leave} \]
Sally-Anne DoTL model: inclusive updates

\[ w_0 : S, t \]

\[ \text{leave} : (\top, \neg S) \]

\[ \text{Leave} \]

\[ \text{enter} : (\top, S) \]

\[ \text{enter} \]

\[ \text{PeekBasket} \]

\[ \neg t \]

\[ \text{Let} \]

\[ h = (e_1, \ldots, e_n) \]

\[ \text{be a most plausible history for agent i}. \]

\[ \text{The event } e_n \text{ is called a } \text{surprise} \text{ to agent } i \text{ in } h \text{ if } (e_1, \ldots, e_{n-1}) \text{ is not a most plausible history for agent } i. \]

\[ \text{The event } \neg t \text{ is a surprise above.} \]
Sally-Anne DoTL model: inclusive updates

\[ w_0 : S, t \]

\[ leave : (\top, \neg S) \]

\[ S \]

\[ skip : (\top, \top) \]

\[ move : (\top, x \land \neg t) \]

\[ \text{Leave} \]

\[ \text{Swap} \]
Sally-Anne DoTL model: inclusive updates

\[ w_0 : S, t \]

\[
\begin{align*}
\text{leave} &: (\top, \neg S) \\
\text{swap} &: (\top, \top) \\
\text{skip} &: (\top, \top) \\
\text{move} &: (\top, x \land \neg t) \\
\text{enter} &: (\top, S) \\
\text{PeekBasket} &: (\neg t, \top) \\
\text{Let} \quad h &= (e_1, \ldots, e_n) \quad \text{be a most plausible history for agent} \quad i. \\
\text{The event} \quad e_n \quad \text{is called a} \quad \text{surprise} \quad \text{to agent} \quad i \quad \text{in} \quad h \quad \text{if} \quad (e_1, \ldots, e_{n-1}) \quad \text{is not a most plausible history for agent} \quad i. \\
\text{The event} \quad \neg t! \quad \text{is a surprise above.}
\end{align*}
\]
Sally-Anne DoTL model: inclusive updates

\[ w_0 : S, t \]

**Leave**

\[ \text{leave} : (\top, \neg S) \]

**Swap**

\[ \text{skip} : (\top, \top) \quad \text{move} : (\top, x \land \neg t) \]

**Enter**

\[ \text{enter} : (\top, S) \]

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Sally-Anne DoTL model: inclusive updates

\[ w_0 : (S, t) \]

\[ \text{leave} : (\top, \neg S) \]

\[ \text{skip} : (\top, \top) \]

\[ \text{move} : (\top, x \land \neg t) \]

\[ \text{enter} : (\top, S) \]

\[ \text{Swap} \]

\[ \text{Leave} \]

\[ \text{Enter} \]

\[ \text{t} \]

\[ \text{x} \]

\[ \text{S} \]

\[ \text{S, t} \]

\[ \text{S, x} \]
Sally-Anne DoTL model: inclusive updates

\[ w_0 : S, t \]

\[ leave : (\top, \neg S) \]

\[ skip : (\top, \top) \quad move : (\top, x \land \neg t) \]

\[ enter : (\top, S) \]

\[ \neg t! : (\neg t, \top) \]

Let \( h = (e_1, \ldots, e_n) \) be a most plausible history for agent \( i \). The event \( e_n \) is called a surprise to agent \( i \) in \( h \) if \((e_1, \ldots, e_{n-1})\) is not a most plausible history for agent \( i \). The event \( \neg t! \) is a surprise above.
Sally-Anne DoTL model: inclusive updates

\[
\text{leave} : (\top, \neg S)
\]

\[
\text{skip} : (\top, \top)
\]

\[
\text{move} : (\top, x \land \neg t)
\]

\[
\text{enter} : (\top, S)
\]

\[
\neg t! : (\neg t, \top)
\]

\[
\text{peekBasket}
\]

\[
\text{w}_0 : S, t
\]

\[
\text{leave}
\]

\[
\text{swap}
\]

\[
\text{move}
\]

\[
\text{enter}
\]

\[
\text{peekBasket}
\]
Let $h = (e_1, \ldots, e_n)$ be a most plausible history for agent $i$. The event $e_n$ is called a **surprise** to agent $i$ in $h$ if $(e_1, \ldots, e_{n-1})$ is not a most plausible history for agent $i$. The event $\neg t!$ is a surprise above.
Language and semantics of DoTL models

We extend the previous language by ∗-free PDL (including inverses) on events/histories:

\[
\phi ::= \cdots | \langle \pi \rangle \phi \\
\pi ::= e | \pi; \pi | \pi \cup \pi | \pi^{-1} \quad \text{where } e \in \Sigma.
\]

Semantics.

\( \mathcal{D}, h \models \langle \pi \rangle \phi \) iff for some \( h' \) with \( hR_\pi h' \) we have \( \mathcal{D}, h' \models \phi \), where

- \( R_e = \{ (h, he) \mid he \in H \} \)
- \( R_{\pi_1 \cup \pi_2} = R_{\pi_1} \cup R_{\pi_2} \)
- \( R_{\pi_1;\pi_2} = R_{\pi_1} \circ R_{\pi_2} \)
- \( R_{\pi^{-1}} = (R_\pi)^{-1} \)

\( \mathcal{D} \models \phi \) iff \( \mathcal{D}, h \models \phi \) for all \( h \in H_{\text{max}} \cap H_d \).

**Example.** Any DoTL model \( \mathcal{D} \) has full synchrony: There is a number \( m \) such that \( \mathcal{D} \models C(\langle (\bigcup_{i \in \Sigma} e_i)^{-m} \rangle T \land [(\bigcup_{i \in \Sigma} e_i)^{-(m+1)}])_\bot) \)
Subjective models

When DoTL models are constructed from the **subjective view** of an agent, it might not be possible to point out a single designated history.

The **associated subjective model** of agent $i$ of a plausibility model/event model/DoTL model is achieved by closing the set of designated points under the epistemic indistinguishability relation of agent $i$.

The DoTL to the right shows the inclusive product updates from the subjective view.
**Restricted inclusive update**

**Inclusive update** of $D$ with $E$ restricted to a subset $I$ of the frontier: Mark frontier histories outside $I$ with “cut”. Only update frontier histories in $I$ with $E$. Formally: Let $D$ be a DoTL model over $\Sigma$ and $E$ an event model with events $E$. Let $I \subseteq H_{max}$ be a set of frontier histories of $D$. The inclusive update of $D$ with $E$ restricted to $I$ is the DoTL model $D \otimes^I \cup E$ over $\Sigma \cup E$ given by the union of $D$ and $(\text{frontier}(D) \upharpoonright I) \otimes E$, and where all $h \in H_{max} - I$ are marked by “cut”.

The **most plausible update** for agent $i$ of $D$ with $E$, denoted $D \otimes^i \cup E$, is the inclusive update restricted to the most plausible histories of the frontier for agent $i$—and closed under more plausible histories for other agents. Formally: For a DoTL model $D$, the histories considered most plausible by agent $i$ are the histories $h' \in H_{max}$ satisfying $D \models \hat{B}_i (h'^{-1})^T$. Let $\preceq = (\cup_{i \in A} \leq_i)^+$. The most plausible update for agent $i$ of $D$ with $E$ is the inclusive update of $D$ with $E$ restricted to \{ $h \in H_{max}$ | $h \preceq h'$ for some most plausible history $h'$ for agent $i$ in $D$ \}.

A restricted inclusive update is called **degenerate** if no designated histories are added (at the new level).
Most plausible updates on the Sally-Anne example

$w_0 : S, t$

$leave : (T, \neg S)$

$leave$
Most plausible updates on the Sally-Anne example

\[
\begin{align*}
\text{leave:} & \ (\top, \neg S) \\
\text{Leave} & \\
\end{align*}
\]

\[
\begin{align*}
\omega_0 & : S, t \\
\text{leave} & \\
\text{t} & \\
\end{align*}
\]
Most plausible updates on the Sally-Anne example

\[
\begin{align*}
\text{leave:} & \ (\top, \neg S) \\
\text{skip:} & \ (\top, \top) \\
\text{move:} & \ (\top, x \land \neg t) \\
\end{align*}
\]

\[
\begin{align*}
\text{S} \\
\text{S} \\
\text{\textit{Swap}} \\
\end{align*}
\]

\[
\begin{align*}
\text{Leave} \\
\text{\textit{Swap}} \\
\text{\textit{move}} \\
\end{align*}
\]
Most plausible updates on the Sally-Anne example

$\text{leave}: (\top, \neg S)$  
$\text{skip}: (\top, \top)$  
$\text{move}: (\top, x \land \neg t)$

$w_0 : S, t$

$\text{leave}$  
$\text{skip}$  
$\text{move}$

$\text{cut}$
Most plausible updates on the Sally-Anne example

- **leave:** $\langle \top, \neg S \rangle$
- **skip:** $\langle \top, \top \rangle$
- **move:** $\langle \top, x \land \neg t \rangle$
- **enter:** $\langle \top, S \rangle$

**Graphical Representation:**

- **Leave**
  - $w_0 : S, t$
  - **leave**
- **Swap**
  - **skip**
  - **move**
- **Enter**
  - $t \leftarrow S \leftarrow x$

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Most plausible updates on the Sally-Anne example

\begin{itemize}
  \item \textbf{leave}: $(\top, \neg S)$
  \item \textbf{skip}: $(\top, \top)$
  \item \textbf{move}: $(\top, x \land \neg t)$
  \item \textbf{enter}: $(\top, S)$
  \item $w_0 : S, t$
  \item $\neg t : (\neg t, \top)$
\end{itemize}
Most plausible updates on the Sally-Anne example

\[ \text{leave} : (\top, \neg S) \]

\[ \text{skip} : (\top, \top) \quad \text{move} : (\top, x \land \neg t) \]

\[ \text{enter} : (\top, S) \]

\[ \neg t! : (\neg t, \top) \]

\[ w_0 : S, t \]

\[ \text{leave} \]

\[ \text{Swap} \]

\[ \text{Enter} \]

\[ \text{PeekBasket} \]
Most plausible updates on the Sally-Anne example

- \text{leave}:(\top, \neg S)
- \text{skip}:(\top, \top)
- \text{move}:(\top, x \land \neg t)
- \text{enter}:(\top, S)
- \neg t!:(\neg t, \top)

\text{PeekBasket: }\neg t!

\text{Degenerate!}

\text{S, t}

\text{enter: }\neg t!

\text{cut: }\neg t!

\text{enter: }S, X

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Degenerate models, surprise and abduction

When an agent $i$ does most plausible updates and ends up with a degenerate model, it corresponds to a surprise: There are no longer any histories consistent with the observed.

In those cases, the agent needs to do abduction: Expand some of the cut histories (histories labelled “cut”).

**Single abduction step:** Choose a cut history, remove the “cut” label, and expand it.

**Example.** Agent $i$ might do most plausible updates on an initial (subjective) plausibility model $\mathcal{M}$ with (subjective) event models $\mathcal{E}_1, \ldots, \mathcal{E}_n$: $\mathcal{M} \otimes^i \mathcal{E}_1 \otimes^i \mathcal{E}_2 \otimes^i \cdots \otimes^i \mathcal{E}_n$. An abduction step is then to pick a cut history $h$ of length $m$ and replace the DoTL model by:

$$
\mathcal{M} \otimes^i \mathcal{E}_1 \otimes^i \mathcal{E}_2 \otimes \cdots \otimes \mathcal{E}_{m-1} \otimes^i \mathcal{E}_m \otimes \cdots \otimes^i \mathcal{E}_n.
$$
Sally-Anne with abduction

\[ \text{leave}(\top, \neg S) \quad \text{Leave} \]

\[ w_0 : S, t \]

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Sally-Anne with abduction

\[
\text{leave} : (\top, \neg S)
\]

\[
\text{leave}
\]

\[
\omega_0 : S, t
\]

\[
\text{leave}
\]

\[
t
\]
Sally-Anne with abduction

\[ w_0 : S, t \]

- **Leave**
  \[ \text{leave}: (\top, \neg S) \]

- **Swap**
  \[ \text{swap}: (\top, \top) \]

- **Move**
  \[ \text{move}: (\top, x \land \neg t) \]

- **Enter**
  \[ \text{enter}: (\top, S) \]
Sally-Anne with abduction

leave: ($\top, \neg S$)

swap: ($\top, \top$)

move: ($\top, x \land \neg t$)

$w_0 : S, t$

leave

skip

move

$S$

$x$
Sally-Anne with abduction

Leave: $(\top, \neg S)$

Swap: $(\top, \top)$

Move: $(\top, x \land \neg t)$

Enter: $(\top, S)$

$w_0 : S, t$

Leave

Swap

Move

Enter

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Sally-Anne with abduction

\[
\begin{align*}
\text{leave}: & (\top, \neg S) \\
\text{skip}: & (\top, \top) \\
\text{move}: & (\top, x \land \neg t) \\
\text{enter}: & (\top, S)
\end{align*}
\]
Sally-Anne with abduction

\[ \text{leave}: (T, \neg S) \]

\[ \text{skip}: (T, T) \quad \text{move}: (T, x \land \neg t) \]

\[ \text{enter}: (T, S) \]

\[ \neg t!: (\neg t, T) \]

\[ \text{PeekBasket} \]

\[ w_0 : S, t \]

\[ \text{leave} \]

\[ \text{swap} \]

\[ \text{move} \]

\[ \text{cut} \]

\[ \text{enter} \]

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Sally-Anne with abduction

\begin{align*}
\text{leave} &: (\top, \neg S) \\
\text{skip} &: (\top, \top) \\
\text{move} &: (\top, x \land \neg t) \\
\text{enter} &: (\top, S) \\
\neg t! &: (\neg t, \top) \\
\text{PeekBasket} \\
\end{align*}

\begin{align*}
\text{Bolander: Multi-agent Abduction, 6 Dec 2018 – p. 17/22}
\end{align*}
Sally-Anne with abduction

\begin{align*}
\text{leave} &: (\top, \neg S) \\
\text{skip} &: (\top, \top) \\
\text{move} &: (\top, x \land \neg t) \\
\text{enter} &: (\top, S) \\
\neg t! &: (\neg t, \top)
\end{align*}

\begin{align*}
\text{Swap} \\
\text{Enter} \\
\text{PeekBasket} \\
\text{Abduct!}
\end{align*}
Sally-Anne with abduction

\[
\begin{align*}
\text{leave}: & (\top, \neg S) \\
\text{skip}: & (\top, \top) \\
\text{move}: & (\top, x \land \neg t) \\
\text{enter}: & (\top, S) \\
\neg t!: & (\neg t, \top) \\
\text{PeekBasket}: & \\
\end{align*}
\]
Sally-Anne with abduction

- **leave**: $(\top, \neg S)$
- **skip**: $(\top, \top)$
- **move**: $(\top, x \land \neg t)$
- **enter**: $(\top, S)$
- **\neg t!**: $(\neg t, \top)$
- **w_0**: $S, t$
- **leave**
- **Swap**
- **Enter**
- **PeekBasket**
Sally-Anne with abduction

\[
\begin{align*}
leave &: (\top, \neg S) \\
S &: \text{Skip: } (\top, \top) \quad \text{move: } (\top, x \land \neg t) \\
enter &: (\top, S) \\
\neg t! &: (\neg t, \top) \\
\end{align*}
\]

\[
\begin{align*}
\text{Swapping:} & \quad w_0 : S, t \\
\text{Leave} & \quad \text{swap} \\
\text{Move} & \quad \text{move} \\
\text{Enter} & \quad \text{enter} \\
\text{Peek Basket} & \quad \neg t! : (\neg t, \top)
\end{align*}
\]
Simple abduction types

The crucial thing about abduction is choosing which cut histories to expand. Some obvious options:

- Expand the most plausible cut histories among the cuts of maximal length. This corresponds to chronological minimisation [Bell 1998].
- Expand the most plausible cut histories of minimal length. This corresponds to inverse chronological minimisation.
- Uniform expansion of all most plausible cut histories of any length.
Simple abduction types

The crucial thing about abduction is choosing which cut histories to expand. Some obvious options:

• Expand the most plausible cut histories among the cuts of maximal length. This corresponds to chronological minimisation [Bell 1998].
• Expand the most plausible cut histories of minimal length. This corresponds to inverse chronological minimisation.
• Uniform expansion of all most plausible cut histories of any length.

If Sally is away for 2 time steps before coming back and peeking into the basket, she will by chronological minimisation come to believe the marble was moved at the second time step. By inverse chronological minimisation, she will come to believe it was moved at the first time step.

Inverse chronological minimisation is consistent with action-priority update: Action-priority update will also give that (leave, move, skip) is a more plausible history than (leave, skip, move) (later actions take precedence over earlier).
Defeasible and indefeasible knowledge and belief

With restricted updates, belief and knowledge becomes “defeasible”. If Sally does most plausible updates, she only has one possible world when returning: the one representing the wrong location of the marble.

We can distinguish between defeasible knowledge and belief ($K^d_i$ and $B^d_i$) and indefeasible knowledge and belief ($K_i$ and $B_i$).

A history $h'$ is said to be $i$-accessible above a history $h$ if $h' \sim_i h_{init}$ for some prefix $h_{init}$ of $h$.

- $\mathcal{D}, h \models B^d_i \phi$ iff $\phi$ holds in the most plausible histories $i$ cannot distinguish from $h$.
- $\mathcal{D}, h \models K^d_i \phi$ iff $\phi$ holds in all histories $i$ cannot distinguish from $h$.
- $\mathcal{D}, h \models B_i \phi$ iff there are no $i$-accessible cuts above $h$ and $\phi$ holds in all most plausible histories $i$ cannot distinguish from $h$.
- $\mathcal{D}, h \models K_i \phi$ iff there are no $i$-accessible cuts above $h$ and $\phi$ holds in all histories $i$ cannot distinguish from $h$.

**Proposition.** Indefeasible knowledge/belief in a restricted update implies knowledge/belief in the unrestricted update. **Proof.** By definition of product update.
Defeasible and indefeasible belief in Sally-Anne

$\text{leave} : (\top, \neg S)$  
$\text{skip} : (\top, \top)$  
$\text{move} : (\top, x \land \neg t)$  
$\text{enter} : (\top, S)$

$w_0 : S, t$

$\text{leave}$  
$\text{swap}$  
$\text{enter}$  
$\text{move}$  
$\text{cut}$

$D, (w_0, \text{leave}, \text{skip}, \text{enter}) \models B^d_S t \land K^d_S t \land \neg B_S t$. 

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Chain abduction

Let \((i_1, \ldots, i_n)\) be sequence of agents. \((i_1, \ldots, i_n)\)-abduction on history \(h\) iteratively takes any cut that is \((i_1, \ldots, i_n)\)-accessible (by the epistemic relation) above \(h\) and expands it.

**Proposition.** Assume \(\mathcal{D}'\) results from \(\mathcal{D}\) by \((i_1, \ldots, i_n)\)-abduction on history \(h\). If \(\mathcal{D}', h \models B_{i_1}^d \cdots B_{i_n}^d \phi\) where \(\phi\) is propositional, then \(\mathcal{D}', h \models B_{i_1} \cdots B_{i_n} \phi\). (also holds for knowledge)
Final comments

Conclusion. Existing approaches to Sally-Anne in DEL don’t offer recovery from false beliefs (but see also [van Eijck 2017]). Using plausibility models solves this, but is computationally unfeasible. Our frameworks sits between these two—seeks to combine the best of both.