Extraire le noyau d’un dialogue de persuasion pour évaluer sa qualité

Extracting the core of a persuasion dialog to evaluate its quality

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Résumé : Un dialogue de persuasion est un dialogue dans lequel les agents échangent des arguments à propos d’un sujet. Dans ce type de dialogue, les agents ne sont pas d’accord sur le statut du sujet et chacun essaie de persuader les autres de changer d’avis. Dans la littérature, plusieurs systèmes fondés sur la théorie de l’argumentation ont été proposés pour modéliser les dialogues de persuasion. Il est important de pouvoir analyser la qualité de ces dialogues. C’est pourquoi des critères de qualité doivent être définis afin d’être à même de réaliser cette analyse.
Cet article aborde cet important problème et propose un critère qui concerne la concision du dialogue. Un dialogue est concis si tous ses coups sont appropriés et utiles pour obtenir le même résultat. À partir d’un dialogue de persuasion, on calcule le dialogue “idéal” lui correspondant. Ce dialogue idéal est concis. Un dialogue de persuasion est considéré intéressant s’il est proche de son dialogue idéal.

Abstract: A persuasion dialog is a dialog in which agents exchange arguments on a subject. In this kind of dialog, the agents disagree about the status of the subject and each one tries to persuade the others to change his mind. Several systems, grounded on argumentation theory, have been proposed in the literature for modeling persuasion dialogs. It is important to be able to analyze the quality of these dialogs. Hence, quality criteria have to be defined in order to perform this analysis.
This paper tackles this important problem and proposes one criterion that concerns the conciseness of a dialog. A dialog is concise if all its moves are relevant and useful in order to reach the same outcome as the original dialog. From a given persuasion dialog, in this paper we compute its corresponding “ideal” dialog. This ideal dialog is concise. A persuasion dialog is thus interesting if it is close to its ideal dialog.
1 Introduction

Persuasion is one of the main types of dialogs encountered in everyday life. A persuasion dialog concerns two (or more) agents who disagree on a state of affairs, and each of them tries to persuade the others to change their minds. For that purpose, agents exchange arguments of different strengths. Several systems have been proposed in the literature for allowing agents to engage in persuasion dialogs (e.g. [4, 5, 7, 8, 10, 11, 13]). A dialog system is built around three main components: i) a communication language specifying the locutions that will be used by agents during a dialog for exchanging information, arguments, etc., ii) a protocol specifying the set of rules governing the well-definition of dialogs such as who is allowed to say what and when? and iii) agents’ strategies which are the different tactics used by agents for selecting their moves at each step in a dialog. All the existing systems allow agents to engage in dialogs that obey to the rules of the protocol. Thus, the only properties that are guaranteed for a generated dialog are those related to the protocol. For instance, one can show that a dialog terminates, the turn shifts equally between agents in that dialog (if such rule is specified by the protocol), agents can refer only to the previous move or are allowed to answer to an early move in the dialog, etc. The properties inherited from a protocol are related to the way the dialog is generated. However, the protocol is not concerned by the quality of that dialog. Moreover, it is well-known that under the same protocol, different dialogs on the same subject may be generated. It is important to be able to compare them w.r.t. their quality. Such a comparison may help to refine the protocols and to have more efficient ones.

While there are a lot of works on dialog protocols (eg. [9]), no work is done on defining criteria for evaluating the persuasion dialogs generated under those protocols, except a very preliminary proposal in [2]. The basic idea of that paper is, given a finite persuasion dialog, it can be analyzed w.r.t. three families of criteria. The first family concerns the quality of arguments exchanged in this dialog. The second family checks the behavior of the agents involved in this dialog. The third family concerns the dialog as a whole. In this paper, we are more interested by investigating this third family of quality criteria. We propose a criterion based on the conciseness of the generated dialog. A dialog is concise if all its moves (i.e. the exchanged arguments) are both relevant to the subject (i.e. they don’t deviate from the subject of the dialog) and useful (i.e. they are important to determine the outcome of the dialog). Inspired from works on proof procedures that have been proposed in argumentation theory in order to check whether an argument is accepted or not [1], we compute and characterize a sub-dialog of the original one that is concise. This sub-dialog is considered as ideal. The closer the original dialog to its ideal sub-dialog, the better is its quality. This report contains all the proofs of the article [3] published in ECSQARU’09.

The paper is organized as follows: Section 2 recalls the basics of argumentation theory. Section 3 presents the basic concepts of a persuasion dialog. Section 4 defines the notions of relevance and usefulness in a dialog. Section 5 presents the concept of ideal dialog founded on an ideal argumentation tree built from the initial dialog.
2 Basics of argumentation systems

Argumentation is a reasoning model based on the construction and the comparison of arguments. Arguments are reasons for believing in statements, or for performing actions. In this paper, the origins of arguments are supposed to be unknown. They are denoted by lowercase Greek letters. In [6], an argumentation system is defined by:

**Definition 1 (Argumentation system)** An argumentation system is a pair \( AS = (A, R) \), where \( A \) is a set of arguments and \( R \subseteq A \times A \) is an attack relation. We say that an argument \( \alpha \) attacks an argument \( \beta \) iff \( (\alpha, \beta) \in R \).

Note that to each argumentation system is associated a directed graph whose nodes are the different arguments, and the arcs represent the attack relation between them.

Since arguments are conflicting, it is important to know which arguments are acceptable. For that purpose, in [6], different acceptability semantics have been proposed. In this paper, we consider the case of grounded semantics. Remaining semantics are left for future research.

**Definition 2 (Defense–Grounded extension)** Let \( AS = (A, R) \) and \( B \subseteq A \).

- \( B \) defends an argument \( \alpha \in A \) iff \( \forall \beta \in A \), if \( (\beta, \alpha) \in R \), then \( \exists \delta \in B \) s.t. \( (\delta, \beta) \in R \).

- The grounded extension of \( AS \), denoted by \( E \), is the least fixed point of a function \( F \) where \( F(B) = \{ \alpha \in A \mid B \text{ defends } \alpha \} \).

When the argumentation system is finite in the sense that each argument is attacked by a finite number of arguments, \( E = \bigcup_{j\geq 0} F^j(\emptyset) \).

Now that the acceptability semantics is defined, we can define the status of any argument. As we will see, an argument may have two possible statuses: accepted or rejected.

**Definition 3 (Argument status)** Let \( AS = (A, R) \) be an argumentation system, and \( E \) its grounded extension. An argument \( \alpha \in A \) is accepted iff \( \alpha \in E \), it is rejected otherwise. We denote by \( Status(\alpha, AS) \) the status of \( \alpha \) in \( AS \).

**Property 1 ([1])** Let \( AS = (A, R) \), \( E \) its grounded extension, and \( \alpha \in A \). If \( \alpha \in E \), then \( \alpha \) is indirectly defended\(^1\) by non-attacked arguments against all its attackers.

3 Persuasion dialogs

This section defines persuasion dialogs in the same spirit as in [4]. A persuasion dialog consists mainly of an exchange of arguments between different agents of the set \( Ag = \{a_1, \ldots, a_n\} \). The subject of such a dialog is an argument, and its aim is to provide the

\(^1\)An argument \( \alpha \) is indirectly defended by \( \beta \) iff there exists a finite sequence of distinct arguments \( a_1, \ldots, a_{2n+1} \) such that \( \alpha = a_1, \beta = a_{2n+1}, \) and \( \forall i \in [1, 2n], (a_{i+1}, a_i) \in R, n \in N^* \).
status of that argument. At the end of the dialog, the argument may be either “accepted” or “rejected”, this status is the output of the dialog. In what follows, we assume that agents are only allowed to exchange arguments.

Each participating agent is supposed to be able to recognize all elements of \( \text{arg}(\mathcal{L}) \) and \( \mathcal{R}_L \), where \( \text{arg}(\mathcal{L}) \) is the set of all arguments that may be built from a logical language \( \mathcal{L} \) and \( \mathcal{R}_L \) is a binary relation that captures all the conflicts that may exist among arguments of \( \text{arg}(\mathcal{L}) \). Thus, \( \mathcal{R}_L \subseteq \text{arg}(\mathcal{L}) \times \text{arg}(\mathcal{L}) \). For two arguments \( \alpha, \beta \in \text{arg}(\mathcal{L}) \), the pair \( (\alpha, \beta) \in \mathcal{R}_L \) means that the argument \( \alpha \) attacks the argument \( \beta \). Note that this assumption does not mean at all that an agent is aware of all the arguments. But, it means that agents use the same logical language and the same definitions of arguments and conflict relation.

**Definition 4 (Moves)** A move \( m \) is a triple \( (S, H, \alpha) \) such that:

- \( S \in \text{Ag} \) is the agent that utters the move, \( \text{Speaker}(m) = S \)
- \( H \subseteq \text{Ag} \) is the set of agents to which the move is addressed, \( \text{Hearer}(m) = H \)
- \( \alpha \in \text{arg}(\mathcal{L}) \) is the content of the move, \( \text{Content}(m) = \alpha \).

During a dialog several moves may be uttered. Those moves constitute a sequence denoted by \( \langle m_1, \ldots, m_n \rangle \), where \( m_1 \) is the initial move whereas \( m_n \) is the final one. The empty sequence is denoted by \( \langle \rangle \). These sequences are built under a given protocol. A protocol amounts to define a function that associates to each sequence of moves, a set of valid moves. Several protocols have been proposed in the literature, like for instance [4, 11]. In what follows, we don’t focus on particular protocols.

**Definition 5 (Persuasion dialog)** A persuasion dialog \( D \) is a non-empty and finite sequence of moves \( \langle m_1, \ldots, m_n \rangle \) s.t. the subject of \( D \) is \( \text{Subject}(D) = \text{Content}(m_1) \), and the length of \( D \), denoted \( |D| \), is the number of moves: \( n \). Each sub-sequence \( \langle m_i, \ldots, m_n \rangle \) is a sub-dialog \( D' \) of \( D \). We will write also \( D' \subseteq D \).

To each persuasion dialog, one may associate an argumentation system that will be used to evaluate the status of each argument uttered during it and to compute its output.

**Definition 6 (AS of a pers. dialog)** Let \( D = \langle m_1, \ldots, m_n \rangle \) be a persuasion dialog. The argumentation system of \( D \) is the pair \( \text{AS}_D = (\text{Args}(D), \text{Confs}(D)) \) such that:

- \( \text{Args}(D) = \{ \text{Content}(m_i) \mid i \in [1, n] \} \)
- \( \text{Confs}(D) = \{ (\alpha, \beta) \mid \alpha, \beta \in \text{Args}(D) \text{ and } (\alpha, \beta) \in \mathcal{R}_L \} \)

In other words, \( \text{Args}(D) \) and \( \text{Confs}(D) \) return respectively, the set of arguments exchanged during the dialog and the different conflicts among those arguments.

**Example 1** Let \( D_1 \) be the following persuasion dialog between two agents \( a_1 \) and \( a_2 \).
\[
D_1 = \langle (a_1, \{ a_2 \}, \alpha_1), (a_2, \{ a_1 \}, \alpha_2), (a_1, \{ a_2 \}, \alpha_3), (a_1, \{ a_2 \}, \alpha_4), (a_2, \{ a_1 \}, \alpha_1) \rangle.
\]
Let us assume that there exist conflicts in \( \mathcal{R}_L \) among some of these arguments. Those conflicts are summarized in the figure below.

\[
\begin{array}{c}
\alpha_3 \\
\alpha_2 \\
\alpha_1 \\
\alpha_4
\end{array}
\]
Here, $\text{Args}(D_1) = \{\alpha_1, \alpha_2, \alpha_3, \alpha_4\}$ and $\text{Confs}(D_1) = \{(\alpha_2, \alpha_1), (\alpha_3, \alpha_2), (\alpha_4, \alpha_2)\}$.

**Property 2** Let $D = \langle m_1, \ldots, m_n \rangle$ be a persuasion dialog. $\forall D_j \subseteq D$, it holds that $\text{Args}(D_j) \subseteq \text{Args}(D)$, and $\text{Confs}(D_j) \subseteq \text{Confs}(D)$.

**Proof** Let $D = \langle m_1, \ldots, m_n \rangle$ be a persuasion dialog, and let $D' = \langle m'_0, \ldots, m'_k \rangle$. Assume that $D' \subseteq D$, this means that each $m'_i$ is also in the sequence of $D$, thus, $\text{Content}(m'_i) \in \text{Args}(D)$. Consequently, $\text{Args}(D') \subseteq \text{Args}(D)$. Moreover, $\text{Confs}(D') \subseteq \text{Confs}(D)$.

The output of a dialog is the status of the argument under discussion (i.e., the subject):

**Definition 7** (Output of a persuasion dialog) Let $D$ be a persuasion dialog. The output of $D$, denoted by $\text{Output}(D)$, is $\text{Status}(\text{Subject}(D), \mathcal{AS}_D)$.

### 4 Criteria for Dialog quality

In this paper, we are interested in evaluating the conciseness of a dialog $D$ which is already generated under a given protocol. This dialog is assumed to be finite. Note that this assumption is not too strong since a main property of any protocol is the termination of the dialogs it generates [12]. A consequence of this assumption is that the argumentation system $\mathcal{AS}_D$ associated to $D$ is finite as well. In what follows, we propose two criteria that evaluate the importance of the moves that are exchanged in $D$, then we propose a way to compute the “ideal” dialog that reaches the same outcome as $D$.

In everyday life, it is very common that agents deviate from the subject of the dialog. The first criterion evaluates to what extent the moves uttered are in relation with the subject of the dialog. This amounts to check whether there exists a path from a move to the subject in the graph of the argumentation system associated to the dialog.

**Definition 8** (Relevant and useful move) Let $D = \langle m_1, \ldots, m_n \rangle$ be a persuasion dialog. A move $m_i$, $i \in [1, n]$, is relevant to $D$ iff there exists a path (not necessarily directed) from $\text{Content}(m_i)$ to $\text{Subject}(D)$ in the directed graph associated with $\mathcal{AS}_D$. $m_i$ is useful iff there exists a directed path from $\text{Content}(m_i)$ to $\text{Subject}(D)$ in this graph.

**Example 2** Let $D_2$ be a persuasion dialog. Let $\text{Args}(D_2) = \{\alpha_1, \alpha_3, \beta_1, \beta_2\}$. The conflicts among the four arguments are depicted in the figure below.

Suppose that $\text{Subject}(D_2) = \alpha_1$. It is clear that the arguments $\alpha_3, \beta_1$ are relevant, while $\beta_2$ is irrelevant. Here $\beta_1$ is useful, but $\alpha_3$ is not.
Property 3 If a move \( m \) is useful in a dialog \( D \), then \( m \) is relevant to \( D \).

Proof Let \( m \) be a given move in a persuasion dialog \( D \). If \( m \) is useful then there exists a directed path from \( \text{Content}(m) \) to \( \text{Subject}(D) \), thus \( m \) is relevant to \( D \). ■

On the basis of the notion of relevance, one can define a measure that computes the percentage of moves that are relevant in a dialog \( D \). In Example 2, \( \text{Relevance}(D_2) = \frac{3}{4} \). It is clear that the greater this degree is, the better the dialog. When the relevance degree of a dialog is equal to 1, this means that agents did not deviate from the subject of the dialog. The useful moves are moves that have a more direct influence on the status of the subject. However, this does not mean that their presence has an impact on the result of the dialog, i.e., on the status of the subject. The moves that have a real impact on the status of the subject are said “decisive”.

Definition 9 (Decisive move) Let \( D = \langle m_1, \ldots, m_n \rangle \) be a persuasion dialog and \( \text{AS}_D \) its argumentation system. A move \( m_i \) \( (i = 1, \ldots, n) \) is decisive in \( D \) iff

\[
\text{Status}(\text{Subject}(D), \text{AS}_D) \neq \text{Status}(\text{Subject}(D), \text{AS}_D \ominus \text{Content}(m_i))
\]

where \( \text{AS}_D \ominus \text{Content}(m_i) = \langle A', R' \rangle \) s.t. \( A' = \text{ArgS}(D) \setminus \{\text{Content}(m_i)\} \) and \( R' = \text{ConfS}(D) \setminus \{(x, \text{Content}(m_i)), (\text{Content}(m_i), x) \mid x \in \text{ArgS}(D)\} \).

It can be checked that if a move is decisive in a dialog, then it is useful. This means that there exists a directed path from the content of this move to the subject of the dialog in the graph of the argumentation system associated with the dialog.

Property 4 If a move \( m \) is decisive in a persuasion dialog \( D \) then \( m \) is useful in \( D \).

Proof Let us suppose that \( m \) is decisive in \( D \), and that \( \text{Subject}(D) \) is accepted in \( \text{AS}_D \). According to Property 1, \( \text{Subject}(D) \) is indirectly defended by a non-attacked argument. Since \( m \) is decisive, \( \text{Subject}(D) \) is rejected in \( \text{AS}_D \ominus \text{Content}(m) \). It means that at least one attacker is no more indirectly defended by a non-attacked argument. Hence the deletion of \( \text{Content}(m) \) has cut a path from a non-attacked argument to this attacker. Hence \( \text{Content}(m) \) is useful.

If \( \text{Subject}(D) \) is rejected in \( \text{AS}_D \), and accepted in \( \text{AS}_D \ominus \text{Content}(m) \). It means that every attacker is defended by a non-attacked argument in \( \text{AS}_D \ominus \text{Content}(m) \). Hence the deletion of \( \text{Content}(m) \) has eliminated every direct or indirect attacker of the subject. It means that \( \text{Content}(m) \) was on a path from an attacker of the subject to the subject hence it was useful in \( D \). ■

From the above property, it follows that each decisive move is also relevant. Note that the converse is not true as shown in the following example.

Example 3 Let \( D_3 \) be a dialog whose subject is \( \alpha_1 \) and whose graph is the following:
The set \( \{ \alpha_1, \alpha_3, \alpha_5 \} \) is the only grounded extension of \( \text{AS}_{D_3} \). It is clear that the argument \( \alpha_4 \) is relevant to \( \alpha_1 \), but it is not decisive for \( D_3 \). Indeed, the removal of \( \alpha_4 \) will not change the status of \( \alpha_1 \) which is accepted.

**Example 4** Let \( D_4 \) be a dialog whose subject is \( \alpha_1 \), and whose graph is the following:

\[ \alpha_2 \rightarrow \alpha_3 \rightarrow \alpha_1 \]

In this example, neither \( \alpha_2 \) nor \( \alpha_3 \) is decisive in \( D_4 \). However, this does not mean that the two arguments should be removed since the status of \( \alpha_1 \) depends on at least one of them (they are both useful).

On the basis of the above notion of decisiveness of moves, we can define the degree of decisiveness of the entire dialog as the percentage of moves that are decisive.

## 5 Computing the ideal dialog

As already said, it is very common that dialogs contain redundancies in the sense that some moves are uttered but these are useless for the subject, or have no impact on the output of the dialog. Only a subset of the arguments is necessary to determine the status of the subject. Our aim is to compute the subset that returns exactly the same status for the subject of the dialogue as the whole set of arguments, and that is sufficient to convince that this result holds against any attack available in the initial dialog. That subset will form the "ideal" dialog. In what follows, we will provide a procedure for finding this subset and thus the ideal dialog.

A subset of arguments that will be convenient for our purpose contains those arguments that belong to a proof tree leading to the status of the subject. This is due to the fact that a proof tree contains every necessary argument for obtaining the status of the subject. When the subject is accepted, the proof tree contains defenders of the subject against any attack. When the subject is rejected, the proof tree contains at least every non-attacked attacker. Hence, proof trees seem adequate to summarize perfectly the dialog. However, it is important to say that not any proof theory that exists in the literature will lead to the ideal dialog. This is due to the fact that some of them are not concise. In [1], a comparison of proof theories for grounded semantics shows that the one used here is the most concise.

### 5.1 Canonical dialogs

Let us define a sub-dialog of a given persuasion dialog \( D \) that reaches the same output as \( D \). In [1], a proof procedure that tests the membership of an argument to a grounded extension has been proposed. The basic notions of this procedure are revisited and adapted for the purpose of characterizing canonical dialogs.
**Definition 10 (Dialog branch)** Let $D$ be a persuasion dialog and $\text{AS}_D = \langle \text{Args}(D), \text{Conf}(D) \rangle$ its argumentation system. A dialog branch for $D$ is a sequence $\langle \alpha_0, \ldots, \alpha_p \rangle$ of arguments s. t. $\forall i, j \in \llbracket 0, p \rrbracket$

1. $\alpha_i \in \text{Args}(D)$
2. $\alpha_0 = \text{Subject}(D)$
3. If $i \neq 0$ then $(\alpha_i, \alpha_{i-1}) \in \text{Conf}(D)$
4. If $i$ and $j$ are even and $i \neq j$ then $\alpha_i \neq \alpha_j$
5. If $i$ is even and $i \neq 0$ then $(\alpha_{i-1}, \alpha_i) \notin \text{Conf}(D)$
6. $\forall \beta \in \text{Args}(D)$, $\langle \alpha_0, \ldots, \alpha_p, \beta \rangle$ is not a dialog branch for $D$.

Intuitively, a dialog branch is a kind of partial sub-graph of $\text{AS}_D$ in which the nodes contain arguments and the arcs represent inverted conflicts. Note that arguments that appear at even levels are not allowed to be repeated. Moreover, these arguments should strictly attack the preceding argument. The last point requires that a branch is maximal. Let us illustrate this notion on examples.

**Example 5** The only dialog branch that can be built from dialog $D_2$ is depicted below:

\[
\begin{array}{c}
\alpha_1 \rightarrow \beta_1
\end{array}
\]

**Example 6** Let $D_5$ be a persuasion dialog with subject $\alpha$ whose graph is the following:

\[
\begin{array}{c}
\alpha
\end{array}
\]

The only possible dialog branch associated to this dialog is the following:

\[
\begin{array}{c}
\alpha \rightarrow \alpha
\end{array}
\]

**Property 5** A dialog branch is non-empty and finite.

This result comes from the definitions of a dialog branch and of a persuasion dialog.

**Proof**

- A dialog branch is non-empty since the subject of the original persuasion dialog belongs to the branch.

- Let us assume that there exists an infinite dialog branch for a given persuasion dialog $D$. This means that there is an infinite sequence $\langle \alpha_0, \alpha_1, \ldots \rangle$ that forms a dialog branch. In this sequence, the number of arguments of even index and of odd index are infinite. According to Definition 5, the persuasion dialog $D$ is finite, thus both sets $\text{Args}(D)$ and $\text{Conf}(D)$ are finite. Consequently, the set of arguments that belong to the sequence $\langle \alpha_0, \alpha_1, \ldots \rangle$ is finite. Hence, there is at least one argument that is repeated at an even index. This is impossible.

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2 An argument $\alpha$ strictly attacks an argument $\beta$ in a argumentation system $\langle A, R \rangle$ iff $(\alpha, \beta) \in R$ and $(\beta, \alpha) \notin R$. 

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Moreover, it is easy to check the following result:

**Property 6**  For each dialog branch \(\langle \alpha_0, \ldots, \alpha_k \rangle\) of a persuasion dialog \(D\) there exists a unique directed path \(\langle \alpha_k, \alpha_{k-1}, \ldots, \alpha_0 \rangle\) of same length\(^3\) \((k)\) in the directed graph associated to \(\mathcal{A}_D\).

**Proof** Let \(D\) be a persuasion dialog. Let \(\langle \alpha_0, \ldots, \alpha_k \rangle\) be a dialog branch of the dialog tree \(D\). From Definition 10.3, it follows that \(\forall i \in \{1, k\}, (\alpha_i, \alpha_{i-1}) \in \text{Conf}_D\). Hence there is a path of length \(k\) in \(\mathcal{A}_D\) from \(\alpha_k\) to \(\alpha_0\). From Definition 10.2, \(\alpha_0 = \text{Subject}(D)\).

In what follows, we will show that when a dialog branch is of even-length, then its leaf is not attacked in the original dialog.

**Theorem 1**  Let \(D\) be a persuasion dialog and \(\langle \alpha_0, \ldots, \alpha_p \rangle\) be a given dialog branch of \(D\). If \(p\) is even, then \(\nexists \beta \in \text{Args}(D)\) such that \((\beta, \alpha_p) \in \text{Conf}_D\).

**Proof** Let \(D\) be a persuasion dialog and \(\langle \alpha_0, \ldots, \alpha_p \rangle\) be a given dialog branch of \(D\). If \(\exists \beta \in \text{Args}(D)\) s.t. \((\beta, \alpha_p) \in \text{Conf}_D\) then a new sequence beginning by \(\langle \alpha_0, \ldots, \alpha_p, \beta \rangle\) would be a dialog branch, which is forbidden by Definition 10.6.

Let us now introduce the notion of a dialog tree.

**Definition 11 (Dialog tree)**  Let \(D\) be a persuasion dialog and \(\mathcal{A}_D = (\text{Args}(D), \text{Conf}_D)\) its argumentation system. A dialog tree of \(D\), denoted by \(D^t\), is a finite tree whose branches are all the possible dialog branches that can be built from \(D\).

We denote by \(\mathcal{A}_D^t\) the argumentation system associated to \(D^t\), \(\mathcal{A}_D^t = (A^t, C^t)\) s.t. \(A^t = \{\alpha \in \text{Args}(D)\) s.t. \(\alpha\) appears in a node of \(D^t\}\) and \(C^t = \{(\alpha, \beta) \in \text{Conf}_D\) s.t. \((\beta, \alpha)\) is an arc of \(D^t\}\).

Hence, a dialog tree is a tree whose root is the subject of the persuasion dialog.

**Example 7**  Let us consider \(D_6\) whose subject is \(\alpha_1\) and whose graph is the following:

```
   α11   α10   α8   α4
   /\     /\     /\   /
α3   α7   α6   α2
   /\     /\     /\   /
α5   α3   α1   α0
```

The dialog tree associated to this dialog is depicted below:

\(^3\)The length of a path is defined by its number of arcs.
Note that the argument $\alpha_0$ does not belong to the dialog tree.

**Property 7** Each persuasion dialog has exactly one corresponding dialog tree.

**Proof** This follows directly from the definition of the dialog tree. Indeed, the root of the tree is the subject of the persuasion dialog. Moreover, all the possible branches are considered.

An important result states that the status of the subject of the original persuasion dialog $D$ is exactly the same in both argumentation systems $\text{AS}_D$ and $\text{AS}_{D'}$ (where $\text{AS}_{D'}$ is the argumentation system whose arguments are all the arguments that appear in the dialog tree $D'$ and whose attacks are obtained by inverting the arcs between those arguments in $D'$).

**Theorem 2** Let $D$ be a persuasion dialog and $\text{AS}_D$ its argumentation system. It holds that $\text{Status(Subject}(D), \text{AS}_D) = \text{Status(Subject}(D), \text{AS}_{D'})$.

**Proof** This theorem explains why dialog trees are sufficient to decide about the status of the subject. The proof of this theorem is based on two theorems given farther that are referring to the notion of canonical tree.

- If $\text{Subject}(D)$ is accepted in $\text{AS}_D$, then using Theorem 4 we get that there exists a canonical tree $D^c_i$ such that $\text{Subject}(D)$ is accepted in $\text{AS}_{D^c_i}$. Moreover, the way $D^c_i$ has been constructed (by an AND/OR process) imposes that $D^c_i$ contains every direct child of the subject in $D'$. Furthermore, Theorem 3 shows that every branch of $D^c_i$ is of even length. Every leaf of this canonical tree, by definition, is non-attacked in $D^c_i$ and by definition in $\text{AS}_{D'}$. Using Definition 10.4 we get that in each branch of $\text{AS}_{D'}$, each even node strictly attacks the previous node. Hence, by construction, for each direct attacker of the subject in $\text{AS}_{D'}$, there exists at least one defender non-attacked in $\text{AS}_{D'}$ (leaf of $D^c_i$), the defense being strict, the subject belongs to the basic extension of $\text{AS}_{D'}$.

- If $\text{Subject}(D)$ is accepted in $\text{AS}^f_D$ then there exists a non-attacked defender against every direct attacker of the subject in $\text{AS}_{D'}$. This means that there exists a canonical tree based on $\text{AS}_{D'}$ having only even length branches. The subject is accepted in this canonical tree using Theorem 3, which implies that the subject is accepted in $D$ using Theorem 4.
In order to compute the status of the subject of a dialog, we can consider the dialog tree as an And/Or tree. A node of an even level is an And node, whereas a node of odd level is an Or one. This distinction between nodes is due to the fact that an argument is accepted if it can be defended against all its attackers. A dialog tree can be decomposed into one or several trees called canonical trees. A canonical tree is a subtree of $D^t$ whose root is Subject($D$) and which contains all the arcs starting from an even node and exactly one arc starting from an odd node.

**Definition 12 (Canonical tree)** Let $D$ be a persuasion dialog, and let $D^t$ its dialog tree. $D^c$ is a canonical tree of $D^t$ if it is a subtree of $D^t$ built by levels as follows:

- Subject($D$) is its root (of level 0)
- and inductively:
  - if $\alpha$ is a node of even level in $D^c$ then for every $\beta \in D^t$ such that $(\alpha, \beta) \in D^t$, the node $\beta$ and the arc $(\alpha, \beta)$ is added to $D^c$.
  - if $\alpha$ is a node of odd level in $D^c$ and if $\alpha$ has at least one attacker in $D^t$ then for exactly one $\beta \in D^t$ such that $(\alpha, \beta) \in D^t$, the node $\beta$ and the arc $(\alpha, \beta)$ is added to $D^c$.

It is worth noticing that from a dialog tree one may extract at least one canonical tree. Let $D^c_1, \ldots, D^c_m$ denote those canonical trees. We will denote by $AS^c_1, \ldots, AS^c_m$ their corresponding argumentation systems. It can be checked that the status of Subject($D$) is not necessarily the same in these different systems.

**Example 8** From the dialog tree of $D_6$, two canonical trees can be extracted:

It can be checked that the argument $\alpha_1$ is accepted in the argumentation system of the canonical tree on the left while it is rejected in the one of the right.

The following result characterizes the status of Subject($D$) in the argumentation system $AS^c_i$ associated to a canonical tree $D^c_i$.

**Theorem 3** Let $D$ be a persuasion dialog, $D^c_i$ a canonical tree and $AS^c_i$ its corresponding argumentation system. Subject($D$) is accepted in $AS^c_i$ iff all the branches of $D^c_i$ are of even-length.

**Proof** Let $D$ be a persuasion dialog, $D^c_i$ a canonical tree and $AS^c_i$ its corresponding argumentation system.
• Assume that Subject\((D)\) is accepted in \(AS^c_i\), and that there is a branch of \(D^c_i\) whose length is odd. This means that the leaf of this branch, say \(\alpha\), indirectly attacks Subject\((D)\) (the root of the branch).

  – Either \(\alpha\) is not attacked in \(AS^c_i\) it means that \(\alpha\) is accepted hence the second node of the branch is a direct attacker of Subject\((D)\) that is not defended by a non attacked argument, i.e., Subject\((D)\) would not be accepted in \(AS^c_i\).

  – Either \(\alpha\) is attacked in \(AS^c_i\) then it can only be attacked by an argument already present in the branch (hence itself attacked), else the branch would not satisfied Definition 10.6. This also means that the second node of the branch is a direct attacker of Subject\((D)\) that is not defended by a non attacked argument.

• Assume now that all the branches of \(D^c_i\) are of even length, then for each branch the leaf is accepted since it is not attacked in \(AS^c_i\) (using Theorem 1). Then iteratively considering each even node from the leaf to the root, they can all be added to the grounded extension since the leaf defends the penultimate even node against the attack of the last odd node and so on and by construction for each odd node attacking an even node there is a deeper even node that strictly defends it (due to Definition 10.5). Hence each even node is in the grounded extension, so Subject\((D)\) is accepted in \(AS^c_i\).

The following result follows immediately from this Theorem and Theorem 1.

**Corollary 1** Let \(D\) be a persuasion dialog, \(D^c_1\) a canonical tree and \(AS^c_i\) its corresponding argumentation system. If Subject\((D)\) is accepted in \(AS^c_i\), then all the leaves of \(D^c_i\) are not attacked in \(D\).

**Proof** According to Theorem 3, since Subject\((D)\) is accepted in \(AS^c_i\), then all its branches are of even-length. According to Theorem 1, the leaf of each branch of even-length is an argument that is not attacked in \(D\). Thus, all the leaves of \(D^c_i\) are not attacked in \(D\).

An important result shows the link between the outcome of a dialog \(D\) and the outcomes of the different canonical trees.

**Theorem 4** Let \(D\) be a persuasion dialog, \(D^c_1, \ldots, D^c_m\) its different canonical trees and \(AS^c_1, \ldots, AS^c_m\) their corresponding argumentation systems. \(Output(D)^4\) is accepted iff \(\exists i \in [1, m]\) s.t. \(\text{Status}(\text{Subject}(D), AS^c_i)\) is accepted.

**Proof**

\(^4\text{Recall that} Output(D) = \text{Status}(\text{Subject}(D), AS_D).\)
Let us assume that there exists $D_j$ with $1 \leq j \leq m$ and $\text{Status}(\text{Subject}(D), AS_j)$ is accepted. According to Theorem 3, this means that all the branches of $D_j$ are of even length. From Corollary 1, it follows that the leaves of $D_j$ are all not attacked in the graph of the original dialog $D$.

Let $2i$ be the depth of $D_j$ (i.e. the maximum number of moves of all dialog branches of $D_j$).

We define the height of a node $N$ in a tree as the depth of the sub-tree of root $N$.

We show by induction on $p$ that $\forall p$ such that $0 \leq p \leq i$, the set $\{y | y \text{ is an argument of even indice and in a node of height } \leq 2p \text{ belonging to } D_j \}$ is included in the grounded extension of $AS_D$.

- Case $p = 0$. The leaves of $D_j$ are not attacked in $D$ (according to Corollary 1). Thus, they belong to the grounded extension of $AS_D$.

- Assume that the property is true to an order $p$ and show that it is also true to the order $p + 1$.

It is sufficient to consider the arguments that appear at even levels and in a node of height $2p + 2$ of $D_j$. Let $y$ be such an argument. Since $y$ appears at an even level, then all the arguments $y'$ attacking $y$ in $AS_D$ appear in $D_j$ as children of $y$ (otherwise the branch would not be maximal or $D_j$ would not be canonical), and each $y'$ is itself strictly attacked in $AS_D$ by exactly one argument $z$ appearing in $D_j$ as a child of $y'$. Thus, each $z$ is at an even level in $D_j$ and appears as a node of height $2p$ of $D_j$. By induction hypothesis, each argument $z$ is in the grounded extension of $AS_D$. Since all attackers of $y$ have been considered, thus the grounded extension of $AS_D$ defends $y$. Consequently $y$ is also in this grounded extension.

Let us assume that $\text{Status}(\text{Subject}(D), AS_D)$ is accepted. Let $i_0$ be the smallest index $\geq 0$ such that $\text{Subject}(D) \in F^{i_0}(C^2)$. Let us show by induction on $i$ that if an argument $\alpha \in \text{Args}(D)$ is in $F^i(C)$ then there exists a canonical tree of root $\alpha$ for $D^6$ having a depth $\leq 2i$ and having only branches of even length.

- Case $i = 0$: if $\alpha \in C$, then $\alpha$ itself is a canonical tree of root $\alpha$ and depth 0.

- Assume that the property is true at order $i$ and consider the order $i + 1$.

Hence, let us consider $\alpha \in F^{i+1}(C)$ and $\alpha \not\in F^k(C)$ with $k < i + 1$.

Let $x_1, \ldots, x_n$ be the attackers of $\alpha$. Consider an attacker $x_j$. $x_j$ attacks $\alpha$, and $\alpha \in F^{i+1}(C) = F(F^i(C))$. According to Proposition 4.1 in [1], it exists $y$ in the grounded extension of $AS_D$ such that $y$ attacks strictly $x_j$. Since $y$ defends $\alpha$ (definition of $F$) then $y \in F^{i}(C)$. By induction hypothesis applied to $y$, there exists a canonical tree whose root is $y$ and the depth is $\leq 2i$. The same construction is done for each $x_j$. So we get a canonical tree whose root is $\alpha$ and its depth is $\leq 2(i + 1)$ and in which each branch has still an even length.

The set $C$ contains all the arguments that are not attacked in $D$.

Here, we consider a "canonical tree of root $\alpha$ for a dialog $D$". Its definition is more general than canonical tree for a dialog $D$ since it does not requires that all the branches start from the subject of the dialog (modifying item 2 of Definition 10) but requires that all the branches start from the node $\alpha$.
Now, from the fact that $\text{Subject}(D) \in F^c\alpha(C)$ we conclude that it exists a canonical tree of root $\text{Subject}(D)$ having each branch of even length. Using Theorem 3, we get that $\text{Subject}(D)$ is accepted in this canonical tree.

This result is of great importance since it shows that a canonical tree whose branches are all of even-length is sufficient to reach the same outcome as the original dialog in case the subject is accepted. When the subject is rejected, the whole dialog tree is necessary to ensure the outcome.

**Example 9** In Example 7, the subject $\alpha_1$ of dialog $D_6$ is accepted since there is a canonical tree whose branches are of even length (it is the canonical tree on the left in Example 8). It can also be checked that $\alpha_1$ is in the grounded extension $\{\alpha_1, \alpha_4, \alpha_5, \alpha_8, \alpha_9, \alpha_{11}\}$ of $\text{AS}_D$.

So far, we have shown how to extract from a graph associated with a dialog its canonical trees. These canonical trees contain only useful (hence relevant) moves:

**Theorem 5** Let $D^c_i$ be a canonical tree of a persuasion dialog $D$. Any move built on an argument of $D^c_i$ is useful in the dialog $D$.

**Proof** By construction of $D^c_i$, there is a path in this tree from the root to each argument $\alpha$ of the canonical tree. According to Property 6, we get that there exists a corresponding directed path in $\text{AS}_D$ from $\alpha$ to $\text{Subject}(D)$, hence a move containing the argument $\alpha$ is useful in $D$.

The previous theorem gives an upper bound of the set of moves that can be used to build a canonical tree, a lower bound is the set of decisive moves.

**Theorem 6** Every argument of a decisive move belongs to the dialog tree and to each canonical tree.

**Proof** If a move $m$ is decisive then, as seen in the proof of property 4,

- if the subject is accepted in $\text{AS}_D$ then it exists at least a direct attacker of the subject that is no more indirectly defended by a non attacked argument in $\text{AS}_D \oplus \text{Content}(m)$. The subject being accepted in $\text{AD}_D$, this means that there is a canonical tree having only branches of even length (according to Theorem 3). By construction, this canonic tree contains every direct attacker of the subject. If $\text{Content}(m)$ does not belong to this canonic tree then there is a defender of the subject on a path that does not contains $\text{Content}(m)$ in $\text{AS}_D$, if it is the case for every direct attacker of the subject then the subject should have been accepted in $\text{AS}_D \oplus \text{Content}(m)$. This is not possible, hence $\text{Content}(m)$ belongs to the canonical tree that accepts the subject.

- if the subject is rejected in $\text{AS}_D$ but accepted in $\text{AS}_D \oplus \text{Content}(m)$ then there exists a canonical tree where all the branches are of even length in $\text{AS}_D \oplus \text{Content}(m)$. Since the adding of $\text{Content}(m)$ leads to reject the subject,
it means that $\text{Content}(m)$ attacks at least one direct or indirect defender of the subject belonging to each canonical tree that accepts the subject in $\text{AS}_D \oplus \text{Content}(m)$. The sequence containing the branch from the subject to that defender can be prolonged with $\text{Content}(m)$ in order to form a new branch of odd length in $D'$. Hence for every canonical tree that rejects the subject, $\text{Content}(m)$ has to belong one of their branch.

The converse is false since many arguments are not decisive, as shown in Example 4. Indeed, there are two attackers that are not decisive but the dialog tree contains both of them (as does the only canonical dialog for this example).

5.2 The ideal dialog

In the previous section, we have shown that from each dialog, a dialog tree can be built. This dialog tree contains direct and indirect attackers and defenders of the subject. From this dialog tree, interesting subtrees can be extracted and are called canonical trees. A canonical tree is a subtree containing only particular entire branches of the dialog tree (only one argument in favor of the subject is chosen for attacking an attacker while each argument against a defender is selected). In case the subject of the dialog is accepted it has been proved that there exists at least one canonical tree such that the subject is accepted in its argumentation system. This canonical tree is a candidate for being an ideal tree since it is sufficient to justify the acceptance of the subject against any attack available in the initial dialog. Among all these candidate we define the ideal tree as the smallest one. In the case the subject is rejected in the initial dialog, then the dialog tree contains all the reasons to reject it, hence we propose to consider the dialog tree itself as the only ideal tree.

**Definition 13 (ideal trees and dialogs)** If a dialog $D$ has an accepted output - then an ideal tree associated to $D$ is a canonical tree of $D$ in which $\text{Subject}(D)$ is accepted and having a minimal number of nodes among all the canonical graphs that also accept $\text{Subject}(D)$ - else the ideal tree is the dialog tree of $D$.

A dialog using once each argument of an ideal graph is called an ideal dialog.

**Example 10** An ideal Dialog for Dialog $D6$ (on the left) has the following graph (on the right):

Given the above definition, an ideal dialog contains exactly the same number of moves that the number of nodes of the ideal graph.
Property 8 Given a dialog $D$ whose subject is accepted. An ideal dialog $ID$ for $D$ is the shortest dialog with the same output, and s.t. every argument in favor of the subject in $ID$ (including $\text{Subject}(D)$ itself) is defended against any attack (existing in $D$).

This property ensures that, when the subject is accepted in the initial dialog $D$, an ideal dialog $ID$ is the more concise dialog that entails an acceptance. In other words, we require that the ideal dialog should contain a set of arguments that summarize $D$.

Proof If the subject is accepted in $D$ then, by construction, a canonical graph of $D$ contains every argument existing in $D$ that directly attacks the subject since they belong to all the possible dialog branches that can be built from $D$. But for any of them it contains only one attacker that is in favor of the subject (this attacker is a son of an “OR” node in the dialog tree), for each chosen argument in favor of the subject, all the attackers are present in the canonical tree (they are the sons of an “AND” node in the dialog tree). Moreover, if the subject is accepted then every branch of the canonical graph is of even length. It means that the leaves are in favor of the subject and not attacked in the initial dialog $D$. This property is true for any canonical graph. Then since the ideal dialog correspond to the smallest canonical graph it means that it is the shortest dialog that satisfy this property.

Note that the ideal dialog exists but is not always unique. Here is an example of an argumentation system of a dialog which leads to two ideal trees (hence it will lead to at least two ideal dialogs).

So far, we have formally defined the notion of ideal dialog, and have shown how it is extracted from a persuasion dialog. It is clear that the closer (in terms of set-inclusion of the exchanged arguments) to its ideal version the dialog is, the better the dialog.

6 Conclusion

In this paper, we have proposed three criteria for evaluating the moves of a persuasion dialog with respect to its subject: relevance, usefulness and decisiveness. Relevance only expresses that the argument of the move has a link with the subject (this link is based on the attack relation of the argumentation system). Usefulness is a more stronger relevance since it requires a directed link from the argument of the move to the subject. Decisive moves have a heavier impact on the dialog, since their omission changes the output of the dialog.

Inspired from works on proof theories for grounded semantics in argumentation, we have defined a notion of “ideal dialog”. More precisely, we have first defined a dialog tree associated to a given dialog as the graph that contains every possible direct and indirect attackers and defenders of the subject. From this dialog tree, it is then possible to extract sub-trees called “ideal trees” that are sufficient to prove that the subject is
accepted or rejected in the original dialog and this, against any possible argument taken from the initial dialog. A dialog is good if it is close to that ideal tree. Ideal dialogs have nice properties with respect to conciseness, namely they contain only useful and relevant arguments for the subject of the dialog. Moreover for every decisive move its argument belongs to all ideal trees.

From the results of this paper, it seems natural that a protocol generates dialogs of good quality if (1) irrelevant and not useful moves are penalized until there is a set of arguments that relate them to the subject (2) adding arguments in favor of the subject that are attacked by already present arguments has no interest (since they do not belong to any ideal tree). By doing so, the generated dialogs are more concise (i.e. all the uttered arguments have an impact on the result of the dialog), and more efficient (i.e. they are the minimal dialogs that can be built from the information exchanged and that reach the goal of the persuasion).

Note that in our proposal, the order of the arguments has not to be constrained since the generated graph does not take it into account. The only thing that matters in order to obtain a conclusion is the final set of interactions between the exchanged arguments. But the criteria of being relevant to the previous move or at least to a move not too far in the dialog sequence could be taken into account for analyzing dialog quality. Moreover, all the measures already defined in the literature and cited in the introduction could also be used to refine the proposed preference relation on dialogs and finally could help to formalize general properties of protocols in order to generate good dialogs.

Furthermore, it may be the case that from the set of formulas involved in a set of arguments, new arguments may be built. This give birth to a new set of arguments and to a new set of attack relations called complete argumentation system associated to a dialog. Hence, it could be interesting to define dialog trees on the basis of the complete argumentation system then more efficient dialogs could be obtained (but this is not guaranteed). However, some arguments of the complete argumentation system may require the cooperation of the agents. It would mean that in an ideal but practicable dialog, the order of the utterance of the arguments would be constrained by the fact that each agent should be able to build each argument at each step.

References


