Multiple-source data fusion problems in spatial information systems

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Abstract

The paper proposes an overview of different information fusion problems in the context of multiple source spatial data. Problems raise from the uncertainty and possible inconsistency of information as in any information fusion situations, but also from the fact that sources may partition the space in different ways, or may use different partitions of attribute domains or even different conceptual taxonomies or ontologies, for assessing symbolic pieces of data. The paper discusses representation issues and outlines possible approaches to these problems.

Keywords: Uncertainty, fusion, spatial information.

1 Introduction

The management of multiple source, symbolic or numerical, information raises different forms of fusion problems due to the uncertainty and the heterogeneity of information [5]. The spatial distribution of information in geographical information systems [1, 12, 14] adds new features to these problems such as the use of different space partitions by the sources or the possible dependencies between the pieces of information pertaining to parcels that are close.

Also spatial information may involve numerical as well as symbolic attributes whose evaluation may use different vocabularies according to the sources. Spatial information fusion issues may more generally take place in a variety of information handling problems such as the management of multiple source spatial databases systems, the prediction or the detection of changes over time of land-cover parameters (e.g., [17]), or in best location decision problems using multiple information sources and multiple criteria.

The paper is organized in the following way. Section 2 discusses issues related to the representation of spatial information, in terms of single or multiple-valued attributes which may be imprecise or fuzzy, including the distinction between positive and negative information. Section 3 introduces the notions of refinement and coarsening of a partition and examines the related issues of attribute values propagation. Discussing these representation issues, which involve label ontologies and space partitions, is clearly a necessary prerequisite for defining meaningful fusion operations. Section 4 and 5 survey different fusion problems raised by the handling of spatial information, for merging multiple-source information pertaining to the same parcels or to different partitionings of the space.

2 Representation of spatial information

In the field of spatial information, as in other fields, information may be imprecise, pervaded with uncertainty or be inconsistent. The specific aspect of spatial information is that information is associated to *parcels* of land (the parcels are defined by a partition of the land).

Since we are dealing with spatial information, attribute values pertain to parcels, but there may be some ambiguity on the way they apply to the parcels (which are themselves sets of "points"). Moreover the values may be not just elements of an attribute domain, but may be symbolic labels taken from an ontology.

A parcel p is described in terms of several attributes (or evaluation criteria) that have values. Their values may be *numerical* or *symbolic*. Numerical attribute values may be imprecise or fuzzy. A symbolic value can be based on *labels* belonging to a given *vocabulary*. The vocabulary is supposed to be organized into an *ontology* that specifies specialization/generalization relations between concepts. For instance, Figure 1 provides an example of (a part of) an ontology about vegetation (where arrows refer to generalization relations).

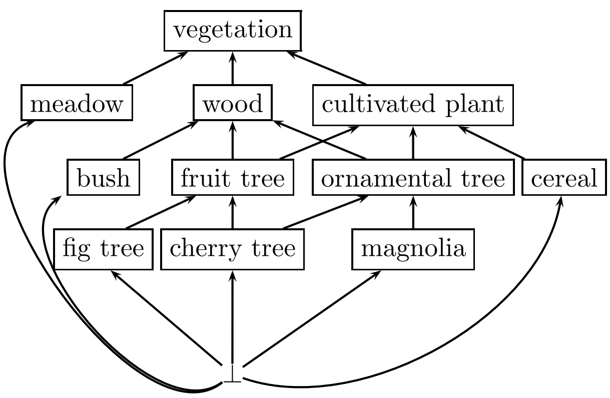

Figure 1: Example of ontology

Observe that an ontology may not have, in general, a lattice structure, which requires that each pair of nodes has a unique least upper bound and a unique greatest lower bound. The ontology of Figure 1 is not a lattice since, for instance, there is no greatest lower bound of {"wood", "cultivated plant"}, indeed there are two elements, namely "fruit tree" and "ornamental tree" which are maximal among the labels that are lower than "wood" and "cultivated plant", but none of them is greater than the other.

A symbolic attribute value may involve several labels. Such a situation may correspond to very different intended meanings. Let b and c be two distinct labels (in the following, for the sake of simplicity, only two labels are considered, but the definitions are similar when considering combinations of any number of labels). Let $a(p)$ denote the available information about the value of attribute a on parcel p. $a(p)$ can be specified by means of combination of terms in the ontology associated with attribute a.

Then we may have to represent imprecise information, or multiple valued information, or information with a positive or negative flavor.

2.1 Single-valued information

Single-valued information refers to the situation when there should be a unique value for a given attribute for a given object (here a parcel). This value may be more or less precisely known. Imprecision refers to the case where several values are still eligible. This value may be numerical or symbolic. In particular in case of symbolic information, the value can be expressed in terms of disjunction or conjunction of symbols. When symbols are labels that belong to an ontology they are themselves more or less precise inasmuch there may exist more specialized labels in the ontology.

A *disjunctive* piece of information, denoted by $a(p) = b \vee c$, expresses imprecision, i.e., either b or c may be appropriate for assessing $a(p)$. Here, $\vee$ is the classical Boolean connective.

For instance, $coverage(p)$ = "rocks" $\vee$ "sand" is to be understood as parcel p is uniformly covered by rocks or uniformly covered by sand, but we do not know what is the real coverage among these two possibilities.

Suppose $b_1, \ldots, b_m$ and $c_1, \ldots, c_n$ be specializations of b and c respectively in the considered ontology. Then, *when the ontology is complete* (i.e., there is no other possible specialization for b than the known $b_1, \ldots, b_m$ and the same holds for c), $a(p) = b \vee c$ also means $a(p) = b_1 \vee \ldots \vee b_m \vee c_1 \vee \ldots \vee c_n$. However, the more concise expression $b \vee c$ is generally preferred. More generally $a(p)$ can be assessed under the form of a possibility distribution over a set of labels, reflecting a weighted disjunction.

In case c generalizes b (or equivalently b specializes c), we should have formally $b \vee c = c$. However in practice, one may state $a(p)$ is "cereal or vegetation" with the intended meaning that $a(p)$ is more plausibly (uniformly) covered by cereals, but it is somewhat possible that the parcel should be covered (uniformly) by another type of vegetation. This might be viewed as a symbolic possibility distribution π semantically defined by

$$\pi(\omega) = \begin{cases} 1 & \text{if } \omega \models b, \\ \alpha < 1 & \text{if } \omega \models c \wedge \neg b, \\ 0 & \text{otherwise.} \end{cases}$$

Attribute values may be also expressed by the linguistic conjunction of labels. This can refer to at least two different uses, one single-valued which is discussed in this subsection and another that refers to a multiple-valued attribute that is considered in the following subsection.

A conjunctive expression, denoted by $a(p) = b \wedge c$, means that the parcel is covered uniformly by something that can be described both by b and c. Here, the symbol $\wedge$ is the classical Boolean conjunction. This is a way to make less imprecise a description (e.g., in Figure 1 "wood" $\wedge$ "cultivated plant" is equivalent to "fruit tree" $\vee$ "ornamental tree").

2.2 Multiple-valued information

A *multiple-valued* piece of information, denoted by $a(p) = b \ \& \ c$, expresses that attribute a may have simultaneously several values (in contrast with the previous subsection) and that the attribute value is the subset $\{b, c\}$, i.e., both b and c applies to parcel p in the sense of attribute a. Here & is a non-classical, idempotent, connective.

Indeed, the subset $\{b, c\}$ has a conjunctive meaning that contrasts with the disjunctive meaning of a possibility distribution used for modeling $b \vee c$ as discussed above [18]. For instance, $coverage(p) =$ "rocks" & "sand" then means that p is both covered by rocks and by sand. We may then want to further distinguish between the case where p is uniformly covered by a mixture of rocks and sand and the case where a subpart of p is only covered by rocks while the other subpart is only covered by sand. In this latter case, the information may be augmented with the respective proportions x and y of the surface of p corresponding to b and c. Namely, $x\%$ of the parcel is b and $y\%$ of the parcel is c where x and y can be known more or less precisely and $x + y \leq 1$ ($x + y < 1$ meaning that the value is not known on the whole part of the parcel).

In case b specializes c, in a multiple-valued pieces of information, one may suspect that since the captor has been able to detect b, the coverage of the rest of the parcel correspond to interpretations of c that exclude b, since otherwise the captor should have been able to detect b on the rest of the parcel as well (provided that the captors have the same performance everywhere).

Lastly, note that pieces of information referring to multiple-valued attributes may be incomplete or imprecise also. For instance, a parcel known to exhibit "rocks" and "sand" may reveal later to also include, say "water". More generally, imprecise multiple-valued information can be represented by means of possibility distributions on the power set of the attribute domain. For instance, the coverage of the parcel is made of "wood and meadow" or "wood and cultivated plant", is represented by associating a possibility equal to 1 to each of the subsets {"wood", "meadow"} and {"wood", "cultivated plant"}. Moreover, a possibility distribution on the power set of an attribute domain can be approximated from above and from below by two possibility distributions on this domain [11].

2.3 Positive vs. negative information

Generally speaking, there are two ways of interpreting information (should it be numerical or symbolic).

- The information can be interpreted "negatively". Namely, any interpretation that is not compatible with the stated pieces of information is judged to be impossible. This kind of information can be termed as "exhaustively closed". This may apply to multiple-valued or single-valued information, e.g., there are "rocks" and "sand" (*and nothing else*), or, e.g., the parcel is uniformly covered by "meadow"' or by "wood" (*but no other coverage is possible*).

- However, there exists also a positive type of understanding that focuses on what is possible for sure (because it has been observed), and does not refer to what is known to be impossible. This kind of information can be viewed as an "open" set of values, as in the "water" example above. The positive understanding is also possible with single-valued disjunctive information (see, e.g., [8]), for instance, $coverage(p) =$"meadow" $\vee$ "wood" may mean that the parcel is uniformly covered by meadow, or by wood, or by something else.

3 Coarsening and refining parcels

As already said, the considered space S is supposed to be partitioned into a set of parcels p_i such that $\forall i, p_i \neq \varnothing$, $\forall i,j, p_i \cap p_j = \varnothing$ and $\cup_i p_i = S$. For the sake of simplicity, we assume that there is no fuzziness about the frontiers between parcels. However, in case of multiple source information, each source may refer to a different partition of S for providing the attribute value information.

Given two partitions $P = \{p_1, \ldots, p_r\}$ and $P' = \{p'_1, \ldots, p'_s\}$, one can define a refinement/coarsening relation, that induces a partial order $\sqsubseteq$ between the partitions. Namely,

Definition 1 (Coarsening and refinement)

$P \sqsubseteq P' \Leftrightarrow \quad \forall p_i \in P, \exists p'_j \in P', \quad p_i \subseteq p'_j$

$P \sqsubset P' \Leftrightarrow \quad P \sqsubseteq P'$ *and not* $P' \sqsubseteq P$

If $P \sqsubset P'$ then P is said to refine P', and P' is said to coarsens P.

Let $ref(P,P')$ be the largest partition in the sense of $\sqsubseteq$ that refines both P and P', and $coar(P,P')$ be the smallest partition that coarsens both P and P'.

Given two distinct partitions P and P', it is always possible to build the two partitions $ref(P,P')$ and $coar(P,P')$. See Figure 2 for an example.

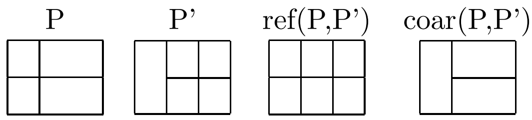

Figure 2: Refinement and coarsening

This raises the problem of propagating attribute values attached to parcels when refining or coarsening a partition.

3.1 Numerical attribute values

We first discuss the case of numerical information. There are different situations according as the attribute is spatially pointwise, or not:

- If the attribute a is "spatially pointwise" then a can be seen as a pointwise function of x where x ranges on the parcel p. For example, altitude is such an attribute: each point of the parcel has an altitude value. Then a parcel p is naturally associated with a pair of minimal and maximal values $\underline{u}(p) = \min_{x \in p} a(x)$ and $\overline{u}(p) = \max_{x \in p} a(x)$.

  Hence propagation of such pairs through refinement/coarsening is easy thanks to:

  $$\underline{u}(p_1 \cap p_2) = \max(\underline{u}(p_1), \underline{u}(p_2))$$
  $$\overline{u}(p_1 \cap p_2) = \min(\overline{u}(p_1), \overline{u}(p_2)).$$

  Note that sometimes intersection leads to an empty interval, which expresses inconsistency. When consistency holds $\underline{u}(p_1 \cap p_2) \leq \overline{u}(p_1 \cap p_2)$.

  $$\underline{u}(p_1 \cup p_2) = \min(\underline{u}(p_1), \underline{u}(p_2))$$
  $$\overline{u}(p_1 \cup p_2) = \max(\overline{u}(p_1), \overline{u}(p_2))$$

  This is the thesis of C-calculus [6, 7].

- If the attribute is not "spatially pointwise", the problem is different. We then need to know how $a(p_1 \cap p_2)$ or $a(p_1 \cup p_2)$ relates to $a(p_1)$ and $a(p_2)$. For instance, a may refer to cardinality (e.g, population) then obviously $a(p_1 \cup p_2) = a(p_1) + a(p_2)$ if $p_1 \cap p_2 = \varnothing$. Average values are more tricky to handle, since an average value over $p_1 \cup p_2$ cannot be computed simply from the averages over p_1 and over p_2 (even if $p_1 \cap p_2 = \varnothing$), but requires the knowledge of other quantities attached to p_1 and p_2, namely their cardinalities (or their ratio).

3.2 Symbolic attribute values

In case of symbolic attribute values, the problem again depends on the precise meaning of the label(s) attached to a parcel, as discussed above (see section 2).

First, assume $a(p_1) = l_1$ and $a(p_2) = l_2$, where l_1 and l_2 are labels in the ontology used for attribute a. We suppose first for simplicity that this ontology has a lattice structure (in contrast with the example of Figure 1) equipped with a partial order $\prec$, where $l \prec l'$ means that l has a more specialized meaning than l', or conversely that l' is more general than l and thus covers more cases.

For a pair (l_1, l_2), there exists two labels $glb(l_1, l_2)$ and $lub(l_1, l_2)$, that are respectively the greatest lower bound and the lowest upper bound of l_1 and l_2 in the lattice with respect to the partial order $\prec$ expressing specialization/generalization. Namely, $glb(l_1, l_2)$ is the most general specialization of both l_1 and l_2, and $lub(l_1, l_2)$ is the most specialized generalization of both l_1 and l_2 in the ontology. Then it is natural to take:

$$a(p_1 \cup p_2) = lub(l_1, l_2) \quad (1)$$
$$a(p_1 \cap p_2) = glb(l_1, l_2). \quad (2)$$

Such situation can take place in a Galois connection construct [2], which describes subsets of objects and their discriminating properties (see, e.g. [15] in a spatial information context). Indeed in a Galois lattice, one label l may have several descendant labels that are associated to different subsets of cases covered by l. But several labels $l_1, \ldots, l_n$ cannot have more than one direct common descendant l' (each label l_i being associated to a set of cases then a common descendant l' is associated to the intersection of the cases of each l_i, since this intersection of cases is unique then the Galois connection can only associate one property to it).

Observe that one may have $glb(l_1, l_2) = \bot$ (bottom of the lattice) in case of inconsistency (i.e., there is no specialization compatible with both l_1 and l_2).

However expressions (1) and (2) are not applicable if a lattice structure is not available. In order to cope with this general case, let us introduce the following notation.

Notation 1 *Let $s(l)$ be the set of labels that heritates from the label l in the ontology (its set of descendants):*
$s(l) = \{l\} \cup \{l' \text{ s.t. } l' \prec l\}$ *where $\prec$ is the partial strict order of the ontology.*

For instance, in Figure 1, "fig tree" belongs to $s(\text{"wood"}) \cap s(\text{"cultivated plant"})$. In this example, there are two nodes, namely "cultivated plant" and "wood" that have two descendants, if the considered vocabulary have been more complete we could have a node representing the two sons, for instance, "cultivated tree", which would have had two sons: "fruit tree" and "ornamental tree" (then the ontology of Figure 1 would have been a lattice). In order to be able to deal with any kind of ontology, we propose (3) and (4) in place of (1) and (2):

$$a(p_1 \cup p_2) = s(l_1) \ \& \ s(l_2) \quad (3)$$
$$a(p_1 \cap p_2) = s(l_1) \cap s(l_2). \quad (4)$$

The first formula expresses the fact that when doing the union of two parcels p_1 and p_2, the obtained region has two parts, one part can be described by l_1 and the other part by l_2. Hence it corresponds to the operator & described above.

The second formula expresses that the area corresponding to the intersection of the two parcels must verify both properties l_1 and l_2. Note that $a(p_1) \cap a(p_2)$ is more concisely described, in a semantically equivalent way, by suppressing every label of $s(l_1) \cap s(l_2)$ that is subsumed.

More generally, any set of labels could be represented more concisely by a minimal set of labels covering them and only them. For instance, considering again the ontology of Figure 1: $s(l_1) \cap s(l_2) = \{$"ornamental tree", "cherry tree"$\}$ would be rewritten as $\{$"cherry tree"$\}$.

4 Fusion problems within the same parcel

Fusion problems refer to the merging of pieces of information coming from different sources. The sources may refer or not to the same ontologies.

4.1 Sources using the same ontology

At the core of any fusion problem, there is a question of handling inconsistency. Namely, assume source 1 says $a(p) = x^1$ and source 2 says $a(p) = x^2$. Then they are basically two situations:

- either x^1 and x^2 are compatible and it is possible to refine the information into $x^1 \cap x^2$, where $x^1 \cap x^2$ refers to:
  - interval intersection in case of numerical information ($x^i = [s^i, t^i]$): $x^1 \cap x^2 = [max(s^1, s^2), min(t^1, t^2)] \neq \varnothing$.
  - in case of symbolic values taken from an ontology, $x^1 = l_1$ and $x^2 = l_2$, and again $x^1 \cap x^2 = s(l_1) \cap s(l_2)$.
- or x^1 and x^2 are not compatible then (at least) one of the sources is wrong, and we do not know which one. Then if we want to keep the information anyway, one can perform a union for combining the information, namely, $x^1 \cup x^2 = l_1 \vee l_2$ (see, e.g., [10]). If $lub(l_1, l_2) = \top$, top element of the lattice, Trung Pham [15] speaks of total conflict since there is no common generic label covering both l_1 and l_2, like, e.g., "water" and "earth".

More generally, it has been shown how to perform any combination operation between uncertain pieces of information at the syntactic level (i.e., at a label level) in agreement with the semantic level that handles the different interpretations of the labels, in the framework of possibilistic logic [3].

4.2 Sources having distinct vocabularies

The problem of combining information provided by two sources using different vocabularies (but using the same spatial partition) is slightly different than the previous one. The ideas outlined below could be extended to fuzzy partitions [9] induced by linguistic terms. For simplicity, let us only consider the case where the two vocabularies V_1 and V_2 used by sources 1 and 2 respectively define two partitions of the domain of attribute a. Namely, $\forall l \in V_i, \forall l' \in V_i, l \wedge l' = \perp$. Thus, as in rough sets theory [13, 9], if sources 1 and 2 states respectively that $a(p) = l_1 \in V_1$ and $a(p) = l_2 \in V_2$, we shall only be able to provide a lower and an upper approximation, $a_\star(p)$ and $a^\star(p)$, using $ref(V_1, V_2)$ and $coar(V_1, V_2)$ respectively, if there exists labels for the cells of these new conceptual partitions. Indeed, given an evaluation $a(p)$ using a vocabulary V_1, there is not an exact translation of it in V_2. In general, the translation of $a(p) \in V_1$ can only be approximated under the form $a_\star(p) \in V_2$, $a^\star(p) \in V_2$, and $[a_\star(p)] \subseteq [a(p)] \subseteq [a^\star(p)]$ in the rough sets sense, where the [ ]'s denote sets of interpretations.

5 Fusion with distinct space partitions

Let us now consider the problems raised by the use of different space partitions, in case of symbolic information. Let us suppose that we have two sources using two different space partitions P and P', which have non trivial refinement and coarsening $ref(P, P')$ and $coar(P, P')$. To each parcel p of P and p' of P', for an attribute a of interest, is attached a symbolic label, namely $a(p) = l$ and

$a(p') = l'$. Here, it is supposed that the l's and the l''s belong to the same ontology. The problem is then to evaluate $a(r)$ and $a(c)$ for $r \in ref(P, P')$ and $c \in coar(P, P')$ from the available information. Let us outline the approach

- for the refined partition. Let us assume that $r = p \cap p'$. Then $a(r) = s(a(p)) \cap s(a(p'))$, if information is consistent. This covers the general case where $a(p)$ and $a(p')$ are possibility distributions [18] over labels of the ontology. For instance, $a(p) = (\lambda_1, l_1) \vee \cdots \vee (\lambda_n, l_n)$ where λ_i express to what extent it is possible that the interpretations of l_i are the precise value of $a(p)$. Note that if $l_i \prec l_j$ then $(\lambda_i, l_i) \vee (\lambda_j, l_j)$ (with $\lambda_i > \lambda_j$ as suggested in section 2) is equivalent to $(\lambda_i, l_i) \vee_{l:l \prec l_j; l \neq l_i} (\lambda_j, l)$. So any possibility distribution over labels for a given attribute can always be written in an expanded form where the labels involved are not a specialization of each other. When handling possibility distributions, $a(r)$ is then computed by a minimum-based intersection. In case of inconsistency between $s(a(p))$ and $s(a(p'))$, maximum-based union should be performed. Trung Pham [15] systematically perform an intersection-based combination and a union-based combination, in the case of non-weighted labels (equivalent to $\{0, 1\}$-possibility distributions), which are respectively called "optimistic" and "pessimistic" fusion.
- for the coarsened partition. Let $c = p \cup p'$ (this could be straightforwardly generalized to the case where p or p' are themselves unions of several parcels in their respective partitions). Then taking $a(p) \vee a(p')$ (or even $lub(a(p), a(p'))$ if we look for a unique label) may be felt inappropriate, because what we need to express is not that the new parcel $p \cup p'$ can be uniformly labeled by the imprecise description "$a(p)$ or $a(p')$", but rather to model that $p \cup p'$ is partly described by $a(p)$ and described by $a(p')$ for the other part. In other words, the attribute becomes multiple-valued, then, as said before, $a(p)\&a(p')$ seems the most appropriate representation.

6 Concluding remarks

This paper intends to provide an introductory discussion of the basic issues related to representation and fusion of multiple source spatial information. Clearly, there are other related problems worth discussing. Let us mention some of them.

Thus, one may be interested not only in retrieving the best information that can be attached to a parcel about some attribute using multiple sources, but also in evaluating some new attribute a that depends on the values of other attributes $a_1, \ldots, a_n$ that can be obtained (maybe with imprecision or uncertainty) from information sources, i.e., formally there is some function f or relation R (maybe described by "if ... then..." rules) such as $a(p) = f(a_1(p), \ldots, a_n(p))$ or $a(p) \in R(a_1(p), \ldots, a_n(p))$. A similar problem is encountered in updating, where the situation at time $t + 1$ generally depends on situation at time t. When the information about the $a_i(p)$'s is represented by possibility distributions, we need to apply the extension principle to f or R, or its syntactic counterpart when dealing with symbolic labels, see [4], for evaluating $a(p)$. This has to be combined with the ideas discussed in this paper about dealing with different sources using different vocabularies or different spatial partitions see, e.g., [16].

Another important issue is the conjoint use of pieces of information pertaining to different parcels together with dependency relations (induced by neighboring properties for instance) between these parcels.

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