Handling enthymemes in time-limited persuasion dialogs*

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Abstract

This paper is a first attempt to define a framework to handle enthymeme in a time-limited persuasion dialog. The notion of incomplete argument is explicited and a protocol is proposed to regulate the utterances of a persuasion dialog with respect to the three criteria of consistency, non-redundancy and listening. This protocol allows the use of enthymemes concerning the support or conclusion of the argument, enables the agent to retract or re-specify an argument. The system is illustrated on a small example and some of its properties are outlined.

1 Introduction

Many persuasion debates have marked human history: Herodotus debate on the three government types, Valiadolid debate, the Bohr-Einstein debate about quantum mechanics, presidential TV debates... The "winner" is often considered as very clever and skilled. Indeed oratory featured in the original Olympics and there exist teaching lessons for being a good orator (e.g. [19]). A good orator is someone who is able to make his point of view adopted by the public whatever the truth is and whatever his adversary may say. This skillness and cleverness is a big challenge for human being in everyday life as well as in History since debates are both very common and very influential. This is why it is important that artificial intelligence focuses on this field of research. This implies to develop at least two features: representing and handling persuasion dialogs, designing good artificial orators (able to find strategies to win a debate).

The first feature has already been widely developed in the literature (see e.g. [3, 5, 8, 10, 11, 17]) but as far as we know the dialog persuasion systems that have been developed either do not define what is an argument or always assume that an argument is a "perfect" minimal proof of a formula. Our purpose is to develop a system in which it is possible to express an argument, called "approximate argument" by [13], that takes into account implicit information. Indeed, it is generally admitted that an argument is composed of two parts: a support and a claim, such that the support is a logical minimal proof of the claim, this kind of argument is called "logical argument" by [13]. In everyday life, there is nearly no "logical argument", since it is not useful and maybe tiring to completely justify a given claim, we often give an argument without mentioning implicit common knowledge. Otherwise argumentation would be very long to express and boring to listen (and could be recursively infinite when each part of the support of a claim should in turn be completely explained). Shortly speaking a logical argument is not into line with Gricean maxims. Approximate arguments, called enthymeme by Aristotle, is a syllogism keeping at least one of the premises or conclusion unsaid. Enthymemes have already been studied in the literature [25, 23, 7, 14], but no formal persuasion dialog system able to handle enthymeme has yet been defined.

Handling enthymeme in persuasion dialog has two advantages: first it allows to deal with more concrete cases where agents want to shorten their arguments, second it may involve strategic matter, namely

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droping a premise may remove a possible attack or may enable to cheat by pretending that implicit knowledge can help to prove a claim while it is not the case... The problem of implicit knowledge was one of the motivation for non-monotonic reasoning which aims at reasonning despite a lack of information. In enthymeme handling, this is not the only aim, it may also be interesting to focus on what is missing. The following example is given by Schopenhauer [19] to exemplify the "extension stratagem" which is the first among the 38 stratagems he designed for taking victory in a dispute (without worrying about the objective truth). Here, the argument of Schopenhauer's opponent is an enthymeme.

Example 1 ([19]) *"I asserted that the English were supreme in drama. My opponent attempted to give an instance to the contrary, and replied that it was a well-known fact that in music, and consequently in opera, they could do nothing at all. I repelled the attack by reminding him that music was not included in dramatic art, which covered tragedy and comedy alone. This he knew very well. What he had done was to try to generalize my proposition, so that it would apply to all theatrical representations, and, consequently, to opera and then to music, in order to make certain of defeating me."*

In the following, we first reintroduce enthymemes in a logical framework, then we present the utterances that can be done in a persuasion dialog with enthymemes. For instance we propose a new speech act, Complete, that allows to precise an already expressed approximate argument. We then develop the protocol that governs a persuasion dialog allowing enthymemes (first defined in [9]). While in concrete dialogs it is often the case that people do not listen to each other or are inconsistent, here the protocol enforces consistency, non redundancy and listening. A novelty of our proposal is the representation of time-limited persuasion dialogs in which each speaker is given a fixed speech-time, this ensures that every dialog has an end and enforces agents to take time into account when uttering their arguments. Although the example we provide is very short and simple, its size allows us to use it along the paper even if it does not show all the strategical aspects of enthymemes (left for further research).

2 Arguments and enthymemes

We consider a logical language $\mathscr{L}$, where Greek letters (e.g. φ, ψ) denote formulas, $\vdash$ the logical inference, $\bot$ the contradiction. Let us recall the definition of [13]:

Definition 1 (logical and approximate arguments [6, 13])
A logical argument *is a pair* $\langle S, \varphi \rangle$ *such that:* $\begin{cases} (1) & S \subseteq \mathscr{L}, \varphi \in \mathscr{L} \\ (2) & S \nvdash \bot, \\ (3) & S \vdash \varphi, \\ (4) & \nexists S' \subset S \text{ s.t. } S' \vdash \varphi \end{cases}$
An approximate argument *is a pair* $\langle S, \varphi \rangle$ *where the* support $S \subseteq \mathscr{L}$ *is a set of propositional formulas, and the* claim *is* $\varphi \in \mathscr{L}$.
Arg_Σ *(resp.* AArg_Σ*) denotes the set of logical (resp. approximate) arguments that can be built from a set of formulas* Σ*, lower-case Latin letters (e.g.* a*,* b*) denote arguments.*

Notation: if A *is a set of arguments, the set of formulas pertaining to the supports and claims of these arguments is denoted by* $\mathsf{form}(A)$*,* $\mathsf{form}(A) = \bigcup_{\langle S, \varphi \rangle \in A} S \cup \{\varphi\}$.

In other words, an approximate argument is simply a pair (support,claim) and when the support is a minimal proof of the claim this argument is called a logical argument. Note that an approximate argument does not need to have a consistent support S and it is not required that its conclusion φ is a logical consequence of S. In order to be able to deal with arguments that have incomplete support or incompletely developed conclusion we first define an incomplete argument and then extend the enthymeme formalization proposed in [7].

Definition 2 (incomplete argument) *An* incomplete argument *is a pair* $\langle S, \varphi \rangle$ *where* $S \subseteq \mathscr{L}$ *and* $\varphi \in \mathscr{L}$ *(i.e.,* $\langle S, \varphi \rangle$ *is an approximate argument) such that:*
$\begin{cases} (1) & S \nvdash \varphi \\ (2) & \exists \psi \in \mathscr{L} \text{ s.t. } \langle S \cup \{\psi\}, \varphi \rangle \text{ is a logical argument} \end{cases}$
IArg_Σ *denotes the set of incomplete arguments that can be built on a set of formulas* Σ*.*

In this definition, the first condition expresses the fact that the argument is strictly incomplete, *i.e.,* the support is not sufficient to infer the conclusion. The second one imposes that it is possible to complete it in order to obtain a logical argument. Logical or incomplete arguments are particular distinct cases of approximate arguments:

Proposition 1 $\mathrm{IArg}_\Sigma \cap \mathrm{Arg}_\Sigma = \varnothing$ *and* $\mathrm{IArg}_\Sigma \cup \mathrm{Arg}_\Sigma \subseteq \mathrm{AArg}_\Sigma$.

Note that the support of an incomplete argument should be consistent or else adding any formula to it would still give an inconsistent support (hence violate condition (2) for logical arguments). Moreover S should be consistent with φ, more formally:

Remark 1 *If $\langle S, \varphi\rangle$ is an incomplete argument then $S \nvdash \bot$ and $S \nvdash \neg\varphi$.*

This remark shows that this notion is a slight variation of Hunter's concept of *precursor*, which he defines as an approximate argument $\langle S, \varphi\rangle$ such that $S \nvdash \varphi$ and $S \nvdash \neg\varphi$. Hence an "incomplete argument" is a "precursor" but the converse is false. The small difference lays in the fact that a completed precursor may not be minimal, for instance $\langle\{a, b, a \wedge b\}, c\rangle$ is a "precursor" and not an "incomplete argument" since any completion would have a non minimal support (i.e., in Definition 1, (4) will not hold).

Example 1 (continued): *The argument proposed by Schopenhauer's opponent is an incomplete argument. Indeed, "in music (m), and consequently in opera (o), English are not supreme ($\neg s$)" maybe transcribed into the following approximate argument: $a_1 = \langle\{m \to \neg s\}, o \to \neg s\rangle$. And by adding the formula $o \to m$ to its support we obtain the following logical argument: $b_1 = \langle\{m \to \neg s, o \to m\}, o \to \neg s\rangle$.*

Definition 3 (enthymeme) *Let $a = \langle S, \varphi\rangle$ and $a' = \langle S', \varphi'\rangle$ being approximate arguments,* $\langle S', \varphi'\rangle$ completes $\langle S, \varphi\rangle$ *iff*[1]
$$\begin{cases} (1) & S \subset S' \text{ and } \varphi = \varphi' \text{ or} \\ (2) & S \subseteq S' \text{ and } \{\varphi'\} \cup S' \vdash \varphi \text{ and } \varphi \neq \varphi' \end{cases}$$

a is an enthymeme *for a' iff a' is a logical argument and a' completes a.*

In other words, there are two ways to "complete" an argument: either by adding premises, then the support should be strictly included in the completed support or by specifying the conclusion, then it should be inferred by the union of the completed conclusion and support but should differ from the previous conclusion. Our definition extends the definition of [7] in the sense that it allows to cover arguments whose conclusion is an implicit claim requiring implicit support (the following example would not be considered as an enthymeme by [7]).

Example 1 (continued): *We may build an infinity of logical arguments decoding an incomplete argument. For instance, a_1 is an enthymeme for the logical argument b_1 but also for the logical argument: $\gamma_1 = \langle\{m \to \neg s, o \to m, o, o \to d\}, \neg(d \to s)\rangle$.*

Note that completion does not necessary give a logical argument since the initial approximate argument may have an inconsistent or redundant support, or the completion may be too weak for being a logical proof of the claim. Moreover, even logical arguments may be completed, since the completion may concern the conclusion.

Remark 2 *When a is an enthymeme for b, it is not necessarily the case that $b \in \mathrm{IArg}$.*

The following function gives the set of logical arguments that can be built from a knowledge base Σ and that are enthymemes for a given argument.

Definition 4 (Decode) *Let $\Sigma \subseteq \mathscr{L}$ and $\langle S, \varphi\rangle \in \mathrm{AArg}$, $\textsf{Decode}_\Sigma(\langle S, \varphi\rangle) = \{\langle S', \varphi'\rangle \in \mathrm{Arg}$ such that $S' \backslash S \subseteq \Sigma$, $\varphi' \in \Sigma$ and $\langle S, \varphi\rangle$ is an enthymeme for $\langle S', \varphi'\rangle\}$.*

In the previous example, it holds that $b_1, \gamma_1 \in \textsf{Decode}_{\mathscr{L}}(a_1)$. It is easy to see that incomplete arguments are particular enthymemes for logical arguments that have the same conclusion ($\varphi = \varphi'$). In other words,

[1] "iff" stands for "if and only if", $\subset$ denotes strict set-inclusion.

Proposition 2 *If $\langle S, \varphi\rangle$ is an incomplete argument then* $\textsf{Decode}_{\mathscr{L}}(\langle S, \varphi\rangle) \neq \varnothing$.

Proof If $a = \langle S, \varphi\rangle$ is an incomplete argument then there exists $\psi \in \mathscr{L}$ such that $\langle S \cup \{\psi\}, \varphi\rangle$ is a logical argument. Besides $\psi \notin S$ since $S \nvdash \varphi$ while $S \cup \{\psi\} \vdash \varphi$, hence $S \subset S \cup \{\psi\}$, in other words, there exists $a' \in$ Arg s.t. a' completes a. ■

3 A protocol for persuasion dialogs

In persuasion dialogs [12, 24], two or more participants are arguing about a claim, and each party is trying to persuade the other participants to adopt its point of view (i.e. to agree with the "right" claim). The set of symbols representing agents is denoted AG but in the following we are going to focus on a dialog between only two agents (named 1 and 2, this assumption is convenient but a generalization for more agents can easily be done).The current agent will often be called x in that case the other agent will be denoted by $\overline{x}$. A communicative act, called move, is defined below:

Definition 5 (Moves) *A* move *is a triplet $(sender, act, content)$, where $sender \in AG$, $act \in \{$Accept, Agree, Argue, Assert, Challenge, Close, Dismantle, Quiz, Quizlink, Complete, Retract$\}$ and ($content = \varnothing$ or $content \in \mathscr{L}$ or $content \in$ AArg or $content \in$ AArg $\times$ AArg).*
For a given move, the functions Sender, Act *and* Content *are returning respectively, the first, second and third element of the triplet. When there is no ambiguity about the sender, the move is denoted by $(act\ content)$.*

A move m is well-formed *if (*Act*(m)* Content*(m)) matches a utterance given in Table 1. Let $\mathscr{M}$ be the set of all well-formed moves.*

We consider a set of eleven speech acts that are described in Table 1 with their three associated effects (locutionary, illocutionary and perlucotionary [4]). Namely, the six usual speech acts used in persuasion dialog are augmented with Quiz and Agree proposed by [7] enabling to handle incomplete arguments, to which we add Quizlink enabling to ask for a completion concerning the conclusion of an argument, Complete allowing to precise an argument and Dismantle for retracting an argument. Although some speech acts are "assertive" (according to Searle [20]) namely Assert and Argue, we claim that they are "commissives" in the sense that they commit the utterer to both be able to explain them when challenged and avoid to contradict them. Moreover the "directive" speech acts such as Challenge or Quiz induce commitments for the hearer to answer to these questions. While Close is clearly a "declarative" speech act, it is less obvious for Retract, Dismantle, Accept and Agree since they not only are "declarative" but also "assertive" (because the accepted or agreed formulas are known as if the utterer had asserted them, the retracted formulas or dismantled arguments correspond to assertion of the form "I assert neither φ nor $\neg\varphi$" "I assert neither that S is a valid proof for φ nor that it is not") and "commissives"(since they are assertive).

These commitments are stored in a base called "commitment store" (first introduced by [12] in the dialog game DC). The protocol is a boolean function that uses it in order to check if a move is acceptable at a given stage of the dialog as follows:

Definition 6 (Persuasion dialog) *Let $p \in \mathbb{N} \cup \{\infty\}$, let (F, A) be a common knowledge base s.t. $F \subseteq \mathscr{L}$ and $A \subseteq$ AArg and $F \cup$ form(A) is consistent.*

A sequence $(m_n)_{n \in [\![1,p]\!]}$ is a persuasion dialog *of* length p *based on (F, A) iff*

- Act$(m_1) =$ Assert *and $\forall n \in [\![1, p]\!], m_n$ is a well formed move*
- *and there is a sequence $(CS_n)_{n \in [\![1,p+1]\!]}$ such that:$\forall n \in [\![1, p+1]\!], CS_n$ is a tuple $(F_1, A_1, R_1, F^\circ, A^\circ, F_2, A_2, R_2) \in 2^{\mathscr{L}} \times 2^{\text{AArg}_{\mathscr{L}}} \times 2^{\mathscr{M}} \times 2^{\mathscr{L}} \times 2^{\text{AArg}_{\mathscr{L}}} \times 2^{\mathscr{L}} \times 2^{\text{AArg}_{\mathscr{L}}} \times 2^{\mathscr{M}}$ called* commitment store at stage n *satisfying:*
- *-Starting condition: $CS_1 = (\varnothing, \varnothing, \varnothing, F, A, \varnothing, \varnothing, \varnothing)$*
- *-Current conditions: $\forall n \in [\![1, p-1]\!]$, let $x =$ Sender(m_n),*
  Close $\notin R_x$ *and* `precond`(m_n) *is true in CS_n and*
  $CS_{n+1} = (F'_1, A'_1, R'_1, F^{\circ\prime}, A^{\circ\prime}, F'_2, A'_2, R'_2)$ *defined by* `effect`(m_n, CS_n)
- *-Ending conditions: let $x =$ Sender(m_p),*
  `precond`(m_p) *is true in CS_p and* Act$(m_p) =$ Close *and* Close $\in R_{\overline{x}}$

Utterance	Meaning	Speaker's intention	Effects on the audience
Accept φ	acceptance of φ	to announce that he accepts φ	the speaker is associated with φ
Agree $\langle S, \varphi \rangle$	acceptance of $\langle S, \varphi \rangle$	to announce that he accepts $\langle S, \varphi \rangle$ as an enthymeme	the speaker is associated with formulas of $S \cup \{\varphi\}$ and knows a logical argument that decodes $\langle S, \varphi \rangle$
Argue $\langle S, \varphi \rangle$	providing a set of formulas S which may support φ if completed	to prove that φ is justified	the speaker is associated with formulas of $S \cup \{\varphi\}$ and knows a logical argument that decodes $\langle S, \varphi \rangle$
Assert φ	statement of assertion φ	to make the hearers believe φ	the speaker is associated with φ
Challenge φ	seeking for arguments supporting φ	to obtain arguments for φ	the receiver must justify φ
Close	closing the dialog	to announce that he has nothing to add	the speaker can no longer participate to the dialog.
Complete (a, b)	providing a new argument b completing a previous one a	to explicit an incomplete argument	the speaker is associated with the support and claim of b and knows a logical argument that decodes b
Dismantle $\langle S, \varphi \rangle$	withdrawal of argument $\langle S, \varphi \rangle$	to renounce to the fact that S is a proof of φ	the speaker is no more associated with $\langle S, \varphi \rangle$
Quiz $\langle S, \varphi \rangle$	seeking for a completion of argument $\langle S, \varphi \rangle$	to obtain a more detailed argument for φ	the receiver must complete $\langle S, \varphi \rangle$
Quizlink $\langle S, \varphi \rangle$	seeking for a link between $\langle S, \varphi \rangle$ and the dialog	to obtain a completion in which the implicit conclusion is disclosed	the receiver must complete at least the conclusion of $\langle S, \varphi \rangle$
Retract φ	withdrawal of assertion φ	to restore consistency or to renounce to prove φ	the speaker is no more associated with φ

Table 1: Speech acts for enthymeme persuasion dialogs

where `precond` *and* `effect` *are given in Table 2* [2].

The commitment store used in this definition is made of three knowledge bases:
- a base containing *common knowledge* divided into two parts: the common formulas, denoted by F°, and the common arguments, denoted by A°,
- and the two commitment stores of *each agents*, each one divided into three parts[4]:
  - the first two parts contain the *assertive commitments* of agent x separated into a set F_x of propositional formulas, and a set A_x of approximate arguments
  - the third one contains the *commitments towards the other agent*, i.e., the requests to which x should answer, denoted by R_x.

In this definition, the "starting condition" is simply an initialisation of the commitment store. The "current conditions" require that, in order to make a move, the sender should not have already closed his participation to the dialog, and ensure that every move is done under the adequate preconditions, each move has an effect on the commitment store (preconditions and effects are described in Table 2). Note that the deterministic definition of the effects of the moves induces that the sequence of commitment store stages associated to a persuasion dialog is *unique*. The "ending conditions" ensure that the last move of the dialog is a Close move and that the other agent has already closed his participation to the dialog. If these "ending conditions" are not possible then the dialog has no end. This may seem not realistic, but it is often the case that the physical end of a debate is not really a true ending of the subject since the participants may still not be convinced by the arguments of their adversary, and the dialog will continue at another occasion. Let us describe more precisely how each move is taken into account according to Table 2:

[2]For shortness, in the "effect" column, the sets that remain unchanged are not mentioned, and K_x denotes the set of formulas $F_x \cup \mathsf{form}(A_x) \cup F^\circ \cup \mathsf{form}(A^\circ)$

[4]The distinction between internal and external commitment is inspired from [1].

An agent x may "accept" a formula φ (respectively "agree" about an argument $\langle S, \varphi \rangle$) only if this formula (resp. argument) has been uttered by the other agent $\overline{x}$ and is consistent with both the common knowledge and all the formulas that x has already uttered. After the Accept move the formula becomes common knowledge and is no more considered as $\overline{x}$ own utterance. Similarly, an Agree move introduces the formulas used in the argument as well as the argument itself into the common knowledge and the argument is removed from $\overline{x}$ own arguments. When these moves are uttered the commitments of the speaker to accept (or agree) this formula (or argument) are fulfilled hence removed from the commitment store, moreover the requests he may have made about this formula (or argument) are nomore committing his adversary.

The protocol is designed for obtaining "rational" dialogs, in terms of *non redundancy*, *self consistency* and *listening*. Hence, in order to Assert a formula (or Argue an argument) this formula (argument) should not have already been asserted and should be consistent with common knowledge and with what the utterer has already said. Each Assert or Argue move commits the receiver to accept or agree with it or to make the sender retract or dismantle it. This commitment is dropped when the move is retracted (or dismantled or completed) by his utterer or accepted (or agreed) by the receiver.

The Challenge is authorized only if the formula has been uttered by the other agent (and if it is not common knowledge) but not already been proved nor challenged. After this move the other agent is committed to give an argument for the challenged formula or to "retract" it, this is translated by adding (Challenge φ) to his requests commitment store. This request will be removed when an argument whose claim is φ will be agreed or if the formula φ is directly accepted.

A Close move requires that all the commitments of the agent are fulfilled. After this move the agent is not allowed to speak anymore: for this purpose an artificial commitment Close is added to his request commitment store, and, as described in the "current condition", no move can be done by agent x if Close is present in R_x.

"Completing" an argument $a = \langle S, \varphi \rangle$ by $b = \langle S', \varphi' \rangle$ should be done by giving a more precise argument b than a (uttered by the current speaker, but not yet agreed by the hearer) *i.e.*, a logical or incomplete argument that completes it. The new argument b should be consistent and not already present. After the utterance, b replaces a in the set of uttered arguments, some commitments of the utterer may nomore be appropriate: namely, Quiz a , Quizlink a. But a request commitment maybe inherited, namely if there was a request Quizlink a and $\varphi = \varphi'$, then it becomes Quizlink b.

Dismantle (or Retract) allows to remove an argument (respectively a formula) from the utterances of agent x. The commitments concerning this argument or formula are also removed from the request commitment stores of the sender and the receiver.

A move (Quiz $\langle S, \varphi \rangle$) can be done by an agent x only if there is no logical argument completing $\langle S, \varphi \rangle$ that can be built from the common knowledge and the formulas already asserted by x (this set is called K_x for short). A (Quizlink a) move requires that there is no obvious link between a and something said by x (positively or negatively) neither with a previous assertion of $\overline{x}$.

Example 1 (continued): *Let us consider the following persuasion sub-dialog:*

$$D = \begin{pmatrix} (Shopenhauer, & \textsf{Assert}, & d \rightarrow s &), \\ (Adversary, & \textsf{Argue}, & \langle \{m \rightarrow \neg s\}, o \rightarrow \neg s \rangle &), \\ (Shopenhauer, & \textsf{Argue}, & \langle \{d \leftrightarrow t \vee s\}, m \rightarrow \neg d \rangle &), \\ (Adversary, & \textsf{Agree}, & \langle \{d \leftrightarrow t \vee s\}, m \rightarrow \neg d \rangle &) \end{pmatrix}$$

Suppose that common knowledge is: $F^\circ = \{o \rightarrow m, o\}$ meaning that "opera is music" and that "opera exists". Table 3 describes the commitment stores of each participant, with a_1, a_2 denoting respectively $\langle \{m \rightarrow \neg s\}, o \rightarrow \neg s \rangle$ and $\langle \{d \leftrightarrow t \vee s\}, m \rightarrow \neg d \rangle$.

After these moves the dialog is not finished since two requests are not yet answered. Schopenhauer has two options either (1) he agrees with his adversary's argument a_1 (since it is consistent with common knowledge) then he would have no more commitments and his adversary will be obliged either to accept the first claim or to provide another argument against it, or (2) he may ask his adversary to precise the link that argument a_1 has with the formulas already asserted. In that case the adversary would not be able to Complete his argument since the logical argument that completes a_1 and that has a link with the subject is γ_1 ($\langle \{m \rightarrow \neg s, o \rightarrow m, o, o \rightarrow d\}, \neg(d \rightarrow s) \rangle$) whose support is now inconsistent with the common knowledge (see Table 3).

Proposition 3 *Two Close moves belonging to a persuasion dialog have distinct senders.*

Proof Due to the definition of the precondition of Close, in order to be able to do it there should not remain any commitment unfulfilled, however a Close move commits the sender by adding Close to its requirement commitment store R_x. ■

Remark 3 *Even if a persuasion dialog has a finite length* p*, it may be the case that, in* CS_{p+1}*, neither* $\mathit{Content}(m_1) \in F^\circ$ *nor* $\neg\mathit{Content}(m_1) \in F^\circ$

Proof Consider the dialog $((1, \mathsf{Assert}, \varphi), (1, \mathsf{Retract}, \varphi))$. ■

Definition 7 (Output) *Let* D *be a persuasion dialog of length* p*, with* $(CS_n)_{n \in [\![1,p+1]\!]}$ *its sequence of commitment stages, the output of the dialog,* Output(D)*, is:*

- *Undecided if* $p = \infty$
- *otherwise* $\begin{cases} \textit{Public agreement that } \varphi \textit{ holds,} & \textit{if in } CS_{p+1} \textit{ it holds that } \varphi \in F^\circ \\ \textit{Public agreement that } \neg\varphi \textit{ holds,} & \textit{if in } CS_{p+1} \textit{ it holds that } \neg\varphi \in F^\circ \\ \textit{No public agreement on } \varphi & \textit{otherwise} \end{cases}$

4 A protocol for time-limited persuasion dialogs

Since a persuasion dialog may be infinite, we introduce particular persuasion dialogs where the speaking time is restricted, indeed it is often the case that the speakers of a public debate have to keep strictly to a given speaking time. This notion requires to define first the duration associated to a move.

Definition 8 (Move duration) *We assume a function* $size : \mathscr{L} \to \mathbb{N}^\star$ *that associates an integer to each formula (e.g., the size of a binary encoding of this formula).*
The duration $d(m)$ *of a move* m *is equal to:*

$$d(m) = \begin{cases} 1 + size(\varphi) & \text{if } m = (\mathit{Assert}\,\varphi) \\ 1 + \sum_{\psi \in S} size(\psi) + size(\varphi) & \text{if } m = (\mathit{Argue}\,\langle S, \varphi\rangle) \\ 1 + \sum_{\psi \in S'} size(\psi) + size(\varphi') & \text{if } m = (\mathit{Complete}\,(\langle S, \varphi\rangle, \langle S'\varphi'\rangle)) \\ 1 & \text{if } \mathit{Act}(m) \notin \{\mathit{Assert}, \mathit{Argue}, \mathit{Complete}\}, \end{cases}$$

There is a link between taking duration into account and allowing enthymemes, since the usual reason to use an enthymeme is for sake of shortness (even if sometimes it is more a strategical choice). In the above definition, the duration of every move is one except for Assert, Argue and Complete moves where it is strictly greater than 1. This may seem artificial but it is based on the fact that those three moves are introducing new formulas (hence requiring time to express them) while other moves refer to already expressed formulas (hence could be shortly expressed by using only a reference). Now, we introduce the time-limited persuasion dialog which is a variant of a persuasion dialog:

Definition 9 (Time-limited persuasion dialog) *Let* $p \in \mathbb{N} \cup \{\infty\}$ *and* $T \in \mathbb{N}$*, let* $(F, A) \in 2^{\mathscr{L}} \times 2^{\mathrm{AArg}}$ *be a common knowledge base such that* $F \cup \mathit{form}(A)$ *is consistent.*

A sequence of moves $(m_n)_{n \in [\![1,p]\!]}$*, is a* T-limited persuasion dialog *of* length p *based on* (F, A) *iff:*

$\left\{\begin{array}{l} \textsf{Act}(m_1) = \textsf{Assert} \textit{ and } \forall n \in [\![1,p]\!], m_n \textit{ is a well formed move and there is a} \\ \textit{sequence } (CS_n)_{n\in[\![1,p+1]\!]} \textit{ where } \forall n \in [\![1,p+1]\!], CS_n \textit{ is a tuple } (F_1, A_1, R_1, \\ dur_1, F^\circ, A^\circ, F_2, A_2, R_2, dur_2) \textit{ called } \text{commitment store at stage } n \textit{ with } dur_1, \\ dur_2 \in \mathbb{N} \textit{ representing the agents remaining speaking time and } F^\circ, F_1, F_2 \subseteq \mathscr{L}, \\ A^\circ, A_1, A_2 \subseteq \mathrm{AArg}_{\mathscr{L}}, R_1, R_2 \subseteq \mathscr{M}, \textit{ satisfying:} \\ \textit{-Starting condition: } CS_1 = (\varnothing,\varnothing,\varnothing,T,F,A,\varnothing,\varnothing,\varnothing,T) \\ \textit{-Current conditions: } \forall n \in [\![1,p-1]\!], \textit{ let } x = \textsf{Sender}(m_n), \\ \quad \textsf{Close} \notin R_x \textit{ and } \texttt{precond}(m_n) \textit{ is true in } CS_n \textit{ and } dur_x \geq d(m_n) \textit{ and} \\ \quad CS_{n+1} = (F'_1, A'_1, R'_1, dur'_1, F^{\circ\prime}, A^{\circ\prime}, F'_2, A'_2, R'_2, dur'_2) \\ \quad \textit{defined by } \texttt{effect}(m_n, CS_n) \textit{ with } dur'_x = dur_x - d(m_n), dur'_{\overline{x}} = dur_{\overline{x}} \\ \textit{-Ending conditions: let } x = \textsf{Sender}(m_p), \\ \quad \textsf{Close} \notin R_x \textit{ and } \texttt{precond}(m_p) \textit{ is true in } CS_p \textit{ and } \textsf{Act}(m_p) = \textsf{Close} \\ \textit{and } dur_x \geq d(m_p) \textit{ and } \left\{\begin{array}{l} \textit{- either } \textsf{Act}(m_p) = \textsf{Close} \textit{ and } \textsf{Close} \in R_{\overline{x}} \\ \textit{- or } dur_x - d(m_p) = 0 \textit{ and } dur_{\overline{x}} = 0 \end{array}\right. \end{array}\right.$ *where* `precond` *and* `effect` *are given in Table 2.*

The last condition expresses the termination condition for the dialog: the dialog may finish either because the two agents agree to close it or because they have no more speaking time.

Proposition 4 *A T-limited persuasion dialog of length p is such that:*

$$\forall x \in AG, \forall k \in [\![1,p]\!] \quad dur_x(k) \geq 0$$

$$\forall x \in AG, \sum_{\{m \in D \,|\, \textsf{Sender}(m)=x\}} d(m) \leq T$$

Proposition 5 *A T-limited persuasion dialog is finite.*

This last property is important since by allowing enthymemes and requests about them, it is possible that a dialog may never end. Indeed an argument may be "Quizlinked" eternally (when there is no common knowledge) since explaining a concrete fact could require to go back to reasons involving the Big-Bang theory.

Definition 10 (Output of a time limited persuasion dialog) *Let D be a T-limited persuasion dialog of length p, with $(CS_n)_{n\in[\![1,p+1]\!]}$ its associated sequence of commitment stages, the output of the dialog, denoted by Output(D), is:*

– *Public agreement that φ holds,*	*if, in $CS_{p+1}, \varphi \in F^\circ$*
– *Public agreement that $\neg\varphi$ holds,*	*if, in $CS_{p+1}, \neg\varphi \in F^\circ$*
– *No public agreement on φ,*	*else*

The following proposition shows that when a dialog is closed by the two participants then they have fulfilled all their commitments (hence their commitment store contains only the Close commitment).

Proposition 6 *If Close appears twice in a time-limited persuasion dialog such that $(CS_n)_{n\in[\![1,p+1]\!]}$ is its associated sequence of commitment stages then in CS_{p+1} it holds that $R_x = R_{\overline{x}} = \{Close\}$ and $F_x = F_{\overline{x}} = \varnothing$ and $A_x = A_{\overline{x}} = \varnothing$.*

Proof Due to Proposition 3, the two Close moves have been done by two distinct agents. Moreover, in order to do a Close move the commitments toward the other agent should be empty, i.e, $R_x = R_{\overline{x}} = \varnothing$. Furthermore, if all their commitments are fulfilled then they either have agreed or accepted all adversary's arguments or he has retracted them. ■

Proposition 7 *If D is a time-limited persuasion dialog of length p and $(CS_n)_{n\in[\![1,p+1]\!]}$ its associated sequence of commitment stages, then $\forall n \in [\![1,p+1]\!]$, $\forall x \in AG$,*

- $F_x \cup \textsf{form}(A_x) \cup F^\circ \cup \textsf{form}(A^\circ)$ *consistent and*

- $F_x \cap F^\circ = \varnothing$ *and*

- $A_x \cap A^\circ = \varnothing$

Corollary 1 *If D is a time-limited persuasion dialog of length p and $(CS_n)_{n\in[\![1,p+1]\!]}$ is its associated sequence of commitment stages, then,*

if $\textsf{Output}(D) = \varphi$ *then* $\left\{ \begin{array}{l} \varphi \cup F_x \cup F_{\overline{x}} \cup F^\circ \textit{ is consistent and} \\ \nexists \langle S, \psi \rangle \in A_1 \cup A_2 \cup A^\circ \textit{ s.t. } \psi \vdash \neg\varphi \end{array} \right.$

The following propositon shows that common knowledge may only increase with the persuasion dialog.

Proposition 8 *If D is a time-limited persuasion dialog of length p based on a common knowledge base (F, A), and if $(CS_n)_{n\in[\![1,p+1]\!]}$ is its associated sequence of commitment stages, then $\forall n \in [\![1, p+1]\!]$, in CS_n, it holds that $F \subseteq F^\circ$ and $A \subseteq A^\circ$*

5 Concluding remarks

This work is a preliminary study on handling enthymemes in persuasion dialogs. The ambition was to deal with incomplete information both in the premises and in the claim of an argument. The latter is more difficult to handle and has required to introduce a new speech act Quizlink allowing to ask for an insight about what is hiding behind the claim. In some cases, one may not disagree with an argument that is not related with the subject but when he understands the underlying implication he wants to reject it. This is why it is necessary to allow the agent to "dismantle" an argument even if this argument is not attacked.

In this work we only represent what is publicly uttered, since we consider that we do not have access to the agent's mind. This way to apprehend the public statements is also done for instance by [10], where a public utterance is called "grounded". Their proposal is a framework to represent and reason about the public knowledge during a persuasion dialog, their approach allows to deal with inconsistent assertions (which is not allowed in our framework) like in Walton & Krabbe's system PPD_0. This feature seems more realistic since it is up to the other agent to detect and denounce inconsistency by asking to its adversary to "resolve" it. Dealing with possible inconsistent assertions is a challenge for further developments of our approach, however we could argue that a well designed protocol should enforce that what is public is consistent (in order to obtain a debate of hight quality that is civilised and respectful of the audience). Note that [10] do not deal with "dark side commitments" [24] (implicit assertions that are difficult to concede explicitely in front of a public) since they do not want to take into account the agent's mind but rather focus on what is observable and objective. In our approach a "dark side commitment" can be encountered when decoding an enthymeme, it may be the necessary piece to add in order to obtain a logical argument and may be revealed by means of a Quizlink or a Quiz move.

Our definition of incomplete argument maybe compared to the notion of "partial argument" given in [22], which is a set of default rules, and conclusion such that there exists a minimal set of strict rules and facts coming from a given knowledge base that allows to defeasibly derive the conclusion. Beyond the fact that we do not consider default reasoning, our incomplete argument, and particularly the ones that are enthymemes are not lacking information because it is not available as in the work of [22] but rather because the lacking information is considered as obvious or worthwile to conceal. The use of enthymemes is not necessarily a proof of weakness but rather "a highly adaptative argumentation strategy, given the need of everyday reasoners to optimize their cognitive resources" as it is claimed in [15, 16]. The computation of the completion of enthymeme, namely how to define our function Decode, is out of the scope of the paper but as already been studied by several authors (see e.g. [23, 14] in which several examples are analysed in order to understand how implicit premises can be discovered, and which provides a set of argumentation schemes that can be used as a guide for finding them). Moreover, it seems important to take into account that in enthymemes, the link between the premises and the claim is not necessarily classical logic inference, this is why [14] proposes to view it as a presumptive type of argumentation scheme. The notion of non-classical inference is also suggested by [18] that defines an argument as a pair where the premises provide backing for the claim but do not necessarily infere it. Indeed in this work, the argument is composed by a set of litterals (for the premises) and a literal (for the claim), with the only constraint that no premise is equal to the claim or its negation. This work is related to our own on another aspect since the argument is evaluated with respect to a knowledge base called "evidence", this base plays a similar role than our "common knowledge base" and represents the context in which arguments are to take into account. The coherence and redundancy notions of an argument with respect to the "evidence" are also introduced, this slightly differs from our approach in which the protocol ensures "self-coherence" of an agent (hence it implies both the common knowledge and the agent utterances), non-redundancy (based on common knowledge but also on agents utterances) and listening (this last is not related to [18] since they do not deal with dialog systems). In [18] the use of evidence it not done in order to complete arguments as we do with the common knowledge but to decide about their status. Besides a very appealing aspect of this approach is that evidence may evolve hence may imply changes in the arguments status while in our proposal, common knowledge may only increase consistently. Again the non-monotonic aspect seems to be interesting to consider in future studies.

During dialogs, the public utterances are stored and may evolve when arguments are retracted or replaced. Since enthymemes are possible and based on implicit information that is often common knowledge, we use a common knowledge base that is public. An advantage of our proposal is that at the end of the dialog all agreed assertions are kept in this common knowledge base that can hence only increase during the dialog, and that can be used for future dialogs.

A very appealing development of this framework concerns the strategical part, we plan to translate our protocol rules into the Game player project language GDL2 [21], indeed in GDL2 it is possible to handle games with imperfect information. With this translation, strategies coming from game theory and strategies dedicated to dialog games (e.g. [2]) coud be compared. Moreover, the move duration would have to be taken into account for the strategical aspect.

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m	$\texttt{precond}(m)$[3]	$\texttt{effect}(m, (F_x, A_x, R_x, F^\circ, A^\circ, F_{\overline{x}}, A_{\overline{x}}, R_{\overline{x}}))$
Accept φ	$\varphi \in F_{\overline{x}}$ and $\{\varphi\} \cup K_x$ consistent	$R'_x = R_x \backslash \{(\text{Accept}\,\varphi)\}, F^{\circ\prime} = F^\circ \cup \{\varphi\}$ $F'_{\overline{x}} = F_{\overline{x}} \backslash \{\varphi\}$ $R'_{\overline{x}} = R_{\overline{x}} \backslash \left\{ \begin{array}{l} (\text{Challenge}\,\varphi), (\text{Quiz}\,\langle S, \varphi\rangle) \\ (\text{Quizlink}\,\langle S, \varphi\rangle) \end{array} \middle\vert\, S \subseteq \mathscr{L} \right\}$
Agree $\langle S, \varphi\rangle$	$\langle S, \varphi\rangle \in A_{\overline{x}}$ and $S \cup \{\varphi\} \cup K_x$ consistent	$R'_x = R_x \backslash \{(\text{Agree}\,\langle S, \varphi\rangle)\}, F^{\circ\prime} = F^\circ \cup \{\varphi\} \cup S,$ $A^{\circ\prime} = A^\circ \cup \{\langle S, \varphi\rangle\}, A'_{\overline{x}} = A_{\overline{x}} \backslash \{\langle S, \varphi\rangle\}$ $R'_{\overline{x}} = R_{\overline{x}} \backslash \{(\text{Challenge}\,\varphi), (\text{Quiz}\,\langle S, \varphi\rangle), (\text{Quizlink}\,\langle S, \varphi\rangle)\}$
Argue $\langle S, \varphi\rangle$	$\langle S, \varphi\rangle \notin A^\circ \cup A_x \cup A_{\overline{x}}$ and $S \cup \{\varphi\} \cup K_x$ consistent	$A'_x = A_x \cup \{\langle S, \varphi\rangle\}, R'_{\overline{x}} = R_{\overline{x}} \cup \{(\text{Agree}\,\langle S, \varphi\rangle)\}$
Assert φ	$\varphi \notin K_x \cup F_{\overline{x}}$ and $\{\varphi\} \cup K_x$ consistent	$F'_x = F_x \cup \{\varphi\},$ $R'_{\overline{x}} = R_{\overline{x}} \cup \{(\text{Accept}\,\varphi)\}$
Challenge φ	$\varphi \in F_{\overline{x}}$ and $(\text{Challenge}\,\varphi) \notin R_{\overline{x}}$ and $\nexists \langle S, \varphi\rangle \in A_x \cup A_{\overline{x}} \cup A^\circ$	$R'_{\overline{x}} = R_{\overline{x}} \cup \{(\text{Challenge}\,\varphi)\}$
Close	$R_x = \varnothing$	$R'_x = \{\text{Close}\}$
Complete $(a = \langle S, \varphi\rangle,$ $b = \langle S', \varphi'\rangle)$	$a \in A_x$ and (b completes a) and $b \notin A^\circ \cup A_x \cup A_{\overline{x}}$ and $S' \cup \{\varphi'\} \cup K_x$ consistent	$A'_x = A_x \backslash \{a\} \cup \{b\}$ $R'_{\overline{x}} = R_{\overline{x}} \backslash \{(\text{Agree}\,a)\} \cup \{(\text{Agree}\,b)\}$ $R'_x = R_x \backslash \{(\text{Quiz}\,a), (\text{Quizlink}\,a)\}$ $\cup \{(\text{Quizlink}\,b)$, if $(\text{Quizlink}\,a) \in R_x$ and $\varphi = \varphi'\}$
Dismantle a	$a \in A_x$	$A'_x = A_x \backslash \{a\}$ $R'_x = R_x \backslash \{(\text{Quiz}\,a), (\text{Quizlink}\,a)\},$ $R'_{\overline{x}} = R_{\overline{x}} \backslash \{(\text{Agree}\,a)\}$
Quiz a	$a \in A_{\overline{x}}$ and $(\text{Quiz}\,a) \notin R_{\overline{x}}$ and $\text{Decode}_{K_x}(a) = \varnothing$	$R'_{\overline{x}} = R_{\overline{x}} \cup \{(\text{Quiz}\,a)\}$
Quizlink a	$a \in A_{\overline{x}}$ and $(\text{Quizlink}\,a) \notin R_{\overline{x}}$ and $\nexists \langle S', \varphi'\rangle \in \text{Decode}_{K_x}(a)$ s.t. $\left\{ \begin{array}{l} \{\varphi', \neg\varphi'\} \cap (F_x \cup \text{form}(A_x)) \neq \varnothing \\ \text{or } \varphi' \in F_{\overline{x}} \cup \text{form}(A_{\overline{x}}) \end{array} \right.$	$R'_{\overline{x}} = R_{\overline{x}} \cup \{(\text{Quizlink}\,a)\}$
Retract φ	$\varphi \in F_x$ [3] with $K_x = F_x \cup \text{form}(A_x) \cup F^\circ \cup \text{form}(A^\circ)$	$F'_x = F_x \backslash \{\varphi\}$ $A'_x = A_x \backslash \{\langle S, \psi\rangle \mid S \subseteq \mathscr{L}, (\varphi \in S \text{ or } \psi = \varphi)\},$ $R'_x = R_x \backslash \left\{ \begin{array}{l} (\text{Challenge}\,\varphi), \\ (\text{Quiz}\,\langle S, \psi\rangle) \\ (\text{Quizlink}\,\langle S, \psi\rangle) \end{array} \middle\vert \begin{array}{l} S \subseteq \mathscr{L}, \\ \varphi \in S \text{ or } \psi = \varphi \end{array} \right\}$ $R'_{\overline{x}} = R_{\overline{x}} \backslash \left\{ \begin{array}{l} (\text{Accept}\,\varphi), \\ (\text{Agree}\,\langle S, \psi\rangle) \end{array} \middle\vert \begin{array}{l} S \subseteq \mathscr{L}, \\ \varphi \in S \text{ or } \psi = \varphi \end{array} \right\}$

Table 2: Effects and conditions of a move from x towards $\overline{x}$

After the third move

Schopenhauer			Common knowledge		Adversary		
Formulas	Arguments	Requests	Formulas (F°)	Arguments (A°)	Formulas	Arguments	Requests
$d \to s$	a_2	(Agree a_1)	$o \to m$ o			a_1	(Accept $d \to s$) (Agree a_2)

After the fourth move

$d \to s$		(Agree a_1)	$o \to m$ o $d \leftrightarrow t \vee s$ $m \to \neg d$	a_2		a_1	(Accept $d \to s$)

If the move $(Shopenhauer$, Quizlink, $a_1)$ is done
Then the move $(Adversary$, Dismantle, $a_1)$ should be done, leading to:

$d \to s$			$o \to m$ o $d \leftrightarrow t \vee s$ $m \to \neg d$	a_2			(Accept $d \to s$)

If the Adversary has no other argument linked with the subject, then he is forced to do the move $(Adversary$, Accept, $d \to s)$ in order to be authorized to close the dialog:

			$o \to m$ o $d \leftrightarrow t \vee s$ $m \to \neg d$ $d \to s$	a_2			
Schopenhauer			Common knowledge		Adversary		

Table 3: Commitments stores of Schopenhauer and his Adversary