Change in argumentation systems: exploring the interest of removing an argument*

Pierre Bisquert Claudette Cayrol Florence Dupin de Saint-Cyr
Marie-Christine Lagasquie-Schiex
IRIT, Université Paul Sabatier, 31062 Toulouse Cedex 9, France,
{bisquert,ccayrol,dupin,lagasq}@irit.fr

July 8, 2011

Abstract

This article studies a specific kind of change in an argumentation system: the removal of an argument and its interactions. We illustrate this operation in a legal context and we establish the conditions to obtain some desirable properties when removing an argument.

1 Introduction

Argumentation is a very active research area, in particular for its applications concerning reasoning ([10, 2]) or negotiation between agents ([4]). It allows to model the exchange of arguments between several agents (dialog), but also allows a single agent to manage incomplete and potentially contradictory information. Hence, argumentation is a way to handle uncertainty about the outcome of a dialog or the conclusion of a reasoning process. The arguments thus emitted are in interaction, generally by means of an attack relation representing the conflicts between arguments (for example, when the conclusion of an argument contradicts an assumption of another one).

Argumentation theory proposes several methods for drawing a conclusion about a set of interacting arguments. One of these methods is the study of *"extensions"*, sets of arguments that are said *acceptable* (*i.e.* a set able to defend itself collectively while avoiding internal conflicts). Another method is the study of the individual status of each argument determined by its membership to one or all extensions. Formal frameworks were proposed for representing argumentation systems, in particular [10] which allows to handle the arguments like purely abstract entities connected by binary relations.

Although dynamics of argumentation systems has been recently explored by several works ([6, 7, 5, 11]), the removal of an argument has scarcely been mentioned. However, there exist practical applications. First of all, a speaker can need *to occult*

*This is a draft version, the article was published In : Proc. of the Fifth International Conference on Scalable Uncertainty Management SUM-2011 (http://www.wright.edu/academicaffairs/sum/) October 10 - 12, 2011, Dayton, Ohio, USA, Springer Lecture Notes in Artificial Intelligence series.

some argument, in particular when he does not want, or is not able, to present this argument in front of a given audience[1]; it is then necessary to know what would be the output of the speaker's argumentation system without this argument: that can be achieved by a removal in his initial system. In addition, this same audience can force the speaker *to remove* an argument, in particular when this last is regarded as illegal in the context. Note that, when the argument is uttered, it is not discarded by default because the audience may not know that this argument is illegal; however, it could be removed later, when proved to be illicit. Moreover, the removal turns out to be useful in order to evaluate *a posteriori* the impact of a precise argument on the output of the system. In particular for evaluating the quality of a dialog, it is important to be able to differentiate the unnecessarily uttered arguments from the decisive ones (see [3]: an argument is decisive if its removal makes it possible to change the conclusion of the dialog). Lastly, it can be interesting to know how to guarantee that one or more arguments are accepted by removing a minimal set of arguments. Note that the removal of an argument X cannot always be reduced to the addition of a new argument Y attacking X, in particular because it may happen that an attacked argument remains acceptable. Furthermore, it is more economic to remove an argument rather than to add one, which might progressively overload the system. We thus propose to study theoretically the impact that the removal of one argument may produce on the initial set of extensions of an argumentation system.

The article is organized as follows. An example illustrating the interests of removing an argument is presented in Section 2. Section 3 gives a brief state of the art about argumentation. Section 4 exposes some properties of the extensions and of the status of some arguments when a particular argument is removed. Lastly, Section 5 establishes the links with related works and concludes this article.

2 Illustrative example

We describe a *four players game* example inspired from the one given by Brewka in [8]. This game involves two speakers (the prosecutor and the lawyer) and two listeners (the judge and the jury). Although the discussion concerns only the two speakers, we choose to also model the audience in order to be able to study the dialog from the point of view of a neutral external observer. The presence of a judge makes it possible to illustrate a case of permanent removal: the objection. Let us note that this example can be expanded easily with more than two speakers.

2.1 Presentation of the game

This game takes place during an oral hearing, gathering four entities which play quite distinct roles and which interact in order to determine whether an argument is acceptable or not.

- the *prosecutor* (P) wants to make accept a particular argument Q which is the subject of the hearing by the court. He has his own argumentation system in

[1]Social norms, or the will to avoid providing information to an adversary, etc.

which he can *occult* arguments threatening Q, *i.e.,* withdraw temporarily some arguments which could prevent Q from being accepted. He can also occult some other arguments not threatening Q, but considered, for example, irrelevant or dangerous in front of a particular jury (according to his strategy of argumentation).

- the *defense lawyer* (D) possesses also its own argumentation system[2], he behaves like the prosecutor, but he tries to make the argument Q defeated.
- the *judge* ensures that the argumentation process takes place under good conditions. He gets involved when one participant makes an objection; he can then accept this objection (thus force the corresponding argument to be deleted), or reject it.
- the *jury*[3] has the last word. Its role is to listen to the prosecutor's and lawyer's arguments and to draw from them a conclusion concerning the acceptability of the argument Q. The jury begins the game with an argumentation system containing only the argument Q and supplement it with the arguments presented successively by the prosecutor and the lawyer (if these arguments are not cancelled by objections). When the hearing is closed (*i.e.* when neither the prosecutor nor the lawyer can give new arguments), the jury can determine whether Q is acceptable or not.

In our example, the subject of the argumentation is the guilt of the defendant concerning the murder of his wife. Table 1 summarizes the set of existing arguments concerning this example and their distribution between the prosecutor and the defense lawyer.

2.2 Arguments of the prosecutor

Let us examine the arguments of the prosecutor. He knows only two arguments attacking his thesis (Argument 1): Arguments 6 and 4. The prosecutor is not over worried about 4 because 5 enables him to defend his thesis against it. The prosecutor knows, on the other hand, no argument which can defeat 6. Not being able to find what could beat this argument, and hoping that the lawyer is not informed about it, the prosecutor decides to occult it in order to ensure the acceptability of his thesis in his argumentation system.

2.3 Arguments of the defense

Now let us examine the arguments of the defense lawyer who aims at preventing the acceptability of Argument 1. The lawyer has two arguments attacking directly 1, namely, 4 and 2. While 2 is not attacked (as far as he knows), it is not the same for 4 which is attacked by 7; unable to find something to oppose to 7, the lawyer thus prefers to occult

[2]In order to have a genuine confrontation, it is necessary that the prosecutor and the lawyer share some arguments. However, these arguments are dealt differently by the two speakers: often considered as positive for one and negative for the other one.

[3]Although being a group of persons, the jury is considered to be only one decision-making entity.

	Argument	Known by
1	*Mr. X is guilty of premeditated murder of Mrs. X, his wife.*	P & D
2	*The defendant has an alibi, his business associate having solemnly sworn that he had seen him at the time of the murder.*	D
3	*The close working business relationships between Mr X. and his associate induce suspicions about his testimony.*	P
4	*Mr. X loves his wife so extremely that he asked her to marry him twice. Now, a man who loves his wife could not be her murderer.*	P & D
5	*Mr. X has a reputation for being promiscuous.*	P
6	*The defendant would not have had any interest to kill his wife, since he was not the beneficiary of the enormous life insurance she had contracted.*	P
7	*The defendant is a man known to be venal and his "love" for a very rich woman could be only lure of profit.*	D

Table 1: Arguments concerning the case of Mr. X.

7 (in order to be sure that 1 will be rejected), hoping that the prosecutor will not utter it.

2.4 The oral hearing in front of the court

Now that we know the arguments of the two speakers, we can consider the exchanges between them during the hearing (see Table 2).
Turn 0 establishes the subject of the dialog; it is a mandatory stage fixing the argument that the prosecutor and the lawyer will try to make, respectively, accept or reject.
Turns 1 to 4 are "normal" exchanges of arguments between the speakers, these arguments are used by the jury to build its argumentation system.
Turn 5 introduces the objection process, *i.e.,* opposition by the adverse party to an

Turn	Player	Action
0	Prosecutor	1
1	Defender	2
2	Prosecutor	3
3	Defender	4
4	Prosecutor	5
5	Defender	Objection
6	Judge	Sustained
7	Prosecutor	Close
8	Defender	Close
9	Jury	Deliberation

Table 2: Successive turns of the hearing.

argument considered as illegal[4]. Here, the defense lawyer utters an objection against Argument 5 because it is based on *hearsay*.
The validity of the objection is examined at **Turn 6**: the judge has to decide if the argument presented in Turn 4 is illegal (by referring to the protocol in force in this context). The judge has chosen to sustain the objection requested by the lawyer, which introduces the mechanism of removal. Indeed, an objection indicates that the targeted argument should not be taken into account anymore nor registered in the official report. Note that adding a new argument is not equivalent to the removal of an argument since addition increases the number of arguments, hence the complexity of the system. Moreover, adding an argument that attacks the illegal argument does not guarantee the rejection of the latter, especially through mechanisms, such as defense, that can lead the illegal argument to remain accepted. Removing the illegal argument thus ensures impossibility of taking this argument into account anymore. Still during Turn 6, the jury proceeds to the removal of the objected argument from his argumentation system, and both speakers are occulting this argument definitely.
Turns 7 to 9 are closing the hearing: none of the two speakers has any new argument to present, which they successively indicate by the action *"Close"*. The deliberation of the jury follows, in order to determine if the subject of the hearing (Argument 1) is accepted or not.

The result of this deliberation will be given in the following section after some reminders about argumentation theory and a formalization of the example.

3 Formal Framework

The work presented in this article uses the formal framework proposed by [10].

Definition 1 (Argumentation System)**.** *An* argumentation system *is a pair* $\langle \mathbf{A}, \mathbf{R} \rangle$*, where* $\mathbf{A}$ *is a finite nonempty set of arguments and* $\mathbf{R}$ *is a binary relation on* $\mathbf{A}$*, called* attack relation*. Let* $A, B \in \mathbf{A}$*,* $A\mathbf{R}B$ *means that* A *attacks* B*.* $\langle \mathbf{A}, \mathbf{R} \rangle$ *will be represented by a graph whose vertices are the arguments and whose arcs correspond to* $\mathbf{R}$*.*

The acceptable set of arguments ("extensions") are determined according to a given semantics whose definition is usually based on the following concepts:

Definition 2 (Conflict-free set, defense and admissibility)**.** *Let* $A \in \mathbf{A}$ *and* $\mathcal{S} \subseteq \mathbf{A}$*,*

- $\mathcal{S}$ *is* conflict-free *iff there does not exist* $A, B \in \mathcal{S}$ *such that* $A\mathbf{R}B$*.*
- $\mathcal{S}$ defends an argument A *iff each attacker of* A *is attacked by an argument of* $\mathcal{S}$*. The set of the arguments defended by* $\mathcal{S}$ *is denoted* $\mathcal{F}(\mathcal{S})$*;* $\mathcal{F}$ *is called the* characteristic function *of* $\langle \mathbf{A}, \mathbf{R} \rangle$*.*
- $\mathcal{S}$ *is an* admissible *set iff it is conflict-free and it defends all its elements.*

[4] Arguments illegality criteria are defined by the protocol governing the hearing and may evolve according to the context; nevertheless arguments that are fallacious, irrelevant or obtained by hearsay, are assumed to be illegal.

In this article, we restrict our study to the most traditional semantics proposed by [10]:

Definition 3 (Acceptability semantics)**.** *Let $\mathcal{E} \subseteq \mathbf{A}$,*

- *$\mathcal{E}$ is a* preferred extension *iff $\mathcal{E}$ is a maximal admissible set (with respect to set inclusion $\subseteq$).*
- *$\mathcal{E}$ is* the only grounded extension *iff $\mathcal{E}$ is the least fixed point (with respect to $\subseteq$) of the characteristic function $\mathcal{F}$.*
- *$\mathcal{E}$ is a* stable extension *iff $\mathcal{E}$ is conflict-free and attacks any argument not belonging to $\mathcal{E}$.*

The status of an argument is determined by its presence in the extensions of the selected semantics. For example, an argument can be "accepted sceptically" (resp. "credulously") if it belongs to all the extensions (resp. at least to one extension) and be "rejected" if it does not belong to any extension.

We now recall the definition given by [9] for the removal of an argument and its interactions:

Definition 4 (Removing an argument)**.** *Let $\langle \mathbf{A}, \mathbf{R} \rangle$ be an argumentation system.* Removing an argument $Z \in \mathbf{A}$ *interacting with other arguments is a change operation, denoted $\ominus_I^a$, providing a new argumentation system such that:*

$$\langle \mathbf{A}, \mathbf{R} \rangle \ominus_I^a Z = \langle \mathbf{A} \setminus \{Z\}, \mathbf{R} \setminus \mathcal{I}_z \rangle$$

where $\mathcal{I}_z = \{(Z, X) \mid (Z, X) \in \mathbf{R}\} \cup \{(X, Z) \mid (X, Z) \in \mathbf{R}\}$ is the set of interactions concerning Z[5].

The set of extensions of $\langle \mathbf{A}, \mathbf{R} \rangle$ is denoted by $\mathbf{E}$ (with $\mathcal{E}_1, \ldots, \mathcal{E}_n$ standing for the extensions). A change creates a new argumentation system $\langle \mathbf{A}', \mathbf{R}' \rangle$ represented by a graph $\mathcal{G}'$, whose set of extensions is denoted by $\mathbf{E}'$ (with $\mathcal{E}'_1, \ldots, \mathcal{E}'_m$ standing for the extensions). It is assumed that the change does not concern semantics, *i.e.*, the semantics remains the same after the change.

Note that if an argumentation system $\langle \mathbf{A}', \mathbf{R}' \rangle$ is obtained by removing an argument Z in the argumentation system $\langle \mathbf{A}, \mathbf{R} \rangle$, then $\langle \mathbf{A}, \mathbf{R} \rangle$ can be obtained by adding Z to $\langle \mathbf{A}', \mathbf{R}' \rangle$. The study of the duality of addition with respect to removal is left for future work, along with another kind of duality, evoked below, concerning the change properties.

A change operation has an impact on the structure of the set of extensions and thus on the status of particular arguments. The reader may refer to [9] for a presentation of these properties and for their detailed analysis in the case of the addition of an argument. Among all these properties, one may find for example the expansive change that occurs when the number of extensions remains the same, whereas each extension of $\mathcal{G}'$ includes strictly an extension of $\mathcal{G}$, and any extension of $\mathcal{G}$ is strictly included in an extension of $\mathcal{G}'$[6].

[5] In the symbol $\ominus_I^a$, the a stands for "argument" and I for "interactions", meaning that the removal concerns an argument and its interactions.

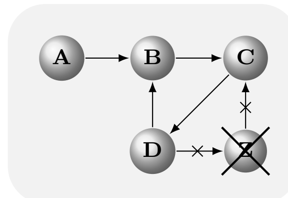
[6] The notation $\subset$ stands for strict set-inclusion.

Definition 5 (Expansive change)**.** *The change from $\mathcal{G}$ to $\mathcal{G}'$ is expansive*[7] *iff*

$$\begin{cases} (1) & \mathbf{E} \neq \varnothing, |\mathbf{E}| = |\mathbf{E}'|, \\ (2) & \forall \mathcal{E}'_j \in \mathbf{E}', \exists \mathcal{E}_i \in \mathbf{E}, \mathcal{E}_i \subset \mathcal{E}'_j \textit{ and} \\ (3) & \forall \mathcal{E}_i \in \mathbf{E}, \exists \mathcal{E}'_j \in \mathbf{E}', \mathcal{E}_i \subset \mathcal{E}'_j \end{cases}$$

Example 1.

Under preferred semantics, the change $\ominus_I^a Z$ with $\mathcal{I}_z = \{(Z, C), (D, Z)\}$ is expansive because $\mathbf{E} = \{\{A\}\}$ and $\mathbf{E}' = \{\{A, C\}\}$.



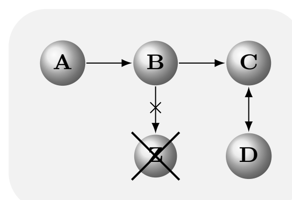
In the following, we introduce a property which could be considered as dual of the previous one. Indeed, a narrowing change occurs when the number of extensions remains the same while any extension of $\mathcal{G}'$ is strictly included in an extension of $\mathcal{G}$ and any extension of $\mathcal{G}$ includes strictly an extension of $\mathcal{G}'$.

Definition 6 (Narrowing change)**.** *The change from $\mathcal{G}$ to $\mathcal{G}'$ is narrowing iff*

$$\begin{cases} (1) & \mathbf{E} \neq \varnothing, |\mathbf{E}| = |\mathbf{E}'|, \\ (2) & \forall \mathcal{E}'_j \in \mathbf{E}', \exists \mathcal{E}_i \in \mathbf{E}, \mathcal{E}'_j \subset \mathcal{E}_i \textit{ and} \\ (3) & \forall \mathcal{E}_i \in \mathbf{E}, \exists \mathcal{E}'_j \in \mathbf{E}', \mathcal{E}'_j \subset \mathcal{E}_i \end{cases}$$

Example 2.

The change $\ominus_I^a Z$ with $\mathcal{I}_z = \{(B, Z)\}$ is narrowing under the preferred and stable semantics because $\mathbf{E} = \{\{A, C, Z\}, \{A, D, Z\}\}$ and $\mathbf{E}' = \{\{A, C\}, \{A, D\}\}$, and also under the grounded semantics because $\mathbf{E} = \{\{A, Z\}\}$ and $\mathbf{E}' = \{\{A\}\}$.



Back to the example

The previous definitions enable us to deal with our example. Indeed, the argumentation systems of the prosecutor and lawyer at the beginning of the hearing are represented by Turn 0 of Table 3 and, for each semantics recalled in Definition 3, the prosecutor's (resp. lawyer's) system admits only one extension $\mathcal{E} = \{1, 3, 5\}$ (resp. $\mathcal{E} = \{2, 4\}$). Note that it could be the case that each agent uses her own semantics since her reasoning (and thus her argumentation system and semantics) is personal. Nevertheless, in the current example, we consider that all agents use the same semantics because it seems natural to assume that the prosecutor and the lawyer know which semantics the jury is using and so they use the same one.

Let us note that at the beginning of the hearing, some existing attacks between arguments may not belong to any of the two speakers' argumentation systems; here, for example, the attack from 3 to 2, observable at Turn 1 of Table 3, was not present

[7] We give here a more restrictive definition than the one given by [9] (the third condition has been added). Strict inclusion is used in order to avoid overlap with other properties.

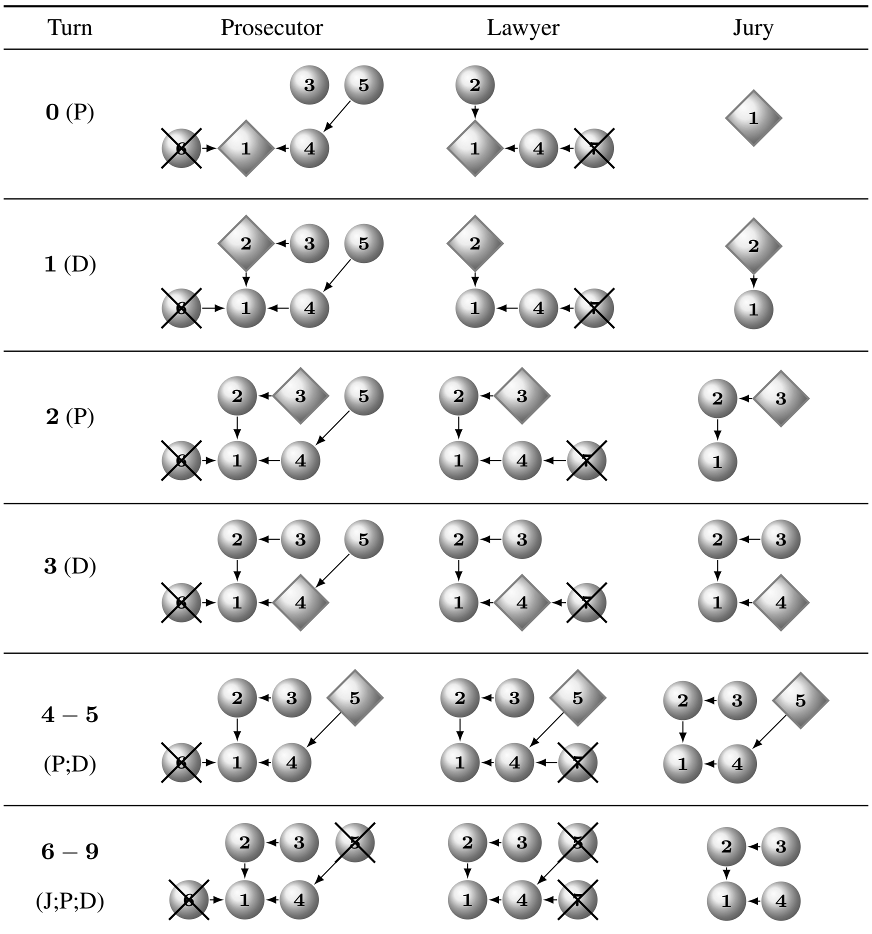


Table 3: Argumentation systems throughout the hearing (a diamond is surrounding the current turn argument)

for the prosecutor nor for the lawyer at Turn 0. Nevertheless, at each turn, the latter update their argumentation systems when they encounter a new argument that they did not know (the jury is supposed to ignore every argument at the beginning but after that, it proceeds in the same way). Table 3 shows the evolution of the various argumentation systems throughout the hearing.

At the deliberation time, the jury must decide upon the case of the hearing, namely Argument 1. For that, the jury should determine its status (accepted or rejected) by computing the extension(s) of its argumentation system with respect to a selected semantics. Whatever the semantics adopted by the jury among those recalled in Definition 3, its argumentation system has only one single extension $\mathcal{E} = \{3, 4\}$. Thus the jury can found the defendant "not guilty" since 1 do not belong to this extension.

Let us note that the removal of 5, to which the objection is related, has an influence on the acceptability of 1. Indeed, if the objection had been rejected, 5 could have defended 1 and ensured its presence in the extension, making it possible to convict the defendant. Moreover, let us notice that the lawyer was quite right to occult 7 because by doing so he has saved his client.

4 First steps towards a decision of removal

In this section, we study some properties characterizing the operation of removal. This kind of work may help a user to decide with full knowledge, in which situation and how to make a removal according to its strategic objectives. Note that the properties presented here are the first results of our study of the removal operation and will have to be deepened.

4.1 Some properties concerning "monotony"

In the framework of change in argumentation, "monotony" is related to the conservation of the extensions. More precisely, [9] defines it as follows: a change from $\mathcal{G}$ to $\mathcal{G}'$ satisfies monotony iff any extension of $\mathcal{G}$ is included into at least one extension of $\mathcal{G}'$. The following property gives the conditions under which a set of arguments that was jointly accepted remains so after the change.

Proposition 1 (Sufficient conditions for monotony/non-monotony)**.** *When removing an argument Z (according to Definition 4) under the preferred, stable or grounded semantics,*

- *if $\exists \mathcal{E}_i \in \mathbf{E}$ such that $Z \in \mathcal{E}_i$ then $\exists \mathcal{E}_j \in \mathbf{E}$ such that $\forall \mathcal{E}' \in \mathbf{E}'$ $\mathcal{E}_j \not\subseteq \mathcal{E}'$;*
- *if $\nexists \mathcal{E}_i \in \mathbf{E}$ such that $Z \in \mathcal{E}_i$ then $\forall \mathcal{E}_j \in \mathbf{E}$ $\exists \mathcal{E}' \in \mathbf{E}'$ such that $\mathcal{E}_j \subseteq \mathcal{E}'$.*

Since before the change, the removed argument may belong to an extension, it can be interesting to also consider the conditions for a "weak monotony" (*i.e.,* conservation of an extension without taking the removed argument into account), as defined by [6]:

Proposition 2. *When removing an argument Z, if Z does not attack any argument then*

- *$\forall \mathcal{E}$ preferred extension of $\mathcal{G}$, $\left\{ \begin{array}{l} \mathcal{E} \setminus \{Z\} \text{ is admissible in } \mathcal{G}' \text{ and thus} \\ \exists \mathcal{E}' \text{ preferred extension of } \mathcal{G}' \text{ s.t. } \mathcal{E} \setminus \{Z\} \subseteq \mathcal{E}' \end{array} \right.$*
- *$\forall \mathcal{E}$ stable extension of $\mathcal{G}$, $\mathcal{E} \setminus \{Z\}$ is a stable extension of $\mathcal{G}'$.*

Proposition 3. *When removing an argument Z under the preferred, stable or grounded semantics, if Z does not attack any argument of $\mathcal{G}$, then $\forall \mathcal{E}$ extension of $\mathcal{G}$ such that $Z \notin \mathcal{E}$, $\mathcal{E}$ is an extension of $\mathcal{G}'$.*

4.2 Some properties of the expansive change

An expansive change increases the size of the extensions and thus allows to obtain a larger number of arguments in the extensions. The following properties give the conditions under which a change cannot be expansive.

Proposition 4. *It is impossible to have an expansive change $\ominus_I^a$ under stable semantics.*

Proposition 5. *When removing Z under the preferred or grounded semantics, if this change is expansive then* $\begin{cases} Z \text{ does not belong to any extension of } \mathcal{G}, \\ \text{and } Z \text{ attacks at least one element of } \mathcal{G}. \end{cases}$

4.3 Some properties of the narrowing change

The narrowing change may be considered as dual of the expansive change since it decreases the size of the extensions. This can be desirable when one wishes to reduce the possibilities of argumentation of the adverse party. The following property merges three properties (one for each semantics) that provide a necessary condition for obtaining a narrowing change.

Proposition 6. *When removing Z under the preferred, stable or grounded semantics, if the change is narrowing then there exists one extension $\mathcal{E}$ of $\mathcal{G}$ s.t. $Z \in \mathcal{E}$.*

5 Discussion and conclusion

In this article, we studied a particular kind of change in argumentation: the removal of an argument and its interactions. First, we have presented an example coming from the legal world. This example illustrates the need to remove arguments in an argumentation system. In this application, at least two reasons are invoked in order to remove an argument: namely *objection* and *occultation*. After having pointed out the theoretical bases of argumentation, we have been able to model our example while showing the impact of these removals. Then we have studied some properties of the operation of removal.

Although the removal of an argument has been given little attention in the literature, at least three papers have focused on it. Namely, [6] has studied the removal of arguments and attacks (called "abstraction") in a quite particular case since the authors were interested in the situations where there exists only one single extension which one wants to preserve at identical after removal[8]. The results given by [6] make it possible to characterize the set of attacks to remove in order to "conserve the extension" under a given semantics (generally the grounded semantics). This paper also characterizes the set of interactions relating an argument to the argumentation system when the removal of this argument should respect the property of conservation. This may give birth to promising lines of research when developing further the study of the properties defined by [9] applied to removal.

[5] deals with the question (called *"enforcement"*) of how to modify an argumentation system so as to guarantee that a given set of arguments would belong to an extension. The modifications they considered are additions of arguments (with their associated interactions) and semantics switches. Results of impossibility and results concerning monotony are proposed. The authors stress that the removal of argument presents little interest for the problem considered, since it would offer the trivial solution of removing all the arguments that differs from those which one wants to guarantee. However, with a minimal change criterion, it could be interesting to compute the

[8]This "conservation of the extension" preserves all the arguments except for the removed argument.

minimal set of arguments to remove so as to guarantee a given set of arguments. It is one prospect of our work.

Let us note finally that [9] gives also examples of removal illustrating various properties of change. Besides, this same article shows that doing a parallel between addition in an argumentation system and revision in the sense of [1] (*AGM*) is not convenient (the formalisms are different and the concept of consistency which is central in the work of *AGM* does not have any equivalent in argumentation). For the same reasons, a parallel between removal of argument and the *AGM* contraction is not really meaningful (even if some concepts of *AGM* have inspired our work).

Several issues are to be specified and improved, we next describe some future orientations of our research.

- Many properties about change in argumentation are to be discovered or deepened, particularly for the removal change.
- Intuitively, it seems that an objected argument, and thus removed one, makes nevertheless its effect on the audience; the jury cannot instantaneously delete this argument from its mind and is likely to be influenced about it. It would be interesting to study the impact that such an argument can have on the preferences, or on the moral values, of the jury.
- The narrowing change seems to be a dual property of the expansive change. This concept of *duality* between operations and changes should be studied more deeply.
- In the illustrative example, we have seen that it may be beneficial not to reveal some arguments. One of our prospects is to characterize situations, in the way of [12], where it is crucial to select which arguments to reveal or to hide. This will allow us to develop strategies to maximize the chances that the audience accepts a specific argument. Furthermore, it would be interesting to focus particularly on the cases where the participants do not share the same semantics, and on the strategic choices which might arise consequently.

Acknowledgements

We would like to thank the reviewers for their help and valuable suggestions.

References

[1] C. Alchourrón, P. Gärdenfors, and D. Makinson. On the logic of theory change : partial meet contraction and revision functions. *Journal of Symbolic Logic*, 50:510–530, 1985.

[2] L. Amgoud and C. Cayrol. Inferring from inconsistency in preference-based argumentation frameworks. *International Journal of Automated Reasoning*, 29(2):125–169, 2002.

[3] L. Amgoud and F. Dupin de Saint Cyr. Extracting the core of a persuasion dialog to evaluate its quality. In *Proc. of ECSQARU*, volume LNAI 5590, pages 59–70. Springer-Verlag, 2009.

[4] L. Amgoud, N. Maudet, and S. Parsons. Modelling dialogues using argumentation. In *Proc. of ICMAS*, pages 31–38, 2000.

[5] R. Baumann and G. Brewka. Expanding argumentation frameworks: Enforcing and monotonicity results. In *Proc. of COMMA*, pages 75–86. IOS Press, 2010.

[6] G. Boella, S. Kaci, and L. van der Torre. Dynamics in argumentation with single extensions: Abstraction principles and the grounded extension. In *Proc. of ECSQARU (LNAI 5590)*, pages 107–118, 2009.

[7] G. Boella, S. Kaci, and L. van der Torre. Dynamics in argumentation with single extensions: Attack refinement and the grounded extension. In *Proc. of AAMAS*, pages 1213–1214, 2009.

[8] G. Brewka. Dynamic argument systems: A formal model of argumentation processes based on situation calculus. *Journal of Logic and Computation*, 11(2):257–282, 2001.

[9] C. Cayrol, F. Dupin de Saint Cyr, and M.-C. Lagasquie-Schiex. Change in Abstract Argumentation Frameworks: Adding an Argument. *Journal of Artificial Intelligence Research*, 38:49–84, 2010.

[10] P. M. Dung. On the acceptability of arguments and its fundamental role in nonmonotonic reasoning, logic programming and n-person games. *Artificial Intelligence*, 77(2):321–358, 1995.

[11] M. O. Moguillansky, N. D. Rotstein, M. A. Falappa, A. J. García, and G. R. Simari. Argument theory change through defeater activation. In *Proc. of COMMA 2010*, pages 359–366. IOS Press, 2010.

[12] I. Rahwan, K. Larson, and F. Tohmé. A characterisation of strategy-proofness for grounded argumentation semantics. In *Proc. of IJCAI 2009*, pages 251–256, 2009.

Annex: Proofs

Proof of Proposition 1. For the first item of this proposition, under any semantics, if there exists an extension $\mathcal{E} \in \mathbf{E}$ such that $Z \in \mathcal{E}$ then $\forall \mathcal{E}' \in \mathbf{E}'$, $\mathcal{E} \not\subset \mathcal{E}'$, since, the change being a suppression, Z does not belong to any extension of $\mathcal{G}'$. For the second item of this proposition, we consider each semantics separately:

Preferred semantics: let us suppose that Z does not belong to any extension of $\mathcal{G}$. We show that any extension $\mathcal{E}$ of $\mathcal{G}$ is admissible in $\mathcal{G}'$. Let $\mathcal{E} \in \mathbf{E}$:

- $\mathcal{E}$ is conflict-free in $\mathcal{G}$ and thus still conflict-free in $\mathcal{G}'$.

- Let us show that $\mathcal{E}$ defends its elements in $\mathcal{G}'$. If $X \in \mathcal{E}$ such that X is attacked by Y in $\mathcal{G}'$, then X is also attacked by Y in $\mathcal{G}$, but $X \in \mathcal{E}$, therefore it is defended by an argument T which attacks Y in $\mathcal{G}$. Since, we had assume that $Z \notin \mathcal{E}$, we know that $T \neq Z$, therefore $T \in A'$ and thus T attacks also Y in $\mathcal{G}'$. Thus, $\mathcal{E}$ defends X in $\mathcal{G}'$.

$\mathcal{E}$ is thus admissible. In conclusion, since $\mathcal{E}$ is admissible in $\mathcal{G}'$, it is included in one of the preferred extensions of $\mathcal{G}'$.
Stable semantics: let us suppose that Z does not belong to any extension of $\mathcal{G}$. We show that any stable extension $\mathcal{E}$ of $\mathcal{G}$ is a stable extension in $\mathcal{G}'$. Let $\mathcal{E} \in \mathbf{E}$:

- $\mathcal{E}$ is conflict-free in $\mathcal{G}$ and thus still conflict-free in $\mathcal{G}'$.

- If $Y \in A'$ and $Y \notin \mathcal{E}$ then $Y \in A$ and $Y \notin \mathcal{E}$. Since the extension $\mathcal{E}$ is stable in $\mathcal{G}$, $\mathcal{E}$ attacks Y in $\mathcal{G}$. Therefore, there exists $T \in \mathcal{E}$ such that T attacks Y. As we had assumed that $Z \notin \mathcal{E}$, we know that $Z \neq T$, so T attacks Y in $\mathcal{G}'$.

$\mathcal{E}$ is thus stable in $\mathcal{G}'$, hence $\mathcal{E} \in \mathbf{E}'$ so $\mathcal{E}$ is included in a stable extension of $\mathcal{G}'$.
Grounded semantics: *Case where* $\mathbf{E} = \{\{\}\}$: Let us proceed similarly to preferred semantics: it is known that $\mathbf{E}'$ is nonempty (since we are under the grounded semantics). Thus there exists $\mathcal{E}' \in \mathbf{E}'$. Since $\mathcal{E} = \varnothing \subseteq \mathcal{E}'$, hence the proposition is true.
Case where $\mathbf{E} \neq \{\{\}\}$: Let us suppose that Z does not belong to the grounded extension of $\mathcal{G}$. It is enough to show that the extension $\mathcal{E}$ of $\mathcal{G}$ is included in the grounded extension $\mathcal{E}'$ of $\mathcal{G}'$. We know, thanks to Definition 1, that the binary relation $\mathbf{R}$ is finite. However, according to [10], if $\mathbf{R}$ is finite then $\mathcal{E} = \bigcup_{I \geq 1} \mathcal{F}^i(\varnothing)$ and $\mathcal{E}' = \bigcup_{I \geq 1} \mathcal{F}'^i(\varnothing)$. Let us prove by induction on $i \geq 1$ that $\mathcal{F}^i(\varnothing) \subseteq \mathcal{F}'^i(\varnothing)$.

- $i = 1$: for any argument Y, if $Y \in \mathcal{F}(\varnothing)$ then Y is not attacked in $\mathcal{G}$. Removing Z does not change anything about that, Y is thus not attacked in $\mathcal{G}'$, and thus $Y \in \mathcal{F}'(\varnothing)$.

- Induction assumption (for $1 \leq I \leq p, \mathcal{F}^i(\varnothing) \subseteq \mathcal{F}'^i(\varnothing)$): Let $\mathcal{S} = \mathcal{F}^p(\varnothing)$ and $\mathcal{S}' = \mathcal{F}'^p(\varnothing)$. First of all, let us prove that $\mathcal{F}(\mathcal{S}) \subseteq \mathcal{F}'(\mathcal{S})$. Let $Y \in \mathcal{F}(\mathcal{S})$. By definition, $\mathcal{F}(\mathcal{S}) \subseteq \mathcal{E}$, therefore $Y \in \mathcal{E}$. If Y is attacked by X in $\mathcal{G}'$ then Y is attacked by X in $\mathcal{G}$. But since $Y \in \mathcal{F}(\mathcal{S})$, $\mathcal{S}$ defends Y, therefore $\exists T \in \mathcal{S}$ such that T attacks X in $\mathcal{G}$. By assumption, $Z \notin \mathcal{E}$, therefore $Z \notin \mathcal{S}$, therefore $T \neq Z$ and thus $T \in \mathbf{A}'$. Thus, $\mathcal{S}$ defends Y in $\mathcal{G}'$. Thus $Y \in \mathcal{F}'(\mathcal{S})$.

  We have just shown that $\mathcal{F}(\mathcal{S}) \subseteq \mathcal{F}'(\mathcal{S})$ and we also have, using the induction assumption, $\mathcal{S} \subseteq \mathcal{S}'$. Knowing that $\mathcal{F}'$ is monotonous (by definition), we have $\mathcal{F}(\mathcal{S}) = \mathcal{F}^{p+1}(\varnothing) \subseteq \mathcal{F}'(\mathcal{S}) \subseteq \mathcal{F}'(\mathcal{S}') = \mathcal{F}'^{p+1}(\varnothing)$. Therefore, $\mathcal{E} \subseteq \mathcal{E}'$. ■

Proof of Proposition 2. **Preferred semantics:** Let us suppose that $\mathcal{E} \setminus \{Z\}$ is not admissible in $\mathcal{G}'$. $\mathcal{E}$ being an extension of $\mathcal{G}$, there is no conflict in $\mathcal{E} \setminus \{Z\}$, therefore it exists an argument $Y \in \mathcal{E} \setminus \{Z\}$ such that Y is not defended by $\mathcal{E} \setminus \{Z\}$ in $\mathcal{G}'$. Thus there exists an argument $T \in \mathcal{G}'$ such that T attacks Y in $\mathcal{G}'$. Since we are removing Z, Z can be neither Y, nor T, therefore T also attacks Y in $\mathcal{G}$. Moreover, Y cannot be defended by Z in $\mathcal{G}$ since Z does not attack any argument, therefore Y is not defended by $\mathcal{E} \setminus \{Z\}$ in $\mathcal{G}$, and thus $\mathcal{E} \setminus \{Z\}$ is not admissible in $\mathcal{G}$, which contradicts our starting assumption. Thus, $\mathcal{E} \setminus \{Z\}$ is admissible in $\mathcal{G}'$ and is thus contained in a preferred extension of $\mathcal{G}'$.

Stable semantics: let Y be an argument such that $Y \notin \mathcal{E} \setminus \{Z\}$, and $Y \in \mathcal{G}'$. Then, $Y \neq Z$ and thus $Y \notin \mathcal{E}$. However, $\mathcal{E}$ is a stable extension of $\mathcal{G}$, therefore $\mathcal{E}$ attacks Y in $\mathcal{G}$. As Z does not attack any argument, $\mathcal{E} \setminus \{Z\}$ attacks also Y in $\mathcal{G}$. Besides since the change is a removal, $\mathcal{E} \setminus \{Z\}$ attacks also Y in $\mathcal{G}'$ and thus $\mathcal{E} \setminus \{Z\}$ is stable in $\mathcal{G}'$. ■

Lemma 1. *When removing an argument Z under the preferred semantics, if Z does not attack any argument, any extension of $\mathcal{G}'$ is admissible in $\mathcal{G}$.*

Proof of Lemma 1. Let $\mathcal{E}'$ be a preferred extension of $\mathcal{G}'$. $\mathcal{E}'$ is conflict-free in $\mathcal{G}'$ and thus in $\mathcal{G}$ also. If an argument $Y \in \mathcal{E}'$ is attacked in $\mathcal{G}$ by another argument X then $X \neq Z$ and $X \in \mathcal{G}'$, therefore Y is also attacked by X in $\mathcal{G}'$. $\mathcal{E}'$ is a preferred extension of $\mathcal{G}'$ which contains Y, hence $\mathcal{E}'$ attacks X in $\mathcal{G}'$, so in $\mathcal{G}$. ■

Proof of Proposition 3. **Preferred semantics:** let $\mathcal{E}$ be a preferred extension of $\mathcal{G}$. According to Proposition 2, there exists an extension $\mathcal{E}'$ of $\mathcal{G}'$ such that $\mathcal{E} \setminus \{Z\} \subseteq \mathcal{E}'$. However, $Z \notin \mathcal{E}$, therefore $\mathcal{E} = \mathcal{E} \setminus \{Z\} \subseteq \mathcal{E}'$. In addition, according to Lemma 1, since Z does not attack any argument, any extension of $\mathcal{G}'$ is admissible in $\mathcal{G}$, therefore there exists an extension $\mathcal{E}_i$ of $\mathcal{G}$ such that $\mathcal{E}' \subseteq \mathcal{E}_i$. Thus $\mathcal{E} \subseteq \mathcal{E}' \subseteq \mathcal{E}_i$. However, $\mathcal{E}$ is a maximal admissible set for set-inclusion in $\mathcal{G}$. Thus $\mathcal{E} = \mathcal{E}' = \mathcal{E}_i$. Thus, $\mathcal{E}$ is an extension of $\mathcal{G}'$.
Stable semantics: it is directly due to Proposition 2 and to the fact that $\mathcal{E} \setminus \{Z\} = \mathcal{E}$.
Grounded semantics: let $\mathcal{E}$ be the single grounded extension of $\mathcal{G}$ and let $Z \in \mathcal{G}$ be an argument such that $Z \notin \mathcal{E}$. According to Proposition 1, we get $\mathcal{E} \subseteq \mathcal{E}'$, where $\mathcal{E}'$ is the grounded extension of $\mathcal{G}'$.

It remains to establish that $\mathcal{E}' \subseteq \mathcal{E}$. For this purpose, we show that $\mathcal{E}$ is a fixpoint of $\mathcal{F}'$ and since $\mathcal{E}'$ is the least one, we have $\mathcal{E}' \subseteq \mathcal{E}$. Thus let us show that $\mathcal{F}'(\mathcal{E}) = \mathcal{E}$.

- First, let us prove that $\mathcal{F}'(\mathcal{E}) \subseteq \mathcal{E}$. Let $Y \in \mathcal{F}'(\mathcal{E})$ then $Y \neq Z$ and $\mathcal{E}$ defends Y in $\mathcal{G}'$. Since Z does not attack any argument, the only attackers of Y in $\mathcal{G}$ are those of $\mathcal{G}'$, therefore $\mathcal{E}$ defends Y in $\mathcal{G}$ and $Y \in \mathcal{F}(\mathcal{E}) = \mathcal{E}$.

- Conversely, let us show now that $\mathcal{E} \subseteq \mathcal{F}'(\mathcal{E})$. Let $Y \in \mathcal{E} = \mathcal{F}(\mathcal{E})$ then $\mathcal{E}$ defends Y in $\mathcal{G}$. We know that $Z \notin \mathcal{E}$ thus $Y \neq Z$. Since Z attacks no argument, $\mathcal{E}$ thus defends Y in $\mathcal{G}'$ and $Y \in \mathcal{F}'(\mathcal{E})$.

Thus $\mathcal{E} = \mathcal{F}'(\mathcal{E})$ and we have $\mathcal{E}' \subseteq \mathcal{E}$. In conclusion, $\mathcal{E}$ is an extension of $\mathcal{G}'$. ■

Proof of Proposition 4. Let us suppose that there exists an expansive suppression. It is thus assumed that $\mathbf{E} \neq \varnothing$, $|\mathbf{E}| = |\mathbf{E}'|$, and for any extension $\mathcal{E}'$ of $\mathcal{G}'$, there exists an extension $\mathcal{E}$ of $\mathcal{G}$ such that $\mathcal{E} \subset \mathcal{E}'$. Let us consider any extension $\mathcal{E}'_j$ of $\mathcal{G}'$ then there exists an extension $\mathcal{E}_i$ of $\mathcal{G}$ such that $\mathcal{E}_i \subset \mathcal{E}'_j$. Thus there exists an argument $Y \in \mathcal{E}'_j$ such that $Y \notin \mathcal{E}_i$. Let us note that $Y \in \mathcal{G}$ since we are in the case of the suppression of an argument. $\mathcal{E}_i$ being stable in $\mathcal{G}$, there exists an argument $T \in \mathcal{E}_i$ such that T attacks Y in $\mathcal{G}$. However, $\mathcal{E}_i \subset \mathcal{E}'_j$, therefore $T \in \mathcal{E}'_j$, and, by assumption, $Y \in \mathcal{E}'_j$. Thus T attacks Y in $\mathcal{G}'$ and thus $\mathcal{E}'_j$ is not conflict-free, which contradicts our starting assumption. ■

Proof of Proposition 5. The first item of this proposition comes directly from the definition of the expansive change and Proposition 1, both for preferred semantics or grounded semantics.

For the second item, **Preferred semantics:** let us suppose that there exists an expansive change and that Z does not attack any argument from $\mathcal{G}$. It is thus supposed that $\mathbf{E} \neq \varnothing$, $|\mathbf{E}| = |\mathbf{E}'|$ and for any extension $\mathcal{E}'$ of $\mathcal{G}'$, there exists an extension $\mathcal{E}$ of $\mathcal{G}$ such that $\mathcal{E} \subset \mathcal{E}'$. Due to the first item of Proposition 5, we know that Z does not belong to any extension of $\mathcal{G}$. If Z does not attack any argument of $\mathcal{G}$ then $\forall \mathcal{E} \in \mathbf{E}$, $\mathcal{E} \setminus \{Z\} = \mathcal{E}$ and, according to Proposition 2, $\mathcal{E}$ is an admissible set in $\mathcal{G}'$. Therefore, $\mathcal{E} \subseteq \mathcal{E}'$, where $\mathcal{E}'$ is a maximal admissible set of $\mathcal{G}'$. However, according to Lemma 1, since Z does not attack anything, $\mathcal{E}'$ is also an admissible set in $\mathcal{G}$, therefore $\mathcal{E}' \subseteq \mathcal{E}$ and thus $\mathcal{E} = \mathcal{E}'$, which contradicts our starting assumption.
Grounded semantics: let us suppose that there exists an expansive change and that Z attacks no argument of $\mathcal{G}$. According to item 1 of Proposition 5, Z does not belong to the grounded extension of $\mathcal{G}$ and, due to Proposition 3, it holds that $\mathcal{E} = \mathcal{E}'$, where $\mathcal{E}'$ is the grounded extension of $\mathcal{G}'$, which contradicts the expansive change. ■

Proof of Proposition 6. **Grounded semantics:** let us suppose that Z does not belong to any extension of $\mathcal{G}$. According to Proposition 1, we have $\mathcal{E} \subseteq \mathcal{E}'$, where $\mathcal{E}$ (resp. $\mathcal{E}'$) is the single grounded extension of $\mathcal{G}$ (resp. $\mathcal{G}'$), which is contradictory with the definition of the narrowing change. **Preferred and stable semantics:** let us suppose that Z does not belong to any extension of $\mathcal{G}$. According to Proposition 1, $\forall \mathcal{E} \in \mathbf{E}$, $\exists \mathcal{E}' \in \mathbf{E}'$, $\mathcal{E} \subseteq \mathcal{E}'$. However, the change being narrowing, $\mathbf{E} \neq \varnothing$ and $\mathbf{E}' \neq \varnothing$. Let $\mathcal{E}_i \in \mathbf{E}$ be an extension, thus it exists an extension $\mathcal{E}'_j \in \mathbf{E}'$ such that $\mathcal{E}_i \subseteq \mathcal{E}'_j$. In addition, still due to the definition of the narrowing change, there exists an extension $\mathcal{E}_k \in \mathbf{E}$ such that $\mathcal{E}'_j \subset \mathcal{E}_k$. We get $\mathcal{E}_i \subset \mathcal{E}_k$. In the case of the **Preferred semantics**, $\mathcal{E}_i$ is not a maximal admissible set and consequently, is not an extension of $\mathcal{G}$, which contradicts our assumption. In the case of the **Stable semantics**, each stable extension being also preferred, this is also impossible under the stable semantics. ■