Approximation of densities via Lloyd's algorithm

– M.Sc. proposal in machine learning and applied mathematics –

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Abstract

The approximation of a probability density by means of an optimization problem based on an optimal transport loss is deeply related to the quantization problem and is usually solved with LLoyd's algorithm, a generalization of k-means algorithm. However, as efficient as it is, this algorithm has drawbacks, the main one being that it only finds critical points, i.e. local minima (which can however be good ones under certain assumptions [2]). New algorithms have also been designed based on Lloyd's to generalize the quantization problem to the problem of projecting a given density on a set of structured measures as in [1], or to model block copolymer experiments [3]. The objective of this internship will be focused on Lloyd's algorithm, either on numerical aspects by proposing variations of the algorithm, or on a more theoretical analysis on its properties of local and global convergence and its variants in relation to the quantization problem and its structure (hexagonal patterns).

Keywords

Optimal transport, Lloyd's algorithm, approximation of probability measures.

Scientific environment & Period

The M.Sc. student will be co-advised by the following researchers within IRIT laboratory (UMR CNRS 5505) in Univeristy of Toulouse

— Elsa Cazelles, CNRS researcher, elsa.cazelles@irit.fr
— Édouard Pauwels, Assistant professor, edouard.pauwels@irit.fr

This internship shall take place in 2023. The precise starting and ending dates can be adjusted according to the availability of the selected candidate. Applicants are invited to send a detailed curriculum to the co-advisors.

In addition, **this internship may lead to a Ph.D.**

References

[1] F. de Gournay, J. Kahn, L. Lebrat, and P. Weiss. Optimal transport approximation of 2-dimensional measures. *SIAM Journal on Imaging Sciences*, 12(2), 762-787 (2019).

[2] Q. Mérigot, F. Santambrogio, and C. Sarrazin. Non-asymptotic convergence bounds for Wasserstein approximation using point clouds. *Advances in Neural Information Processing Systems*, 34, 12810-12821 (2021).

[3] D.P. Bourne, M. A. Peletier and S.M. Roper. Hexagonal patterns in a simplified model for block copolymers. SIAM Journal on Applied Mathematics, 74(5), 1315-1337. *Communications in Mathematical Physics*, 329(1), 117-140 (2014).