Knowledge, actions, and tests

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Abstract

We study a modal logic of knowledge and action, focussing on epistemic tests. We view an epistemic test as an action undertaken by an agent in order to establish whether a given formula is true. Such tests increase the knowledge of agents. We propose a semantics, and associate an axiomatics and a proof procedure.

1 Introduction

Imagine a robot that wants to open a door that might be locked. If the robot is cute enough, he starts by checking whether the door is effectively locked up. Such test actions are an important form of interaction. They are central e.g. in diagnosis in order to discriminate the possible fault configurations) or in decision under uncertainty.

Tests are a one-sided form of communication: the agent acquires knowledge about the environment, while that knowledge-gathering action does not change the environment. (There are two simplifying hypotheses we make here: first, we suppose that the environment of the agent doesn't change while the test is done; second, we suppose that tests do not change the environment.)

What do we test? We can test the physical objects of the world, e.g. a battery or a computer program. Here we are rather interested in tests of facts, i.e. to check *whether* the battery is empty, or to establish *that* the battery is empty.

These two actions are different: we may suppose at least in certain domains that an agent can always check whether a given fact is true or false, while we consider that the action of establishing a fact only succeeds if the fact we try to establish is indeed the case: an agent can only establish that a battery is empty if it really is; in the opposite case we consider that the action cannot be executed. Nevertheless, in the sequel we shall see that we can intertranslate these notions.

In this paper we restrict our analysis to the propositional case. We first present the standard logics of knowledge and action (sections 2 and 3). Then we integrate these two concepts in a single logic, and we investigate axiomatization and automated theorem proving (section 4).

2 Epistemic logic

The analysis of the notions of knowledge and belief in terms of possible states of affairs has been proposed by Hintikka [Hintikka, 1962, Fagin *et al.*, 1995]. We adopt S5 as our logic of knowledge. In order to simplify the reading of the formulas we suppose w.l.o.g. that there is only one agent.

The language of epistemic logic is constructed from a set of atomic formulas FML_0, the usual logical operators of classical logic, and the modal operator $\mathcal{K}$. An example of a formula is $\neg\mathcal{K}p \wedge \neg\mathcal{K}\neg p$. It is read "the agent neither knows p nor $\neg p$ and thus expresses the agent's ignorance w.r.t. the truth of p. The formula $p \wedge \neg\mathcal{K}p$ means that the agent ignores that p, while the formula $p \wedge \mathcal{K}\neg p$ means that the agent is wrong about p. That last formula is inconsistent if we view knowledge true belief, which is what we shall do here.

The semantics of epistemic logic is in terms of possible states. A model of S5 is a triple $M = \langle W, R_{\mathcal{K}}, V \rangle$ where

- W is a set of states (or possible worlds);
- $R_{\mathcal{K}}$ is an equivalence relation on W;
- V associates to each state a valuation: $V(w) \subseteq FML_0$; we often write V_w instead of $V(w)$.

We shall sometimes identify $R_{\mathcal{K}}$ with the function $R_{\mathcal{K}} : W \longrightarrow 2^W$ by stipulating $R_{\mathcal{K}}(w) = \{v : wR_{\mathcal{K}}v\}$.

Given a model $M = \langle W, V \rangle$, we define as usual truth in a state $w \in W$. In particular :

- $\models_{M,w} p$ if $p \in V_w$;
- $\models_{M,w} \mathcal{K}A$ if for every state $v \in R_{\mathcal{K}}(w)$ we have $\models_{M,v} A$.

It is part of the classical results in modal logics that the set of valid formulas of S5 is axiomatized by

MP	$\dfrac{A \ , \ A \ \rightarrow \ B}{B}$
N($\mathcal{K}$)	$\dfrac{A}{\mathcal{K}A}$
Class	The set of theorems of classical logic
K($\mathcal{K}$)	$(\mathcal{K}A \wedge \mathcal{K}(A \rightarrow C)) \ \rightarrow \ \mathcal{K}C$
T($\mathcal{K}$)	$\mathcal{K}A \rightarrow A$
4($\mathcal{K}$)	$\mathcal{K}A \rightarrow \mathcal{K}\mathcal{K}A$
5($\mathcal{K}$)	$\neg\mathcal{K}A \rightarrow \mathcal{K}\neg\mathcal{K}A$

3 Dynamic logic

There exists already a well-known logic containing a test operator, viz. dynamic logic [Harel, 1984]. To the presentation in the latter we prefer that of [Goldblatt, 1992] in terms of standard models, because it is more appropriate for our purposes.

The language of propositional dynamic logic PDL is constructed from a set of atomic formulas FML_0, the classical logic operators $\to, \wedge, \vee, \neg$, a set of atomic actions ACT_0, the action operators $\lambda, \cup, ;\, ?$, and the modal operator $[.]$.[1] An example of a formulas are $\neg p \wedge [p?]p$. We read the formula $[p?]q$ as "after establishing p, q is true", or "after checking that p, q is true".

$\alpha;\beta$ means "execute α and then β", and $\alpha \cup \beta$ means "choose nondeterministically between α and β, and then execute the chosen action".

We define the action $A??$ of checking *whether* A as an abbreviation of the complex action $A? \cup (\neg A)?$. This formally expresses that to check whether A is true amounts to nondeterministically choose between trying to establish that A and trying to establish that $\neg A$.

Remark Note that if we are only interested in tests of the type $A??$, formulas written using $??$ will explode exponentially if we expand the abbreviation. Therefore it is of interest to consider the other way round that $??$ is primitive. In this case we can define the formula $[A?]B$ to be an abbreviation of $[A??](A \to B)$.

Semantics is in terms of a transition system between states: a model is a triple $M = \langle W, \{R_\alpha : \alpha \in ACT\}, V\rangle$ where W is a set of states and V is a valuation as for epistemic logic, and

- each R_α is a relation between states : $R_\alpha \subseteq W \times W$ (called transition relation or accessibility relation).

As we did for epistemic logic, we shall sometimes view R_α as a function.

Given a model M as above, we define as usual the truth of complex formulas in a state, in particular :

- $\models_{M,w} [\alpha]A$ if for every state $v \in R_\alpha(w)$, $\models_{M,v} A$.

As we want the transition relations to reflect the intended meanings of complex actions, we restrict our attention to *standard models*, which satisfy

- $R_\lambda(w) = \{w\}$
- $R_{\alpha\cup\beta} = R_\alpha \cup R_\beta$
- $R_{\alpha;\beta} = R_\alpha \circ R_\beta$
- $R_{A?}(w) = (\text{if} \models_{M,w} A \text{ then } \{w\} \text{ else } \emptyset)$

The notion of validity is that of validity in the class of standard models.

We give a somewhat unusual axiomatization of the set of formulas of PDL that are valid in the class of standard models, in order to take profit of it lateron.

MP	$\dfrac{A\ ,\ A \to B}{B}$
N($[\alpha]$)	$\dfrac{A}{[\alpha]A}$
Class	The set of theorems of classical logic
K($[\alpha]$)	$([\alpha]A \wedge [\alpha](A \to B)) \ \to\ [\alpha]B$
Def(λ)	$[\lambda]A \ \leftrightarrow\ A$
Def(;)	$[\alpha;\beta]A \ \leftrightarrow [\alpha][\beta]A$
Def($\cup$)	$[\alpha \cup \beta]A \ \leftrightarrow ([\alpha]A \wedge [\beta]A)$
Id(?)	$[A?]A$
Exec(?)	$A \to \neg[A?]\neg A$
Pres(?)	$B \to [A?]B$

[1] To simplify we have dropped the iteration operator *.

The axioms Def(;), Def(λ) and Def($\cup$) can be viewed as formulating abbreviations of the respective action constructors. In the standard presentations of PDL, not only the former, but also the test operator ? is defined by

Def(?) $\quad [A?]B \ \leftrightarrow\ (A \to B)$

At least we obtain the same set of provable formulas:

Theorem 1 *Given the rest of the axiomatics, the axioms Id(?), Exec(?) and Pres(?) are equivalent to Def(?).*

A corollary of that result is that our axiomatization is complete w.r.t. standard PDL models.

4 An epistemic dynamic logic

It is the combination of epistemic logic and dynamic logic which will permit us to speak about tests done by agents in order to augment their knowledge. We call that logic epistemic dynamic logic EDL . In such a framework we must consider that actions are accomplished by agents. In consequence, after having established that A an agent knows that A, i.e. the formula $[A?]\mathcal{K}A$ should be valid.

But, while being a conservative extension of epistemic logic, our logic cannot be simply a conservative extension of dynamic logic. Indeed, suppose p is true, and suppose the agent ignores that p is true. Then the agent's ignorance cannot be preserved after establishing that p. Formally, this means that the instance

$$\neg\mathcal{K}p \to [p?]\neg\mathcal{K}p$$

of Pres(A?) should not be valid.

In the sequel we shall make an important *restriction*: we shall suppose that all complex actions are constructed from tests. This hypothesis will allow us to simplify the models and the completeness proof. It will be relaxed in future work.

4.1 Language

We combine the languages of epistemic logic and dynamic logic. Actions and complex formulas are defined by mutual recursion in a way similar to dynamic logic. An example of formula is $\neg\mathcal{K}p \wedge \neg\mathcal{K}\neg p \wedge [p?]\mathcal{K}p$.

As said above, we suppose in the rest of the paper that $ACT_0 = \emptyset$.

We respectively note ACT and *FML* the set of actions and formulas thus defined. We say that a formula from *FML* is *objective* if it contains no occurrence of $\mathcal{K}$.

4.2 Semantics

Without surprise, EDL -models are combinations of S5 and PDL models: a model is a 4-tuple $M = \langle W, R_\mathcal{K}, \{R_\alpha : \alpha \in ACT\}, V\rangle$ where W and V are as before. $R_\mathcal{K}$ is an equivalence relation as for epistemic logic, and the R_α are transition relations as for dynamic logic. Moreover M must satisfy

- if $R_\alpha(w) \neq \emptyset$ then $R_\mathcal{K} \circ R_\alpha = R_\alpha \circ R_\mathcal{K}$.

This condition expresses that the agent is aware of his actions: if A is true and it is possible for him that action $A?$ results in some state v, then v is possible for him after the execution of $A?$, and vice versa.

Truth in a state $w \in W$ is defined as before. What differs is the notion of a *standard model*: as before it must satisfy that

- $R_\lambda(w) = \{w\}$
- $R_{\alpha;\beta} = R_\alpha \circ R_\beta$
- $R_{\alpha\cup\beta} = R_\alpha \cup R_\beta$

but the conditions for ? are slightly weaker than that for PDL:

- if $wR_{A?}u$ then $V_w = V_u$
- if $\models_{M,w} \neg A$ then $R_{A?}(w) = \emptyset$

The first condition expresses that a test has no effect on the physical world, while the second condition says that the action of establishing that A can only be executed if A holds.

4.3 Axiomatization

Now our extensive presentation of dynamic logic turns out to be useful. Indeed, the PDL axiom Def($A?$) is not valid, only $[A?]B \rightarrow (A \rightarrow B)$ is. The axioms Id($A?$), Exec($A?$) and Pres($A?$) allow us to fine-tune: Id($A?$) and Exec($A?$) are valid, while Pres($A?$) must be restricted.

We give the following axiomatization of EDL :

MP	$\dfrac{A\ ,\ A \rightarrow B}{B}$
N($\mathcal{K}$)	$\dfrac{A}{\mathcal{K}A}$
N($[\alpha]$)	$\dfrac{A}{[\alpha]A}$
Class	The set of theorems of classical logic
K($\mathcal{K}$)	$(\mathcal{K}A \wedge \mathcal{K}(A \rightarrow C)) \rightarrow \mathcal{K}C$
T($\mathcal{K}$)	$\mathcal{K}A \rightarrow A$
4($\mathcal{K}$)	$\mathcal{K}A \rightarrow \mathcal{K}\mathcal{K}A$
5($\mathcal{K}$)	$\neg\mathcal{K}A \rightarrow \mathcal{K}\neg\mathcal{K}A$
K($[\alpha]$)	$([\alpha]A \wedge [\alpha](A \rightarrow C)) \rightarrow [\alpha]C$
Def(λ)	$[\lambda]A \leftrightarrow A$
Def(;)	$[\alpha;\beta]A \leftrightarrow [\alpha][\beta]A$
Def($\cup$)	$([\alpha \cup \beta]A \leftrightarrow ([\alpha]A \wedge [\beta]A)$
Id(?)	$[A?]A$
Exec(?)	$A \rightarrow \neg[A?]\neg A$
Pres(?)	$C \rightarrow [A?]C$ if C is an objective formula
Det(?)	$\neg[A?]C \rightarrow [A?]\neg C$
Perm(?, $\mathcal{K}$)	$A \rightarrow ([A?]\mathcal{K}C \leftrightarrow \mathcal{K}[A?]C)$

The axioms Id(?), Exec(?) and Pres(?) are as before. The axiom Det(?) (which is a theorem in PDL) is added here explicitly. Perm(?, $\mathcal{K}$) relates the knowledge of the agent before and after the test.

It follows from Id(?), N($\mathcal{K}$), and Perm(?, $\mathcal{K}$) that the agent does tests consciously, i.e. $[A?]\mathcal{K}A$.

Property 1 *The following equivalences are theorems of EDL .*

1. $[A?]\mathcal{K}C \leftrightarrow (A \rightarrow \mathcal{K}[A?]C)$
2. $[A?]\neg C \leftrightarrow (A \rightarrow \neg[A?]C)$
3. $[A?](C_1 \wedge C_2) \leftrightarrow ([A?]C_1 \wedge [A?]C_2)$
4. $[A?](C_1 \vee C_2) \leftrightarrow ([A?]C_1 \vee [A?]C_2)$
5. $[A?]C \leftrightarrow (A \rightarrow C)$ *if C is an objective formula*
6. $A \leftrightarrow \neg[A?]\bot$

The proof of these equivalences is straightforward.

Note that although the formula $\mathcal{K}B \rightarrow [A?]\mathcal{K}B$ seems to be a theorem at first glance (expressing something like "knowledge is preserved under tests") this is not the case. This is due to the negative introspection axiom 5($\mathcal{K}$).

We postpone the completeness proof, and consider first of all a method of automated theorem proving for our logic.

4.4 Automated theorem proving

We reduce in this section the problem of proving theorems in EDL to that of proving theorems in the standard modal logic S5. The reduction is done by rewrite rules.

Indeed, a glance at the four first equivalences of the above property shows us that applying these equivalences from the left to the right we can 'push down' the modal operator of test through all the other connectives $\mathcal{K}, \neg, \wedge, \vee$. When $[A?]$ reaches an objective formula then we can apply $[A?]C \leftrightarrow (A \rightarrow C)$, and thus eliminate one modal operator of test from the formula. (We suppose here that we start with an operator $[A?]$ with no other $[B?]$ in its scope, and that the other action construction operators have been eliminated using axioms Def(λ), Def(;), and Def($\cup$).)

Iterating these rewrite steps we can obtain formulas without occurrences of test operators.

Theorem 2 *Let A be a formula of EDL . Then there exists a formula A' without test operators such that $A \leftrightarrow A'$ is a theorem of EDL .*

4.5 Soundness and completeness

Each of the axioms that we have given is valid, and the inference rules preserve validity. Hence our axiomatics is sound.

The above theorem gives us completeness.

Theorem 3 *Let A be a formula of EDL . A is EDL -valid iff A is a EDL -theorem.*

Proof Let A be consistent. According to the preceeding theorem there exists a formula A' without test operators such that $A \leftrightarrow A'$ is a EDL -theorem. Hence A' is consistent. Now A' is in the language of S5, and given that the axiomatics of EDL contains that of the epistemic logic S5, A' is as well consistant in S5. Via the completeness of S5 there must therefore exist a S5-model containing a state w where A' is true. Then from that model it is straightforward to extend that model to a EDL -model where A' is true in w. Finally, given that (due to soundness) the equivalences that we have used to rewrite formulas are valid, that EDL -model must also satisfy A in w.

4.6 Complexity

The fragment of EDL without nested tests has an interesting complexity. In this case our rewriting procedure is a polynomial transformation into S5. The problem of deciding whether a given S5-formula is a theorem is coNP-complete: it follows that the decision problem for the fragment of EDL without nested tests is also coNP-complete.

5 Related work

A lot of logics of knowledge and action exist. Closest to ours is the work of Gerbrandy and Groeneveld [Gerbrandy, 1997, Gerbrandy et Groeneveld, 1997, Groeneveld, 1995]. Their Dynamic Epistemic Logic has two sorts of test, the first of which is noted $?A$[2] and is the standard dynamic logic test: it "succeeds [...] when A is true, and fails otherwise". Consequently $[?A]C$ is an abbreviation of $A \to C$. The second one is noted $U\alpha$ and "corresponds to [the] agent [...] learning that program α has been executed". (We have slightly adapted notation.) This means that agents act *a priori* unconsciously and must explicitly learn about the executions of their actions. While this might be considered to be unnatural (in particular for artificial agents), it leaves more flexibility than our language e.g. to speak about agent i learning that agent j learned that A has been tested (expressible here as $U_a U_b ?A$).

$U_b?A$ is similar to our $A?$. More precisely, our logic can be mapped into Gerbrandy's logic of [Gerbrandy, 1997]: our action $A?$ can be translated into their $?A; U?A$.

In [Gerbrandy, 1997] there is given an axiomatics, which is similar to ours. Nevertheless there are subtle differences. We have already mentioned the first one: there is a non-epistemic test $?A$ supplementing the epistemic test $U?A$.

The second main difference is that there, the logic of knowledge is K, while ours is S5. Hence there are no axioms $\mathrm{T}(\mathcal{K})$, $4(\mathcal{K})$, and $5(\mathcal{K})$. It seems to be problematic to add these axioms to the logic. This will be detailed after our next point.

The third main difference is that there, instead of axiom Exec(?) $A \to \neg[A?]\neg A$ there is an equivalence

$$A \leftrightarrow \neg[A?]\neg A$$

(axiom 5 in [Gerbrandy, 1997]). This means that an agent can always successfully learn about the execution of some action.[3] This leads to difficulties at least if we suppose that the epistemic notion under concern satisfies a consistency requirement as expressed by the modal axiom $\mathrm{D}(\mathcal{K})$ $\mathcal{K}A \to \neg\mathcal{K}\neg A$ (that is a consequence of axiom $\mathrm{T}(\mathcal{K})$). Indeed, suppose p is an atom. Then $[U?p]\mathcal{K}p$ is derivable in their logic, as well as $\mathcal{K}\neg p \to [U?p]\mathcal{K}\neg p$. But from these two we can derive $\mathcal{K}\neg p \to [U?p]\mathcal{K}(p \wedge \neg p)$. While in our logic this means that the test action fails, in theirs the test $U?p$ always succeeds, and therefore axiom $\mathrm{D}(\mathcal{K})$ cannot be added to their logic as it stands.

Finally a more technical difference are the respective completeness proofs. While ours basically uses a reduction to a modal logic without tests, theirs is a (much longer) Henkin type proof. Nevertheless, our technique also applies to their logic, and permits thus to obtain a much simpler proof. To wittness, the K-axiom for $[U\alpha]$ together with the above equivalence $A \leftrightarrow \neg[U\alpha]\neg A$ permit to pass the modal operator $[U\alpha]$ through conjunction, disjunction, and negation, and their axiom 7 $[U\alpha]\mathcal{K}A \leftrightarrow \mathcal{K}[\alpha][U\alpha]A$ permits to pass through the epistemic operator $\mathcal{K}$. Finally their axiom 6 permits to eliminate the $[U\alpha]$ operator from formulas. Thus one can follow the same line of reasoning as in our completeness proof.

In a series of articles Segerberg has developped a logic of belief and action called Doxastic Dynamic Logic (DDL) [Segerberg, 1995, Segerberg,]. There are three types of modalities $+A$, $-A$, and $*A$ the first of which corresponds to our $A?$. He discusses axioms for $+A$ that are similar to ours, but nevertheless closer to Gerbrandy and Groeneveld's work. To witness, he also considers that tests are always executable and deterministic, i.e. he has the axiom $[+A]\neg C \leftrightarrow \neg[+A]C$ (his axiom 13), as well as a preservation axiom in terms of equivalence (his axiom 10). Therefore our above remarks also apply to this approach.

Another line of research has been developped in the AI field of reasoning about actions around the concept of knowledge gathering actions [Scherl et Levesque, 1993, Levesque, 1996, Lakemeyer et Levesque, 1998]. We here focus on the latter approach of Lakemeyer and Levesque. The logic AOL proposed there has similarities to our EDL . The main difference is that our logic does not contain the concept of only knowing. To wittness we consider an example given in their paper. "Suppose we have a robot that knows nothing about the initial state

[2]The authors consider several agents and groups of agents. We abstract from that here.

[3]This makes it also possible to write the preservation axiom Pres(?) as an equivalence.

of the environment, but that there is asensing action, reading a sonar, which tells the robot when it is getting close to a wall." Let us read the atomic formulas c and s respectively as 'the robot is close to the wall' and 'the sonar works', and let us interpret mc and ma respectively as the atomic actions of moving closer and moving away from the wall. In our language (allowing actions other than tests), what they then want to prove is

1. $[c??](\mathcal{K}c \vee \mathcal{K}\neg c)$
2. $\mathcal{K}w \to \mathcal{K}[c??](\mathcal{K}c \vee \mathcal{K}\neg c)$
3. $\mathcal{K}\neg c \to [ma](\mathcal{K}\neg c)$
4. $\mathcal{K}\neg c \to [mc](\neg\mathcal{K}c \wedge \neg\mathcal{K}\neg c)$

It is only the last formula that requires the non-monotonic only knowing notion.

6 Conclusion

We have defined a logic of knowledge and action EDL , to which we have associated an automated theorem proving procedure.

As we have noted in section 3, if we are only interested in tests of the type $A??$, formulas written using $??$ will explode exponentially if we expand the abbreviation $A??$ to $A? \cup (\neg A)?$. It is nevertheless possible to give a polynomial reduction into S5 similar to that for tests of the type $A?$, which makes that complexity of theoremhood stays within co-NP.

We plan to continue that work in two directions.

First, our logic allows to reason about the evolution of knowledge by tests, but it does not allow planning of test sequences. This might be achieved in a way similar to our approach in [Castilho *et al.*, 1997b, Castilho *et al.*, to appear].

Second, our actions being restricted to tests (and their sequential and nondeterministic composition), our aim is to relax that restriction. This will probably require to move from a rewriting-based proof procedure towards a semantic tableaux procedure. Here we shall also make use of previous work [Castilho *et al.*, 1997a].

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