

# Review of “Dynamic Epistemic Logic”\*

Andreas Herzig

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The problem of how to extend epistemic logic (EL) in order to allow for reasoning about knowledge and belief in dynamic contexts gained increasing attention during the last 30 years in the fields of artificial intelligence, theoretical computer science and philosophical logic. EL is a branch of modal logic that was designed in the 60ies by Hintikka in order to study reasoning about knowledge and belief. It allows to model static situations, where we do not have to reason about what holds at different points of time, or about both what holds before and after an event or action.

Several approaches were explored to ‘make EL walk’, depending on the field.

First, *theoretical computer science people* mainly worked on combinations of EL and linear-time temporal logic. They proposed a model based on runs (complete sequences of states) and epistemic accessibility relations between runs. They identified the main dichotomies concerning the interaction between knowledge and time, such as: synchronous vs. asynchronous time; perfect recall or no; partial vs. full observability; etc. A complete account of the state of the art in 1995 can be found in the book by Fagin et al. [8].

Second, *artificial intelligence people* worked on combinations of EL and propositional dynamic logic (PDL), as well as on rivals of PDL such as situation calculus, fluent calculus and event calculus. Just as for the non-epistemic case, the approaches concentrated on what might be called practical problems, most importantly the so-called *frame problem*: how can we describe the effects of actions in terms of logical theories without having to describe all its non-effects, and how can we reason efficiently under such descriptions? The first combination of EL and PDL was proposed by Moore [15]. Scherl and Levesque’s so-called epistemic solution to the frame problem extended Reiter’s solution to the frame problem to EL, by defining epistemic accessibility relations on the set of situations (initial sequences of states). A complete account of the 2001 state of the art can be found in Reiter’s book [17].

Finally, *philosophical logic people* mainly worked on the evolution of belief when the agent learns that some proposition  $\varphi$  is true. Such a formula  $\varphi$  is called the *epistemic input*. To ‘learn that  $\varphi$  is true’ is a particular kind of event,

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\*Hans van Ditmarsch, Wiebe van der Hoek, Barteld Kooi, “Dynamic Epistemic Logic”, Springer Verlag, Synthese Library 337, 2007.

that might be called ‘purely epistemic’: there is no change in the physical world, but only in the agent’s mental state. While most of the preceding theoretical computer science and artificial intelligence approaches only consider the notion of knowledge, Alchourrón, Gärdenfors and Makinson (AGM) focussed on belief and the various kinds of change it may undergo [2]. They identified three kinds of purely epistemic events:

- belief expansion by  $\varphi$ ;
- belief contraction by  $\varphi$ ;
- belief revision by  $\varphi$ .

A complete account of the 1988 state of the art can be found in Gärdenfors’s book [10]. Shortly later, Katsuno and Mendelzon studied a fourth kind of event that takes the same logical form (viz. a proposition) but is not purely epistemic: belief updates, where an agent learns that  $\varphi$  is true, and also learns that some event has occurred after which  $\varphi$ ’s truth is observed.<sup>1</sup> Until the mid-1990ies, researchers in belief revision only considered a single agent, call it  $Y$  (like ‘you’), and propositional input  $\varphi$ :  $\varphi$  was not allowed to be about another agent’s beliefs. Such a restriction allows to drop the epistemic operator, and to only consider propositional belief bases  $K$  (alias sets of propositional formulas, representing the belief state of agent  $Y$ ), that are revised by a propositional input formula  $\varphi$ . In his paper “Two traditions in the logic of belief: bringing them together”, Segerberg proposed to integrate the AGM approach with doxastic logic, resulting in dynamic doxastic logic [18, 19, 20]. Aucher recently analyzed these different viewpoints in more depth: the AGM approach takes an internal point of view, while EL takes an external point of view [3].

Independently, efforts were under way to combine EL with dynamic logic modalities, among others by Groenendijk and Stokhof and by Veltman [25]. Crucially, these approaches did not consider abstract events (as available in PDL) but what may be called ‘concrete’, purely epistemic events: events of agents learning that some formula  $\varphi$  is true. In the simplest case such an event is *public*: all agents simultaneously learn that  $\varphi$  is true [16]. More complex cases of so-called private announcements were then explored [11, 24]. The efforts culminated in Baltag, Moss and Solecki’s 1998 TARK paper [7, 6] proposing possible events models:<sup>2</sup> just as Hintikka’s models have worlds that are (statically) possible for an agent, Baltag et al.’s event models allow to model events that are possible for the agent. The outcome situation resulting from such a complex event is modelled by a restricted product of the Hintikka model and that event

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<sup>1</sup>To illustrate Katsuno and Mendelzon’s distinction consider the case of a dictionary: when it is revised then flaws of the preceding version are corrected; when it is updated then either a new word appeared in the ‘real world’, or the meaning of an existing word shifted since the preceding version of the dictionary.

<sup>2</sup>Somewhat misleadingly these models are often called action models in the literature, although there is no notion of agency involved here.

model. The term *dynamic epistemic logics* (DEL) was coined to designate the family of such logics having event models.<sup>3</sup>

The 2007 book “Dynamic Epistemic Logic” by van Ditmarsch, van der Hoek and Kooi is written from the perspective of dynamic epistemic logics, and situates it w.r.t. the other ‘dynamifications’ of epistemic logic that I have mentioned. In the sequel I will briefly summarize the chapters of the book.

The first chapter introduces the subject, and the second chapter provides the necessary background: epistemic and doxastic logic. The third chapter recalls AGM belief revision, including Segerberg’s dynamic doxastic logic. The remaining chapters are about DEL. The fourth chapter is about Plaza’s public announcement logic, both without and with common knowledge. The presentation is both semantic and axiomatic, and is illustrated by several examples, such as a complete analysis of the dynamics of the ‘muddy children’ example, and the intriguing ‘Russian cards’ example. The fifth chapter introduces Gerbrandy’s and van Ditmarsch’s event languages that allow to go beyond public announcement logic and describe private announcements. Again, the concepts are nicely illustrated by means of card games. The sixth chapter finally contains event models as invented by Baltag, Moss and Solecki. It presents the restricted product construction and its axiomatization. By means of an example the authors sketch the relation between these models and Gerbrandy and van Ditmarsch’s event languages of the fifth chapter, but the precise relationship remains to be established. The last two chapters collect more technical material that the authors rightly put there. Chapter 7 contains relevant completeness proofs, while chapter 8 collects results about expressivity. Finally, solutions to exercises are collected in the annex.

To sum it up, the book exhaustively establishes the 2006 state of the art in DEL, and provides relevant pointers towards the artificial intelligence and theoretical computer science literature. Such a collection did not exist up to now for that dynamic extension of EL, and the book therefore fills a gap. As such it is a precious and highly welcome tool for researchers in all these fields. My only worry is that it was probably written a bit too early: DEL is a vibrant and rapidly evolving field, and several contributions and proposals were made at the TARK, LOFT, AAMAS, KR and IJCAI conferences in 2006, 2007 and 2008. Here are a couple of relevant examples:

- Lutz characterized the complexity of public announcement logic PAL [14].
- Van Benthem and Pacuit exploited links with temporal extensions of EL [23, 22].
- Van Benthem, Liu, Roy and others recently extended DEL in order to account not only for belief change, but also for preference change [21, 12, 13].

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<sup>3</sup>It has to be noted that the term update has different meanings in the DEL literature and in the belief revision literature. The DEL update of a belief state corresponds to an expansion in belief revision. While the DEL updates implement purely epistemic events that only change the agents’ beliefs and leave the ‘physical’ world unchanged, belief updates à la AGM on the contrary correspond to a change in the world.

- Balbiani, Baltag and others extended DEL with modal operators similar to Pauly’s coalition logic allowing to reason about the epistemic change that might be brought about by announcements [4, 9, 1, 5];

While the book briefly mentions some of the corresponding papers (mostly referring to preliminary versions), several of them would deserve a more in-depth presentation.

To sum it up, even if the book came a little bit too early, it nevertheless provides the indispensable background for the recent developments, awaiting a more stable ‘Handbook of DEL’ that might be ripe in 5 years or so.

## References

- [1] Thomas Ågotnes and Hans van Ditmarsch. Coalitions and announcements. In *Proc. AAMAS’08*. ACM Press, 2008.
- [2] Carlos Alchourrón, Peter Gärdenfors, and David Makinson. On the logic of theory change: Partial meet contraction and revision functions. *J. of Symbolic Logic*, 50:510–530, 1985.
- [3] Guillaume Aucher. *Perspectives on belief and change*. PhD thesis, Université de Toulouse, July 2008.
- [4] Philippe Balbiani, Alexandru Baltag, Hans van Ditmarsch, Andreas Herzig, Tomohiro Hoshi, and Tiago de Lima. What can we achieve by arbitrary announcements? A dynamic take on Fitch’s knowability. In Dov Samet, editor, *Proceedings of the eleventh conference Theoretical Aspects of Rationality and Knowledge (TARK)*, pages 42–51, Brussels, Belgium, June 25 – 27 2007. Presses universitaires de Louvain.
- [5] Philippe Balbiani, Alexandru Baltag, Hans van Ditmarsch, Andreas Herzig, Tomohiro Hoshi, and Tiago de Lima. What can we achieve by arbitrary announcements? A dynamic take on Fitch’s knowability. *Review of Symbolic Logic*, to appear.
- [6] Alexandru Baltag and Lawrence S. Moss. Logics for epistemic programs. *Synthese*, 139(2):165–224, 2004.
- [7] Alexandru Baltag, Lawrence S. Moss, and Slawomir Solecki. The logic of public announcements, common knowledge, and private suspicions. In *Proc. TARK’98*, pages 43–56. Morgan Kaufmann, 1998.
- [8] Ronald Fagin, Joseph Y. Halpern, Yoram Moses, and Moshe Y. Vardi. *Reasoning about knowledge*. MIT Press, 1995.
- [9] Tim French and Hans P. van Ditmarsch. Undecidability for arbitrary announcement logic. In *Proc. AiML*. College Publications, 2008.

- [10] Peter Gärdenfors. *Knowledge in Flux: Modeling the Dynamics of Epistemic States*. MIT Press, 1988.
- [11] Jelle Gerbrandy. *Bisimulations on Planet Kripke*. PhD thesis, University of Amsterdam, 1999.
- [12] Fenrong Liu. *Changing for the better: preference dynamics and agent diversity*. PhD thesis, University of Amsterdam, ILLC dissertation series, 2008.
- [13] Fenrong Liu. *Thinking before acting: intentions, logic, rational choice*. PhD thesis, University of Amsterdam, ILLC dissertation series, 2008.
- [14] Carsten Lutz. Complexity and succinctness of public announcement logic. In *Proceedings of the Fifth International Joint Conference on Autonomous Agents and Multiagent Systems (AAMAS'06)*, pages 137–144, 2006.
- [15] Robert C. Moore. A formal theory of knowledge and action. In J.R. Hobbs and R.C. Moore, editors, *Formal Theories of the Commonsense World*, pages 319–358. Ablex, Norwood, NJ, 1985.
- [16] Jan A. Plaza. Logics of public communications. In M. L. Emrich, M. Z. Pfeifer, M. Hadzikadic, and Z. W. Ras, editors, *Proc. 4th Int. Symposium on Methodologies for Intelligent Systems*, pages 201–216, 1989.
- [17] R. Reiter. On knowledge-based programming with sensing in the situation calculus. *ACM Trans. on Computational Logic*, 2001.
- [18] Krister Segerberg. Belief revision from the point of view of doxastic logic. *Bulletin of the IGPL*, 3:534–553, 1995.
- [19] Krister Segerberg. Two traditions in the logic of belief: bringing them together. Technical Report 9, Uppsala Prints and Preprints in Philosophy, 1996.
- [20] Krister Segerberg. Two traditions in the logic of belief: bringing them together. In Hans Jürgen Ohlbach and Uwe Reyle, editors, *Logic, Language and Reasoning: essays in honour of Dov Gabbay*, volume 5 of *Trends in Logic*, pages 135–147. Kluwer Academic Publishers, Dordrecht, 1999.
- [21] Johan van Benthem. Dynamic logic of preference upgrade. *Journal of Applied Non-Classical Logics*, 17(2):157–182, 2007.
- [22] Johan van Benthem, Jelle Gerbrandy, and Eric Pacuit. Merging frameworks for interaction: DEL and ETL. In Dov Samet, editor, *Proceedings of Theoretical Aspects of Rationality and Knowledge (TARK 2007)*. Presses universitaires de Louvain, 2007.
- [23] Johan van Benthem and Eric Pacuit. The tree of knowledge in action: Towards a common perspective. In *Advances in Modal Logic*, pages 87–106, 2006.

- [24] Hans P. van Ditmarsch. *Knowledge games*. PhD thesis, Groningen University, ILLC Dissertation Series, Grafimedia, 2000.
- [25] Frank Veltman. Defaults in update semantics. *J. of Philosophical Logic*, 25:221–261, 1996.