Regression in Modal Logic

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ABSTRACT. *In this work we propose an encoding of Reiter's Situation Calculus solution to the frame problem into the framework of a simple multimodal logic of actions. In particular we present the modal counterpart of the regression technique. This gives us a theorem proving method for a relevant fragment of our modal logic.*

KEYWORDS: *reasoning about actions, regression, modal logic, dependence.*

1. Introduction

In the reasoning about actions field most approaches use the Situation Calculus formalism [MCC 69]. Among those, Reiter's [REI 91] has turned out to be most fruitful. His basic formalism is restricted to deterministic actions without ramifications. In order to solve the frame problem he makes use of so-called successor state axioms (SSAs). The latter enable regression [REI 91], which has interesting computational properties.

The Situation Calculus is a dialect of predicate logic, having situations and actions as objects, and where actions are viewed as mappings on the set of situations. At first glance this is very close to possible worlds semantics for Deterministic PDL [HAR 84]. But the precise relation between Reiter's approach and dynamic logic is not as obvious as that. One of the reasons why his formalism cannot be translated straightforwardly into modal logics of action such as PDL is that the Situation Calculus allows quantifying over actions. Worse, such quantifications are central to Reiter's approach.

In [DEM 03] there has been presented a technique to translate Reiter's approach into dynamic logic. In this paper we present a different approach. We solve the problem using an extension of dynamic logic that has been introduced in [CAS 99]. There, dynamic logic is combined with a causal notion based on a dependence relation, resulting in a family of logics $\mathcal{LAP}_{\leadsto}$. $\mathcal{LAP}_{\leadsto}$ is a simple yet powerful account to the frame and ramification problems, with the advantage of having a decision procedure in terms of tableau systems (while the Situation Calculus contains second-order axioms and is a priori not even semi-decidable). We propose an encoding of Reiter's approach into the formalism of $\mathcal{LAP}_{\leadsto}$. Having such a result provides some degree of optimization in doing inference tasks for some classes of problems in the area.

This work is organized in the following way: in Section 2 we present a slightly modified version of PDL, which will serve as the basis for developing the central ideas of this paper. Section 3 is devoted to introduce the basic hypotheses concerning the knowledge we have about actions. In Section 4 we present Reiter's solution to the frame problem in the logical basis of Section 2 and in Section 5 we summarize Reiter's regression technique. We then revisit De Giacomo and Lenzerini's account for encoding domain descriptions into a variant of dynamic logic that avoids quantification over actions (Section 6). In Section 7 we present our modal logic of actions $\mathcal{LAP}_{\leadsto}$. In Section 8 we show how we can do regression in $\mathcal{LAP}_{\leadsto}$. Finally we sketch possible extensions to this work (Section 9) and then give some concluding remarks.

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2. Deterministic PDL with quantification and equality

In this section we introduce a slightly extended version of deterministic PDL containing quantification over actions and the equality predicate.

We use $P_1, P_2, \ldots$ for propositional constants, and $P, Q, \ldots$ as metavariables for propositional constants. $A_1, A_2, \ldots$ denote action constants and $a, b, \ldots$ action variables. We will use $A, B, \ldots$ as metavariables ranging over action constants and variables. PRP is the set of all propositional constants, and ACT is the set of all action constants. Examples of propositional constants are *Loaded* ("the gun is loaded") and *Alive* ("the agent is alive"). Examples of action constants are *shoot* ("shooting the agent") and *strangle* ("strangling the agent"). $L_1, L_2, \ldots$ denote literals. If $L = \neg P$ then we shall identify $\neg L$ with P. $\phi, \psi, \ldots$ denote formulas that are constructed in the usual way from PRP using the classical propositional operators. We shall also call them *classical formulas*. Hence they do not contain modal operators, quantifiers or the equality predicate that is to be introduced below.

We will use modal operators $[A]$ (resp. $[a]$) for each action constant $A \in ACT$ (resp. action variable a). Φ, Ψ will denote complex formulas possibly involving modal operators, quantification, and equality between actions. $[A]\Phi$ is read "after executing

A, Φ". We also use the dual $\langle A \rangle$ of $[A]$. The formula $\langle A \rangle \top$ can be read as "A is executable".

The nonstandard feature of our logic is that we allow for *quantification over actions*, and for *equality between actions*. Hence, in this version of dynamic logic we allow for formulas of the form $\forall a \Phi$, with Φ a complex formula as defined above. In the Yale shooting scenario (YSS) [HAN 86], one can e.g. write

$$\forall a(\mathit{Alive} \wedge \neg[a]\mathit{Alive} \rightarrow (a = \mathit{shoot} \wedge \mathit{HasGun} \wedge \mathit{Loaded})).$$

This is an *explanation closure axiom* [SCH 90] expressing that the only way to make *Alive* false is by the shooting action.

A model is a triple $M = \langle W, R, I \rangle$ where W is a set of Kripke possible worlds, R is a set of binary relations on W, and I is an interpretation function mapping propositional constants to subsets of W, and action constants and variables to elements of R. We will sometimes write $w' \in (I(A))(w)$ instead of $wI(A)w'$, and similarly for variables a.

We say that the interpretation I agrees with I' except possibly on a if and only if

– $I(P) = I'(P)$ for every propositional constant P;

– $I(A) = I'(A)$ for every action constant A;

– $I(b) = I'(b)$ for every action variable b different from a.

For a given model $M = \langle W, R, I \rangle$, $w \models_M \forall a \Phi$ if for every I' such that I agrees with I' except possibly on a, $w \models_{\langle W,R,I' \rangle} \Phi$. $w \models_M [A]\Phi$ if for every $w' \in (I(A))(w)$, $w' \models_M \Phi$. $w \models_M [a]\Phi$ if for every $w' \in (I(a))(w)$, $w' \models_M \Phi$. We say that a formula Ψ is a consequence of the set of global axioms $\{\Phi_1, \ldots, \Phi_n\}$ in the class of models $\mathcal{M}$ (noted $\{\Phi_1, \ldots, \Phi_n\} \models_{\mathcal{M}} \Psi$) if and only if for all $M \in \mathcal{M}$, if $\models_M \Phi_i$ for every Φ_i, then $\models_M \Psi$.

We will use $\mathcal{K}$ to denote the class of all possible models. $\mathrm{D}\mathcal{K} = \{\langle W, R, I \rangle \in \mathcal{K} : R$ is a partial function$\}$ is the class of models where actions are *deterministic*, i.e., $(I(A))(w)$ is either a singleton or empty. Thus, for all action constants A and all formulas Φ

$$\models_{\mathrm{D}\mathcal{K}} \langle A \rangle \Phi \rightarrow [A]\Phi \tag{1}$$

If all actions are deterministic, then every formula without quantification can be brought into a normal form where there are neither conjunctions nor disjunctions in the scope of modal operators. Apart from classical equivalences, this uses the following ones from the left to the right:

$$\models_{\mathrm{D}\mathcal{K}} [A](\Phi \wedge \Psi) \leftrightarrow ([A]\Phi \wedge [A]\Psi) \tag{2}$$

$$\models_{\mathrm{D}\mathcal{K}} [A](\Phi \vee \Psi) \leftrightarrow ([A]\Phi \vee [A]\Psi) \tag{3}$$

3. Describing actions

Reiter (and more generally the reasoning about actions community) focuses on deductions from a theory describing a given set of actions in terms of preconditions and effects. In dynamic logic such an action theory corresponds to a set of global axioms in Fitting's sense [FIT 83]. We have for example,

$$\{[\mathit{load}]\mathit{Loaded}, \mathit{Loaded} \rightarrow [\mathit{shoot}]\neg \mathit{Alive}\} \models_{\mathcal{K}} [\mathit{load}][\mathit{shoot}]\neg \mathit{Alive}$$

In the Situation Calculus, the same result is obtained by quantifying over situations. For our running example we have $\forall s \mathit{Loaded}(\mathit{do}(\mathit{load}, s))$ and $\forall s(\mathit{Loaded}(s) \rightarrow \neg \mathit{Alive}(\mathit{do}(\mathit{shoot}, s)))$, where s is a variable of sort situation, *do* is a function symbol, *load*, *shoot* are constants of sort action, and *Loaded*, *Alive* predicate symbols.

In describing an action theory it is more or less explicitly supposed that the following pieces of information are given. About these items some assumptions of complete information are made.

3.1. *Action preconditions*

For each action constant A there is a classical formula $Poss(A)$ describing the action precondition of A, i.e. the condition under which A can be executed. For example $Poss(\mathit{shoot}) = \mathit{HasGun}$, and $Poss(\mathit{strangle}) = \top$.

It is supposed that *the action preconditions are complete*: A is executable if and only if $Poss(A)$ is true.

In terms of dynamic logic, completeness of action preconditions means that for every $A \in ACT$ we have a global axiom $Poss(A) \leftrightarrow \neg[A]\bot$.

3.2. *Set of possible causes*

For each propositional constant P there are two finite sets of action constants $Cause^+(P)$ and $Cause^-(P)$ describing the positive and negative causes of P. (Note that *ACT* may be infinite.) $Cause^+(P)$ contains the actions in ACT which in some circumstances might cause P to become true, while $Cause^-(P)$ contains those actions that may cause P false. For example $Cause^+(\mathit{Alive}) = \emptyset$ (no action makes an agent alive), $Cause^-(\mathit{Alive}) = \{\mathit{shoot}, \mathit{strangle}\}$, and $Cause^-(\mathit{Loaded}) = \{\mathit{shoot}\}$.[1]

It is also supposed that $Cause^+(P)$ and $Cause^-(P)$ are small, in the sense that $Cause^+(P)$ and $Cause^-(P)$ are much smaller than ACT.

Moreover, we suppose that these two sets are *complete*: whenever $A \notin Cause^+(P)$ then the execution of A can never make P true. In terms of dynamic

1. In Reiter's presentation these functions retrieved from his functions γ^+ and γ^-.

logic, causal completeness means that we have a global axiom $\neg P \rightarrow [A]\neg P$ in that case. Similarly, for every B such that $B \notin \mathit{Cause}^-(P)$ we have a global axiom $P \rightarrow [B]P$. Axioms of that form are called *frame axioms*. In our example, as $\mathit{strangle} \notin \mathit{Cause}^-(\mathit{Loaded})$, we have $\mathit{Loaded} \rightarrow [\mathit{strangle}]\mathit{Loaded}$.

The next piece of information specifies the causal relation in more detail.

3.3. *Effect preconditions*

For all propositional constant $P \in PRP$ and every action constant $A \in \mathit{Cause}^+(P)$ there is a classical formula $\mathit{Cond}^+(A, P)$ describing the *positive effect precondition* of action A. Similarly, for every $A \in \mathit{Cause}^-(P)$ there is a $\mathit{Cond}^-(A, P)$ describing its *negative effect precondition*. For example $\mathit{Cond}^-(\mathit{strangle}, \mathit{Alive}) = \top$, and $\mathit{Cond}^-(\mathit{shoot}, \mathit{Alive}) = \mathit{Loaded}$.[2]

It is supposed that *the effect preconditions are complete*: in situations where the formula $\mathit{Cond}^+(A, P)$ does not hold the execution of A can never make P true. Symmetrically, when $\mathit{Cond}^-(A, P)$ does not hold then the execution of A can never make P false.

In terms of dynamic logic, to every effect precondition $\mathit{Cond}^+(A, P)$ one can associate a global axiom $\mathit{Cond}^+(A, P) \rightarrow [A]P$, and to every effect precondition $\mathit{Cond}^-(A, P)$ one can associate a global axiom $\mathit{Cond}^-(A, P) \rightarrow [A]\neg P$. As an example, consider the formula $\mathit{Loaded} \rightarrow [\mathit{shoot}]\neg \mathit{Alive}$.

Completeness of effect preconditions means that we moreover have a global axiom $(\neg \mathit{Cond}^+(A, P) \wedge \neg P) \rightarrow [A]\neg P$ for every $A \in \mathit{Cause}^+(P)$. Symmetrically, for every B such that $B \in \mathit{Cause}^-(P)$ we have a global axiom $(\neg \mathit{Cond}^-(B, P) \wedge P) \rightarrow [B]P$. For example we have $(\neg \mathit{Loaded} \wedge \mathit{Alive}) \rightarrow [\mathit{shoot}]\mathit{Alive}$.

3.4. *Comments*

The last two completeness assumptions of Sections 3.2 and 3.3 express in modal logic what Reiter calls “explanation closure” and “Clark completion”.

Most importantly, the three pieces of information together with the completeness assumptions make that the possible world resulting from the execution of action A in a possible world w is completely determined: for every model M and world w of M, if $w \not\models_M \mathit{Poss}(A)$ then $(I(A))(w) = \emptyset$. Else the truth value of every P in every w' accessible from w via $I(A)$ is as follows. Suppose w.l.o.g. that $w \models_M P$. Then:

– if $A \notin \mathit{Cause}^-(P)$ then $w' \models_M P$;

– if $A \in \mathit{Cause}^-(P)$ and $w \not\models_M \mathit{Cond}^-(A, P)$ then $w' \models_M P$;

– if $A \in \mathit{Cause}^-(P)$ and $w \models_M \mathit{Cond}^-(A, P)$ then $w' \not\models_M P$.

2. These functions correspond to Reiter’s γ^+ and γ^-.

As all truth values are thus determined, it follows that the set of worlds accessible via $I(A)$ is either empty, or it can be considered to be a singleton. This fits with the assumption that all actions are deterministic.

As we have noted, the action preconditions and effect preconditions appear explicitly in Reiter's formalization, while the sets of possible causes $Cause^+(P)$ and $Cause^-(P)$ only appear implicitly there.

Note that in Reiter's Situation Calculus it is supposed that actions always lead to some state: even in states where the agent has no gun in his hands, the state resulting from the execution of *shoot* exists. The technical reason is that just as every function in predicate logic, his successor function do is total. This means that the logic of each action operator $[A]$ should be KD. We have nevertheless decided to follow the dynamic logic tradition and suppose that the set of worlds accessible via some action A might be empty. Therefore the logic of each $[A]$ is just K.

In fact, inexecutability of the action *shoot* is expressed in Situation Calculus by stating $Poss(\textit{shoot}) \leftrightarrow \textit{HasGun}$, where $Poss(\textit{shoot})$ is a particular propositional constant. In our formulation, $Poss$ is a function associating a classical formula to every action A. $Poss(A)$ can be seen as an abbreviation, such as $Poss(\textit{shoot}) = \textit{HasGun}$. Given a domain description in Reiter's style, we obtain a description in our style if we (1) define our $Poss$-function from Reiter's preconditions $Poss(A) \leftrightarrow \phi$, and (2) replace Reiter's constants $Poss(A)$ by our $\langle A \rangle \top$. The other way round, our version can be translated to Reiter's by (1) defining his preconditions $Poss(A) \leftrightarrow \phi$ from our $Poss$-function, and (2) recursively replacing $[A]\phi$ by $Poss(A) \rightarrow [A]\phi$. Observe that the latter is nothing but the well-known translation from modal logic K to KD [OHL 91, OHL 93].

All this sounds as if action theories could be described in deterministic PDL in a satisfactory manner, but we have not solved the frame problem yet: as by hypothesis $Cause^+(P)$ and $Cause^-(P)$ are small, it follows that the size of the set of frame axioms that we have to state is close to $card(PRP) \times card(ACT)$. This is usually considered to be too big, and a central element in the research program of the reasoning about actions community was to design mechanisms allowing to infer such frame axioms without stating them explicitly.

There was a 20-years-long debate about semantics and theorem proving methods allowing such inferences. Reiter's proposal seems to have closed the debate at least in what concerns deterministic actions without side-effects (also called ramifications). This is going to be presented in the sequel.

4. Reiter's solution to the frame problem

Based on a particular class of models, Reiter proposes to incorporate the basic ingredients of action theories that we have presented in the preceding section into successor state axioms which given a state and an action completely determine the next state.

4.1. *Reiter models*

Reiter requires that names are unique and that models are trees. Thus, given a model $M = \langle W, R, I \rangle$, we say that M is a *Reiter model* if and only if $\langle W, \bigcup_{r \in R} r \rangle$ is a tree, and if $I(A_i) = I(A_j)$, then $i = j$. $\mathcal{RTR}$ will denote the class of all Reiter models.

4.2. *Successor state axioms*

Suppose that all the $Poss(A)$, $Cause^+(P)$, $Cause^-(P)$, $Cond^+(A, P)$ and $Cond^-(A, P)$ are given, and that the completeness assumptions are made. We then can associate with that an action theory R from which the relevant frame axioms will follow. In dynamic logic R is made of the following axioms:

– for every $A \in ACT$, there is an executability axiom $Poss(A) \leftrightarrow \neg[A]\bot$;

– for every $P \in PRP$, if $Cause^+(P) = \{A_1, \ldots, A_n\}$ and $Cause^-(P) = \{B_1, \ldots, B_m\}$ then there is a successor state axiom

$$
\begin{array}{l}
\forall a([a]P \leftrightarrow \\
(\neg Poss(a) \vee \\
(a = A_1 \wedge Cond^+(A_1, P)) \vee \ldots \vee (a = A_n \wedge Cond^+(A_n, P)) \vee \\
(P \wedge \neg(a = B_1 \wedge Cond^-(B_1, P)) \wedge \ldots \wedge \neg(a = B_m \wedge Cond^-(B_m, P)))))
\end{array}
$$

Note that the successor state axiom is well defined because we have supposed that $Cause^+(A)$ and $Cause^-(A)$ are finite.

For the cases where $n = 0$ or $m = 0$, conjunction of the elements of an empty set is identified with $\top$, and disjunction with $\bot$. The latter can be illustrated with our running example, where $Cause^+(\mathit{Alive}) = \emptyset$. The successor state axiom for *Alive* is:

$$
\begin{array}{l}
\forall a([a]\mathit{Alive} \leftrightarrow \\
(\neg Poss(a) \vee \bot \vee (\mathit{Alive} \wedge \neg(a = \mathit{shoot} \wedge \mathit{Loaded}) \wedge \neg(a = \mathit{strangle} \wedge \top))))
\end{array}
$$

We abbreviate $\mathit{Reg}(a, P)$ the right hand side of the equivalence. The successor state axiom for P therefore has the form $\forall a([a]P \leftrightarrow \mathit{Reg}(a, P))$.

Successor state axioms can be equivalently stated for negative literals as:

$$\forall a([a]\neg P \leftrightarrow$$
$$(\neg Poss(a) \vee (a = B_1 \wedge \mathit{Cond}^-(B_1, P)) \vee \ldots \vee (a = B_m \wedge \mathit{Cond}^-(B_m, P)) \vee$$
$$(\neg P \wedge \neg(a = A_1 \wedge \mathit{Cond}^+(A_1, P)) \wedge \ldots \wedge \neg(a = A_n \wedge \mathit{Cond}^+(A_n, P)))))$$

We abbreviate $\mathit{Reg}(a, \neg P)$ the right hand side of this equivalence. For example the successor state axiom for $\neg \mathit{Alive}$ is:

$$\forall a([a]\neg \mathit{Alive} \leftrightarrow$$
$$(\neg Poss(a) \vee (a = \mathit{shoot} \wedge \mathit{Loaded}) \vee (a = \mathit{strangle} \wedge \top) \vee (\neg \mathit{Alive} \wedge \neg\bot)))$$

4.3. *Comments*

Reiter's original axiom [REI 91] is slightly different from ours:

$$\forall a(Poss(a) \rightarrow ([a]P \leftrightarrow$$
$$((a = A_1 \wedge \mathit{Cond}^+(A_1, P)) \vee \ldots \vee (a = A_n \wedge \mathit{Cond}^+(A_n, P)) \vee$$
$$(P \wedge \neg(a = B_1 \wedge \mathit{Cond}^-(B_1, P)) \wedge \ldots \wedge \neg(a = B_m \wedge \mathit{Cond}^-(B_m, P)))))$$

Our version can be proved to be equivalent to his.

In his book [REI 01] Reiter excluded the precondition $Poss(a)$ from the right hand side $\mathit{Reg}(a, P)$ of the SSA, and just writes

$$\forall a([a]P \leftrightarrow$$
$$((a = A_1 \wedge \mathit{Cond}^+(A_1, P)) \vee \ldots \vee (a = A_n \wedge \mathit{Cond}^+(A_n, P)) \vee$$
$$(P \wedge \neg(a = B_1 \wedge \mathit{Cond}^-(B_1, P)) \wedge \ldots \wedge \neg(a = B_m \wedge \mathit{Cond}^-(B_m, P))))$$

Therefore we would have e.g. $[\mathit{shoot}]\neg \mathit{Alive} \leftrightarrow (\mathit{Loaded} \vee (\neg \mathit{Alive} \wedge \neg\bot))$, from which it follows by classical principles that $\neg \mathit{HasGun} \wedge \mathit{Alive} \wedge [\mathit{shoot}]\neg \mathit{Alive} \rightarrow \mathit{Loaded}$. This means that such SSAs do not take into account inexecutability: this issue must be managed "by hand" by introducing $Poss(\mathit{shoot})$ atoms in the right places when proving consequences of SSAs in their recent version.

Finally, we note that Reiter's presentation also contains precondition axioms of the form $Poss(A) \leftrightarrow \phi$. This is not needed here because we view $Poss(A)$ as a function returning a classical formula ϕ, which is directly integrated into our successor state axiom (cf. also our comments in section 3.4).

5. Reiter's regression

Successor state axioms are crucial when it comes to the reasoning aspect of the frame problem, to which we turn now.

Given a Reiter's style action theory R, what can be deduced from it? Suppose Φ is a complex formula without quantification, action variables, and equality, such as *HasGun* $\rightarrow$ [*load*][*shoot*]$\neg$*Alive*. In order to decide whether $R \models_{\mathcal{RTR}} \Phi$, Reiter proposes to rewrite Φ using the successor state axioms from the left to the right. This is what he calls *regression*, and it consists in syntactical substitutions whose iteration reduces a given formula with action symbols into another one with just propositional constants.

At each regression step we have to put formulas in normal form such that there are neither conjunctions nor disjunctions in the scope of modal operators (using the hypothesis that all actions are deterministic). Hence the innermost modal operators have just literals in their scope. For the above example, Φ gets $\neg$*HasGun* $\vee$ [*load*][*shoot*]$\neg$*Alive*.

ALGORITHM 1 (REITER'S REGRESSION). —

`input:`

– *a formula without variables* Φ.

– $Poss(A)$, $Cause^+(P)$, $Cause^-(P)$, $Cond^+(A, P)$ *and* $Cond^-(A, P)$.

`output:` *a classical formula REG*(Φ).

`begin`

`while` Φ *is not classical*

put Φ *in normal form*

choose a subformula $[A]L$*, where* L *is either* P *or* $\neg P$*, for* $P \in PRP$

`if` $L = P$ `then` *replace* $[A]P$ *by* $Reg(A, P)$

`else` *replace* $[A]\neg P$ *by* $Reg(A, \neg P)$.

`end`

Notice that the action variable a of the successor state axiom is instantiated by the constant A.

In our example, the regression of the subformula [*shoot*]$\neg$*Alive* is

$$\neg HasGun \vee (shoot = shoot \wedge Loaded) \vee$$
$$(shoot = strangle \wedge \top) \vee (\neg Alive \wedge \neg\bot)$$

This can be simplified to $\neg$*HasGun* $\vee$ *Loaded* $\vee$ $\neg$*Alive*. Hence the result of the regression of Φ is $\neg$*HasGun* $\vee$ [*load*]($\neg$*HasGun* $\vee$ *Loaded* $\vee$ $\neg$*Alive*).

Each rewriting step thus eliminates a modal operator, and iterated application results in a formula without modal operators. If we iterate regression in our example, we first put the formula

$$\neg HasGun \vee [load](\neg HasGun \vee Loaded \vee \neg Alive)$$

into normal form, obtaining

$$\neg \textit{HasGun} \vee [\textit{load}]\neg \textit{HasGun} \vee [\textit{load}]\textit{Loaded} \vee [\textit{load}]\neg \textit{Alive}.$$

The regression of subformula $[\textit{load}]\neg \textit{HasGun}$ is equivalent to $\neg \textit{HasGun}$, that of subformula $[\textit{load}]\textit{Loaded}$ to $\top$, and that of $[\textit{load}]\neg \textit{Alive}$ to $\neg \textit{Alive}$. We therefore obtain

$$\neg \textit{HasGun} \vee \neg \textit{HasGun} \vee \top \vee \neg \textit{Alive},$$

which is valid in classical propositional logic. Thus the original formula $\textit{HasGun} \rightarrow [\textit{load}][\textit{shoot}]\neg \textit{Alive}$ can be deduced from Reiter's action theory R.

As regression is proved to be sound [REI 01, Theorem 4.5.2], checking validity of the original formula amounts to checking satisfiability of the regressed one in the initial state of the world:

THEOREM 2. — $R \models_{\mathcal{RTR}} \Phi \leftrightarrow \textit{REG}(\Phi)$.

COROLLARY 3. — $R \models_{\mathcal{RTR}} \Phi$ *if and only if* $\textit{REG}(\Phi)$ *is valid in Classical Propositional Logic.*

The rest of the paper explores whether regression can be performed in a simpler framework, in particular without quantifying over actions.

6. De Giacomo and Lenzerini's encoding into PDL

Reiter's Situation Calculus based solution has raised the natural question of at what extent it could be possible to do the same in dynamic logic. Given the expressiveness limitations of the latter (originally it did not allow for quantification over actions), many researchers [ZHA 01] have turned to other ways of facing the problems in the area. There has been others [GIA 95], however, who have tried on the first steps in that direction.

De Giacomo and Lenzerini have expressed Reiter's solution in a slightly modified version of PDL. This is what we take up in this section.

Here we simplify their account a bit and suppose that the set of atomic actions is the finite $\textit{ACT} = \{A_1, A_2, \ldots, A_n\}$. Then their approach can be said to have the following ingredients ($\alpha, \beta, \ldots$ denote complex actions):

– Nondeterministic choice $\alpha \cup \beta$;

– Converse α^-;

– A particular nondeterministic atomic action $\mathbf{any}$ that can be thought of as the nondeterministic composition of all atomic actions of $\textit{ACT}$: $\mathbf{any} = A_1 \cup A_2 \cup \ldots \cup A_n$;

– Complement $\neg\alpha$ w.r.t. $\mathbf{any}$, where $\alpha = B_1 \cup \ldots \cup B_m$ for some $B_1, \ldots, B_m \in \textit{ACT}$.

Moreover it is supposed that the past is deterministic, as expressed by the logical axiom $\neg[\mathbf{any}^-]\neg\Phi \rightarrow [\mathbf{any}^-]\Phi$.

Consider our example theory. Its representation in De Giacomo and Lenzerini's framework is:

$$[\mathbf{any}](\neg Alive \rightarrow \langle \mathbf{any}^- \rangle \neg Alive \vee \langle shoot^- \rangle Loaded \vee \langle strangle^- \rangle \top)$$

$$[\mathbf{any}](Alive \rightarrow \langle \mathbf{any}^- \rangle Alive)$$

Just as for PDL, reasoning in De Giacomo and Lenzerini's logical framework is EXPTIME-complete. While their encoding certainly preserves the spirit of Reiter's successor state axioms, they did not give the counterpart of Reiter's regression, and hence did not investigate whether reasoning for syntactically restricted theories is "cheaper" than EXPTIME.

In the next section we show how this can be simulated without quantification in a simple modal logic of actions augmented by a dependence relation.

7. Solving the frame problem without quantification

7.1. *Adding dependence information to PDL*

In [CAS 99] we have augmented a very simple version of PDL (basically multi-modal K) with metalogical causal information represented by a dependence relation $\leadsto$ between *actions* and *literals*. $A \leadsto L$ means "action A *may* cause literal L". The nonexistence of such a $A \leadsto L$ in $\leadsto$ (noted $A \not\leadsto L$) means that "L will never get true due to A".[3]

$A \leadsto P$ is just another way of writing down that $A \in Cause^+(P)$, and $A \leadsto \neg P$ that $A \in Cause^-(P)$.

Suppose $\leadsto$ is given. Semantically, if $I(A)$ is the accessibility relation associated to action A, the relation $\leadsto$ constrains possible worlds models in the following way:

– if $A \not\leadsto P$ and $w' \in (I(A))(w)$ and $w \notin I(P)$ then $w' \notin I(P)$;

– if $A \not\leadsto \neg P$ and $w' \in (I(A))(w)$ and $w \in I(P)$ then $w' \in I(P)$.

The resulting class of models is called $\mathcal{LAP}_{\leadsto}$. We note D$\mathcal{LAP}_{\leadsto}$ the class of $\mathcal{LAP}_{\leadsto}$-models whose accessibility relations are deterministic. It has been shown in [CAS 99] that the validities of $\mathcal{LAP}_{\leadsto}$ are completely axiomatized by the following set of logical axioms:

1) Some axiomatization of classical logic;

2) $[A]\Phi \wedge [A](\Phi \rightarrow \Psi) \rightarrow [A]\Psi$;

3) $\neg L \rightarrow [A]\neg L$ if $A \not\leadsto L$.

3. In [CAS 99] the language moreover contained an $S4$ modal operator $\Box$ which implies all action operators $[A]$. Laws were prefixed with $\Box$, e.g. $\Box(Loaded \rightarrow [shoot]\neg Alive)$. Here we shall achieve the same thing by viewing action laws as global axioms.

plus the Modus Ponens and the necessitation rule.

It has moreover been shown that $\mathcal{LAP}_{\leadsto}$ is decidable and EXPTIME-complete, and a tableau theorem proving method has been given.

7.2. *Solving the frame problem in* $\mathcal{LAP}_{\leadsto}$

Suppose all the ingredients $Poss(A)$, $Cause^+(P)$, $Cause^-(P)$, $Cond^+(A, P)$, $Cond^-(A, P)$ are given, and let us make the completeness assumptions as introduced in Section 3. We define a dependence relation and a set of global axioms S as follows:

– for every $A_i \in Cause^+(P)$ we put $A_i \leadsto P$, and for every $B_j \in Cause^-(P)$ we put $B_j \leadsto \neg P$;

– for every $A \in ACT$, add an executability axiom to S:

$$Poss(A) \leftrightarrow \neg[A]\bot \tag{4}$$

– for every $P \in PRP$ and every $A_i \in Cause^+(P)$ add two effect axioms to S:

$$Cond^+(A_i, P) \rightarrow [A_i]P \tag{5}$$

$$(\neg Cond^+(A_i, P) \wedge \neg P) \rightarrow [A_i]\neg P \tag{6}$$

– for every $P \in PRP$ and every $B_j \in Cause^-(P)$ add two effect axioms to S:

$$Cond^-(B_j, P) \rightarrow [B_j]\neg P \tag{7}$$

$$(\neg Cond^-(B_j, P) \wedge P) \rightarrow [B_j]P \tag{8}$$

Note that these axioms do not resemble successor state axioms. They nevertheless validate the same regression principle as in Reiter's framework, as it will be shown in the sequel.

A point that bears noting is that our representation indeed counts as a solution to the frame problem: the sets $\leadsto$ and S are both "small" (in the sense that they are much smaller than $card(PRP) \times card(ACT)$), and contain no frame axioms.

Now we turn to an important result:

THEOREM 4. — *Let* $\leadsto$ *and* S *be obtained from given sets* $Poss(A)$, $Cause^+(P)$, $Cause^-(P)$, $Cond^+(A, P)$ *and* $Cond^-(A, P)$. *Then the following equivalences are logical consequences of* S *in* $D\mathcal{LAP}_{\leadsto}$.

1) $[A]P \leftrightarrow \neg Poss(A) \vee P$, *if* $A \not\leadsto P$ *and* $A \not\leadsto \neg P$;

2) $[A]P \leftrightarrow \neg Poss(A) \vee (P \wedge \neg Cond^-(A, P))$, *if* $A \not\leadsto P$ *and* $A \leadsto \neg P$;

3) $[A]P \leftrightarrow \neg Poss(A) \vee Cond^+(A, P) \vee P$, *if* $A \leadsto P$ *and* $A \not\leadsto \neg P$;

4) $[A]P \leftrightarrow \neg Poss(A) \vee Cond^+(A, P) \vee (P \wedge \neg Cond^-(A, P))$, *if* $A \leadsto P$ *and* $A \leadsto \neg P$.

PROOF. —

Proving (1):

($\rightarrow$): We are about to prove $[A]P \wedge \neg P \rightarrow \neg Poss(A)$.

1. $\neg P \rightarrow [A]\neg P$, from the hypothesis $A \not\rightsquigarrow P$
2. $[A]P \wedge \neg P \rightarrow [A]P \wedge [A]\neg P$, from 1. by classical logic
3. $[A]P \wedge [A]\neg P \rightarrow [A]\bot$, by K and classical logic
4. $[A]P \wedge \neg P \rightarrow [A](P \wedge \neg P)$, from 2. and 3. by syllogism
5. $[A](P \wedge \neg P) \rightarrow [A]\bot$, by classical logic
6. $[A]P \wedge \neg P \rightarrow [A]\bot$, by syllogism on 4. and 5.
7. $[A]\bot \rightarrow \neg Poss(A)$, from global axiom (4)
8. $[A]P \wedge [A]\neg P \rightarrow \neg Poss(A)$, from 3. and 7. by classical logic
9. $[A]P \wedge \neg P \rightarrow \neg Poss(A)$, from 2. and 8. by classical logic

($\leftarrow$): We now prove $\neg Poss(A) \vee P \rightarrow [A]P$.

1. $P \rightarrow [A]P$, from the hypothesis $A \not\rightsquigarrow \neg P$
2. $\neg Poss(A) \rightarrow [A]\bot$, from global axiom (4)
3. $[A]\bot \rightarrow [A]P$, by K and classical logic
4. $\neg Poss(A) \rightarrow [A]P$, from 2. and 3. by classical logic
5. $\neg Poss(A) \vee P \rightarrow [A]P$, from 1. and 4. by classical logic

Proving (2):

($\rightarrow$): Let's show $[A]P \wedge \neg P \rightarrow \neg Poss(A)$ and $[A]P \wedge Cond^{-}(A, P) \rightarrow \neg Poss(A)$.

1. $\neg P \rightarrow [A]\neg P$, from the hypothesis $A \not\rightsquigarrow P$
2. $[A]P \wedge \neg P \rightarrow [A]P \wedge [A]\neg P$, from 1. by classical logic
3. $[A]P \wedge [A]\neg P \rightarrow [A]\bot$, by K and classical logic
4. $[A]\bot \rightarrow \neg Poss(A)$, from global axiom (4)
5. $[A]P \wedge [A]\neg P \rightarrow \neg Poss(A)$, from 3. and 4. by classical logic
6. $[A]P \wedge \neg P \rightarrow \neg Poss(A)$, from 2. and 5. by classical logic

7. $Cond^{-}(A,P) \rightarrow [A]\neg P$, by global axiom (7)
8. $[A]P \wedge Cond^{-}(A,P) \rightarrow [A]P \wedge [A]\neg P$, from 7. by classical logic
9. $[A]P \wedge Cond^{-}(A,P) \rightarrow [A]\bot$, from 8. and 3. by classical logic
10. $[A]P \wedge Cond^{-}(A,P) \rightarrow \neg Poss(A)$, from 9. and 4. by classical logic

($\leftarrow$): We are going to prove $\neg Poss(A) \vee (P \wedge \neg Cond^{-}(A,P)) \rightarrow [A]P$.

1. $\neg Poss(A) \rightarrow [A]\bot$, from global axiom (4)
2. $[A]\bot \rightarrow [A]P$, by K and classical logic
3. $\neg Poss(A) \rightarrow [A]P$, from 1. and 2. by classical logic
4. $(P \wedge \neg Cond^{-}(A,P)) \rightarrow [A]P$, from global axiom (8)
5. $\neg Poss(A) \vee (P \wedge \neg Cond^{-}(A,P)) \rightarrow [A]P$, from 3. and 4. by classical logic

Proving (3):

($\rightarrow$): We will prove $[A]P \wedge \neg Cond^{+}(A,P) \wedge \neg P \rightarrow \neg Poss(A)$.

1. $\neg Cond^{+}(A,P) \wedge \neg P \rightarrow [A]\neg P$, by global axiom (6)
2. $[A]P \wedge \neg Cond^{+}(A,P) \wedge \neg P \rightarrow [A]P \wedge [A]\neg P$, from 1. by classical logic
3. $[A]P \wedge [A]\neg P \rightarrow [A]\bot$, by K and classical logic
4. $[A]P \wedge \neg Cond^{+}(A,P) \wedge \neg P \rightarrow [A]\bot$, from 2. and 3. by classical logic
5. $[A]\bot \rightarrow \neg Poss(A)$, from global axiom (4)
6. $[A]P \wedge \neg Cond^{+}(A,P) \wedge \neg P \rightarrow \neg Poss(A)$, from 4. and 5. by classical logic

($\leftarrow$): We are about to prove $\neg Poss(A) \vee Cond^{+}(A,P) \vee P \rightarrow [A]P$

1. $\neg Poss(A) \rightarrow [A]\bot$, from global axiom (4)
2. $[A]\bot \rightarrow [A]P$, by K and classical logic
3. $\neg Poss(A) \rightarrow [A]P$, from 1. and 2. by classical logic
4. $P \rightarrow [A]P$, by hypothesis $A \not\rightsquigarrow \neg P$
5. $Cond^{+}(A,P) \rightarrow [A]P$, from global axiom (5)
6. $\neg Poss(A) \vee Cond^{+}(A,P) \vee P \rightarrow [A]P$, from 3., 4. and 5. by classical logic

Proving (4):

($\rightarrow$): We prove $[A]P \wedge \neg Cond^+(A,P) \wedge \neg(P \wedge \neg Cond^-(A,P)) \rightarrow \neg Poss(A)$

1. $\neg Cond^+(A,P) \wedge \neg P \rightarrow [A]\neg P$, from global axiom (6)
2. $[A]P \wedge \neg Cond^+(A,P) \wedge \neg P \rightarrow [A]P \wedge [A]\neg P$, from 1. by classical logic
3. $Cond^-(A,P) \rightarrow [A]\neg P$, by global axiom (7)
4. $[A]P \wedge \neg Cond^+(A,P) \wedge Cond^-(A,P) \rightarrow [A]P \wedge \neg Cond^+(A,P) \wedge [A]\neg P$, from 3. by classical logic
5. $[A]P \wedge \neg Cond^+(A,P) \wedge [A]\neg P \rightarrow [A]P \wedge [A]\neg P$, by classical logic
6. $[A]P \wedge \neg Cond^+(A,P) \wedge Cond^-(A,P) \rightarrow [A]P \wedge [A]\neg P$, from 4. and 5. by classical logic
7. $[A]P \wedge \neg Cond^+(A,P) \wedge \neg P \vee [A]P \wedge \neg Cond^+(A,P) \wedge Cond^-(A,P) \rightarrow [A]P \wedge [A]\neg P$, from 2. and 4. by classical logic
8. $[A]P \wedge \neg Cond^+(A,P) \wedge \neg(P \wedge \neg Cond^-(A,P)) \rightarrow [A]P \wedge [A]\neg P$, from 7. by classical logic
9. $[A]P \wedge [A]\neg P \rightarrow [A]\bot$, by K and classical logic
10. $[A]P \wedge \neg Cond^+(A,P) \wedge \neg(P \wedge \neg Cond^-(A,P)) \rightarrow [A]\bot$, from 8. and 9. by classical logic
11. $[A]\bot \rightarrow \neg Poss(A)$, from global axiom (4)
12. $[A]P \wedge \neg Cond^+(A,P) \wedge \neg(P \wedge \neg Cond^-(A,P)) \rightarrow \neg Poss(A)$, from 10. and 11. by classical logic

($\leftarrow$): We will prove $\neg Poss(A) \vee Cond^+(A,P) \vee (P \wedge \neg Cond^-(A,P)) \rightarrow [A]P$

1. $\neg Poss(A) \rightarrow [A]\bot$, from global axiom (4)
2. $[A]\bot \rightarrow [A]P$, by K and classical logic
3. $\neg Poss(A) \rightarrow [A]P$, from 1. and 2. by classical logic
4. $Cond^+(A,P) \rightarrow [A]P$, from global axiom (5)
5. $(P \wedge \neg Cond^-(A,P)) \rightarrow [A]P$, by global axiom (8)
6. $\neg Poss(A) \vee Cond^+(A,P) \vee (P \wedge \neg Cond^-(A,P)) \rightarrow [A]P$, from 3., 4. and 5. by classical logic

■

8. Regression in D$\mathcal{LAP}_{\leadsto}$

Here is a regression algorithm for D$\mathcal{LAP}_{\leadsto}$. Suppose Φ is a complex formula without quantification and equality, such as *HasGun* $\rightarrow$ [*load*][*shoot*]$\neg$*Alive*. Let us consider $Cond(A, L) = Cond^{+}(A, P)$ if $L = P$, and $Cond(A, L) = Cond^{-}(A, P)$ if $L = \neg P$.

ALGORITHM 5 (REGRESSION WITH DEPENDENCE). —

`input:`
a formula without variables Φ.
$Poss(A)$, $Cause^{+}(P)$, $Cause^{-}(P)$, $Cond^{+}(A, P)$ *and* $Cond^{-}(A, P)$.

`output:` *a classical formula REG*(Φ).

`begin`

`while` Φ *is not classical*

put Φ *in normal form*

choose some subformula $[A]L$, *where* L *is a literal*

`case` $A \not\leadsto L$ `and` $A \not\leadsto \neg L$ `then` *replace* $[A]L$ *by* $\neg Poss(A) \vee L$

`case` $A \not\leadsto L$ `and` $A \leadsto \neg L$ `then`
replace $[A]L$ *by* $\neg Poss(A) \vee (L \wedge \neg Cond(A, \neg L))$

`case` $A \leadsto L$ `and` $A \not\leadsto \neg L$ `then` *replace* $[A]L$ *by* $\neg Poss(A) \vee Cond(A, L) \vee L$

`case` $A \leadsto L$ `and` $A \leadsto \neg L$ `then`
replace $[A]L$ *by* $\neg Poss(A) \vee Cond(A, L) \vee (L \wedge \neg Cond(A, \neg L))$

`end`

In our example, the regression of [*shoot*]$\neg$*Alive* is $\neg$*HasGun* $\vee$ *Loaded* $\vee$ $\neg$*Alive*. Hence the result of the regression step is *HasGun* $\rightarrow$ [*load*]($\neg$*HasGun* $\vee$ *Loaded* $\vee$ $\neg$*Alive*). Putting this into normal form using (3) we obtain the formula *HasGun* $\rightarrow$ ([*load*]$\neg$*HasGun* $\vee$ [*load*]*Loaded* $\vee$ [*load*]$\neg$*Alive*). The regression of [*load*]$\neg$*HasGun* is $\neg$*HasGun*, that of [*load*]*Loaded* is $\top$, and that of [*load*]$\neg$*Alive* is $\neg$*Alive*. We therefore obtain *HasGun* $\rightarrow$ ($\neg$*HasGun* $\vee \top \vee$ $\neg$*Alive*), which is valid in classical propositional logic.

THEOREM 6 (DECIDABILITY, SOUNDNESS AND COMPLETENESS). — *Suppose* S *and* $\leadsto$ *obtained from* $Poss(A)$, $Cause^{+}(P)$, $Cause^{-}(P)$, $Cond^{+}(A, P)$ *and* $Cond^{-}(A, P)$ *as described in Section 7.2. Let* Φ *be an input formula without quantifiers, action variables, and equality. Then, Algorithm 5 terminates returning a classical formula* ϕ *and* $S \models_{\mathrm{D}\mathcal{LAP}_{\leadsto}} \Phi \leftrightarrow \phi$.

PROOF. — Let Φ be an input formula. Termination is straightforward, as each step of the algorithm eliminates exactly one modal operator. Soundness and completeness are also immediate: after putting formula Φ in normal form, it will be made of conjunctions/disjunctions of modal subformulas. In this case, the equivalence between Φ and ϕ follows from the ones given in theorem 4 together with the rule of substitution of equivalences (which is valid in $\mathrm{D}\mathcal{LAP}_{\leadsto}$). ■

For our example, this means that $HasGun \rightarrow [load][shoot]\neg Alive$ can be deduced with our action theory S and dependence relation $\leadsto$ because its regression is valid in classical logic.

Hence, modulo the equality predicate, we obtain the same result as for Reiter's regression algorithm in the case of our example. This generalizes: a close look at the two algorithms shows that if both our S and $\leadsto$ and Reiter's R are obtained from the same $Poss(A)$, $Cause^{+}(P)$, $Cause^{-}(P)$, $Cond^{+}(A, P)$, $Cond^{-}(A, P)$, then the results are logically equivalent.

It follows thus that whenever $Poss(A)$, $Cause^{+}(P)$, $Cause^{-}(P)$, $Cond^{+}(A, P)$, $Cond^{-}(A, P)$ are given, and the completeness assumptions can be made, then Reiter's formulation in terms of successor state axioms and ours in terms of effect axioms and dependence do the same job in their respective logical basis:

COROLLARY 7. — *Let the sets $Poss(A)$, $Cause^{+}(P)$, $Cause^{-}(P)$, $Cond^{+}(A, P)$, $Cond^{-}(A, P)$ be given. Let R be a Reiter theory obtained from them as described in Section 4. Let $\leadsto$ and S be obtained from them as described in Section 7.2. Let Φ be a complex formula without quantification and equality. Then $R \models_{\mathcal{RTR}} \Phi$ iff $S \models_{\mathrm{D}\mathcal{LAP}_{\leadsto}} \Phi$.*

9. The frame problem for knowledge

9.1. *Sensing actions and knowledge*

Reiter's framework does not account for actions which have no effect on the "real" world, but only on the agents' knowledge. Such actions are close to test actions of dynamic logic.

In order to express the effects of sensing actions we need a modal operator of knowledge $\Box$. The logic of $\Box$ is S5. The dual of $\Box$ is noted $\Diamond$.

The extension of Reiter's solution to knowledge and sensing actions has been studied by Scherl and Levesque [SCH 93]. They make some hypotheses about actions and their perception by the agent that permit to simplify the theory.

9.1.1. *Public action*

First, they suppose that the agent perceives action occurrences completely and correctly. For example whenever shooting takes place the agent is aware of that, and whenever the agent believes shooting has taken place then indeed such an action has occurred. (One might imagine that action occurrences are publicly announced to all agents.)

9.1.2. *Action laws known*

Second, they suppose that the agent knows the laws governing the actions. Hence the agent knows that after strangling the effect always is ¬*Alive*, etc.

9.1.3. *Non-informative actions*

We finally make a third hypothesis that is not made by Scherl and Levesque, but which simplifies exposition without too much loss of generality [HER 00]. We shall suppose henceforth that all actions are non-informative. Non-informative actions are actions which are not observed by the agent beyond their mere occurrence. Upon learning that such an action has occurred the agent updates his belief state: he computes the new belief state from the previous one and his knowledge about the action laws. Hence the new belief state neither depends on the state of the world before the action occurrence, nor on the state of the world after the action occurrence.

In our example the *shoot* action is non informative. If the agent learns that the shooting action has been executed then he does not learn whether the victim died or not: if both *Loaded* and ¬*Loaded* were possible for the agent before, then afterwards he envisages both possible outcomes. Nevertheless, by learning that *shoot* occurred the agent learns that *HasGun* was true before the action.

Clearly, the action of observing the outcome of the *shoot* action is informative: the new belief state depends on the truth value of ¬*Alive* in the real world. Other examples of informative actions are that of looking up a phone number, *testing if* a proposition is true, *telling whether* a proposition is true, etc.

Nevertheless, the agent is not disconnected from the world: he may learn that some proposition is true (i.e. that some action of observing that some proposition has some value has occurred). For example, when he learns that it has been observed that the victim is dead (i.e. he learns that the action of observing ¬*Alive* has been executed) then he is able to update his belief state accordingly. Indeed, the observe actions are non-informative according to our definition: when the agent learns that ϕ has been observed then he is able to update his belief state accordingly, and there is no need to further observe the world. Other examples of non-informative actions are that of *learning that* the phone number of another agent is N, *testing that* a proposition is true (in the sense of dynamic logic tests), *telling that* a proposition is true, etc.

9.1.4. *A successor state axiom*

Under the hypotheses we have made, the following logical axiom is reasonable:

$$[A]\Box\Phi \leftrightarrow ([A]\bot \vee \Box[A]\Phi)$$

From the left to the right, this corresponds to a "no forgetting" principle, while the right-to-left direction expresses a "no learning" principle.

Such an axiom has been called a successor state axiom by Scherl and Levesque.[4] It permits to solve what they have called the frame problem for knowledge.

First, note that because actions are supposed to be deterministic, this axiom allows to deduce

$$[A]\Diamond\Phi \leftrightarrow ([A]\bot \vee \Diamond\neg[A]\neg\Phi).$$

Now these two principles enable regression by allowing for the elimination of $\Box$ and $\Diamond$ operators from the scope of action operators. When all such epistemic operators have been moved outward, Reiter's regression can be applied to the remaining non-epistemic formula, resulting in a modality-free formula of classical propositional logic. As a whole, the resulting formula only contains epistemic operators, but no action operators.

To sum it up, to establish whether a complex formula Φ follows from a domain description amounts to

– move all $[A]$ operators inwards, then

– eliminate all $[A]$ operators by regression, and finally

– check whether the resulting formula Ψ is a theorem of S5.

10. Concluding remarks

In this paper we have presented a purely propositional framework for reasoning about actions in modal logic within which Reiter's regression technique can be applied. We have thus shown that regression does not necessarily build on successor state axioms as in Reiter's original theory, which involves quantification.

We have also seen how the ideas here developed could be extended and applied in reasoning about knowledge.

As we have presented it here, Reiter's solution is very constrained. In particular actions must be deterministic and without indirect effects.

Reiter has proposed [REI 01] to implement nondeterministic actions by means of an operator of nondeterministic composition of deterministic atomic actions similar to that of dynamic logic. For example, the action *toss* of tossing a coin can be thought of as the nondeterministic choice *tossHeads*$\cup$*tossTails* between *tossHeads* and *tossTails*, whose respective effects are *Heads* and *Tails*. $[$*tossHeads* $\cup$ *tossTails*$]\Phi$ is defined to be an abbreviation of $[$*tossHeads*$]\Phi \wedge [$*tossTails*$]\Phi$. Such a solution transfers straightforwardly to our modal logic. But our framework also offers a more straightforward way of dealing with actions with indeterminate effects: we can drop the hypothesis that for every $A \in \mathit{Cause}^+(P)$ the condition $\mathit{Cond}^+(A, P)$ is defined. For example,

4. Their successor state axiom contains supplementary conditions in order to account for the informative part of actions.

although $Cause^{+}(Heads) = Cause^{+}(Tails) = \{toss\}$, there is no way of stating the exact conditions when heads or tails results from tossing.

Reiter's solution supposes that domain descriptions only contain executability and effect laws. Thus it does not allow for static laws such as $Walking \rightarrow Alive$. Such laws augment the effects of the *shoot* action: shooting not only has the (direct) effect $\neg Alive$, but also the (indirect) effect $\neg Walking$. Reiter and Lin [LIN 94] have proposed to "compile away" static laws in a mechanical way into effect laws (see also [MCI 98]). Again, this transfers straightforwardly to our modal logic. Nevertheless, the most challenging continuation of our work is the direct integration of so-called state constraints into the framework (instead of compiling them away as done by Lin and Reiter). But things get much harder in this case, all the more in [CAS 02] we have claimed that up to now there is no satisfactory framework allowing for actions with both indirect and indeterminate effects.

We plan to pursue future works analyzing to at what extent the results here presented could be generalized to Lin's [LIN 95, LIN 96] approach in the case of stratified action theories.

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