Cooperative interpersonal communication and relevant information

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Outline

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1. **Cooperativity**
   - Example
   - Definition
2. Relevance
3. Back to cooperativity
4. Conclusion and perspectives

Example

Agent *a*

late?

incident $\rightarrow$ *late*

Agent *b*

late?

$\neg$*incident* $\rightarrow$ $\neg$*late*

Agent *c*

incident

rain

Example

Agent *a*

late?

incident $\rightarrow$ *late*

Agent *b*

late?

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rain

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late?

$\mathit{incident} \rightarrow \mathit{late}$

Agent *b*

late?

$\neg \mathit{incident} \rightarrow \neg \mathit{late}$

Agent *c*

incident

rain

Example

Agent a

late?

$incident \rightarrow late$

Agent b

late?

$\neg incident \rightarrow \neg late$

incident

Agent c

incident

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Example

Agent *a*

late?

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late?

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Agent b

late?

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Agent c

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Example

Agent *a*

late?

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Agent *b*

late?

$\neg$*incident* $\rightarrow$ $\neg$*late*

rain

Agent *c*

incident

rain

Example

Agent a

$late$?

$incident \rightarrow late$

Agent b

$late$?

$\neg incident \rightarrow \neg late$

$incident \wedge rain$

Agent c

$incident$

$rain$

Example

Agent *a*

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Agent *b*

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$incident \wedge rain$

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- agent c believes that φ does not contain elements disconnected with Q for agent a
- agent c believes that φ is true

Then agent c informs agent a about φ

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Outline

1. Cooperativity
2. **Relevance**
   - Formal definition
   - Properties
   - Hierarchy
3. Back to cooperativity
4. Conclusion and perspectives

Relevance

Let a be an agent, Q an objective formula, φ a formula. We say that φ is relevant for agent a concerning Q iff the following formula, denoted $R_a^Q\varphi$, is true :

$$I_a(B_aQ \vee B_a\neg Q) \;\wedge\; \varphi \;\wedge\; (B_a(\varphi \rightarrow Q) \otimes B_a(\varphi \rightarrow \neg Q))$$

- Information need :
  agent a wants to know whether Q or $\neg Q$
- Piece of information truth value :
  the piece of information φ must be true
- Agent's beliefs base :
  agent a believes that $\varphi \rightarrow Q$ or that $\varphi \rightarrow \neg Q$

Axiomatics

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Axiomatics

A few properties

Proposition 1

$$R_a^Q \varphi \rightarrow \neg B_a \varphi \wedge B_a \neg \varphi$$

If φ is a relevant piece of information for agent a, then she does not know neither φ nor $\neg\varphi$.

Proposition 2

- $I_a(B_a Q \vee B_a \neg Q) \rightarrow R_a^Q Q \text{ xor } R_a^Q \neg Q$
- $(Q_1 \leftrightarrow Q_2) \rightarrow (R_a^{Q_1} \varphi \leftrightarrow R_a^{Q_2} \varphi)$
- $R_a^Q \varphi \rightarrow \neg R_a^Q \neg \varphi$
- $\neg(\varphi_1 \wedge \varphi_2) \rightarrow \neg(R_a^{Q_1} \varphi_1 \wedge R_a^{Q_2} \varphi_2)$

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$$R_a^Q \varphi \rightarrow \neg B_a R_a^Q \varphi$$

If φ is a relevant piece of information, then agent a does not believe it.

Proposition 5

$$B_a(\varphi_1, \varphi_2/Q) \rightarrow (\varphi_2 \wedge R_a^Q \varphi_1 \rightarrow R_a^Q(\varphi_1 \wedge \varphi_2))$$

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Minimal explanation

Definition : Explanation

Let Δ be a set of objective formulae and α and β two objective formulae.
β is an explanation of α if and only if $\vdash B\Delta \rightarrow B(\beta \rightarrow \alpha)$ and $\nvdash B\Delta \rightarrow B(\neg\beta)$.

Intuition : Minimal explanation

- for cubes : α is a minimal explanation of β iff there is no other explanation α' of β such that $\alpha \rightarrow \alpha'$ and $\alpha \nrightarrow \alpha'$ (prime implicants)
- for clauses : α is a minimal explanation of β iff there is no other explanation α' of β such that $\alpha' \rightarrow \alpha$ and $\alpha' \nrightarrow \alpha$ (maxima for subsumption)

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Most relevant formulae

Let $\mathcal{R}_a^Q$ be the set of relevant formulae. For all φ in $\mathcal{R}_a^Q$, we have $B_a(\varphi \rightarrow Q)$ or $B_a(\varphi \rightarrow \neg Q)$ and $\neg B_a(\neg\varphi)$, that means that for all φ in $\mathcal{R}_a^Q$, φ is an explanation of Q or $\neg Q$.

Definition

Let $\mathcal{R}m_a^Q$ be the subset of $\mathcal{R}_a^Q$ that contains the minimal explanations of Q and $\neg Q$. We will write $Rm_a^Q\varphi$ to express that the formula φ belongs to $\mathcal{R}m_a^Q$.

Example

Let us consider the following set of relevant pieces of information to agent a concerning her request Q : $\mathcal{R}_a^Q = \{inc \wedge rain, inc \vee strike, strike\}$. Then $\mathcal{R}m_a^Q = \{strike, inc \wedge rain\}$.

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Definition for cooperativity

Intuitively, agent c is **cooperative with regard to agent** a if and only if for all piece of information φ :
agent c believes that φ is relevant for a concerning some Q
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Definition for cooperativity

A more formal definition

Let a and c be two agents. The agent c is cooperative with regard to a iff forall formula φ, c informs a about φ if and only if there is a request Q such that c believes that φ is maximal relevant for a concerning Q. This is represented by :

$$Coop(c, a) \equiv \forall \varphi Inf_{c,a} \varphi \leftrightarrow \exists Q, B_c(Rm_a^Q \varphi)$$

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Conclusion and perspectives

Conclusion

- A first formal definition to agent-oriented relevance
- Characterization of most relevant pieces of information for an agent
- Characterization of cooperativity between two agents
- Comparison to other definitions of cooperativity

Perspectives

- Relevance for other needs that information need
- Introduction of time

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Axiomatics

- Propositionel tautologies and propositional inference rules ;
- KD45 for B_a,

$$
\begin{array}{rl}
\text{(K)} & \vdash B_a(\varphi \to \psi) \wedge B_a\varphi \to B_a\psi \\
\text{(D)} & \vdash B_a\varphi \to \neg B_a\neg\varphi \\
\text{(4)} & \vdash B_a\varphi \to B_aB_a\varphi \\
\text{(5)} & \vdash \neg B\varphi \to B_a\neg B_a\varphi
\end{array}
$$

- (Nec) Necessitation for B_a, $\dfrac{\vdash \varphi}{\vdash B_a\varphi}$
- (UE) Unit exclusion for I_a, $\vdash \neg I_a(\top)$
- *BI* Introspection,

$$
\begin{array}{rl}
\text{(BI1)} & \vdash I_a\varphi \to B_aI_a\varphi \\
\text{(BI2)} & \vdash \neg I_a\varphi \to B_a\neg I_a\varphi \\
\text{(BI3)} & \vdash B_a(\varphi \leftrightarrow \psi) \to (I_a\varphi \leftrightarrow I_a\psi)
\end{array}
$$

Back