How Can Yes-No Questions Be Informative?

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Introduction

An Example of Interrogative Reasoning (I)

Example 1. *Investigating the disappearance of a race horse, Silver Blaze, and the death of his trainer John Straker, Holmes suspects that Straker accidentally died while attempting to wound the horse, in order to rig a forthcoming race, and make a profit betting against Silver Blaze. Holmes has found in Straker's pocket a bill for an expensive dress, and suspects that Straker bought it for his mistress. The expenses incurred by a double life would explain Straker's scheme. Instead of asking the widow whether she owns the dress, Holmes asks her whether they formerly met, feigning to remember her wearing a similar dress. She answers him that it is unlikely, since she does not own such a dress. Holmes then concludes that his hypothesis is correct.*

(from [Conan Doyle, 1960], *Silver Blaze*.)

An Example of Interrogative Reasoning (II)

The example from Conan Doyle's Silver Blaze shows a two-sided situation:

- a situation of inquiry where a certain conclusion has to be established by a combination of deductive and interrogative steps;
- an interaction context in which an inquirer asks questions to a source.

Holme's example shows that asking certain questions may introduce information in inquiry contexts, thus affecting source's answers. Holmes must prevent his source from guessing his intention in order to get a true information.

A Model of Empirical Reasoning

Empirical inquiry can be modeled as a game in which a player, Inquirer, tries to establish a given **conclusion** C from a set of **premises** T, using both deductive steps and interrogative steps in an **underlying model** M.

Inquiry games reasoning can be formalized in *semantic tableaux systems* (see [Genot, 2009]) with the following features:

- The tableau procedure must be modified for 'checking' model M rather than model-building;
- Rules for questions and answers are added;
- Rules for 'revisions' are added too.
- Open subtableaux represent possibilities left so far uneliminated by the premises and answers.

Answers narrow down the range of alternatives left open (ways M might be) given the information assumed (premises) and obtained (answers) by Inquirer.

Features of the Interrogative Model of Inquiry

Hintikka and his associates have focused on *pure discovery*:

- Premises are true in M;
- A unique source (Oracle) is truthful and *non-strategic*;
- All answers are accepted when recieved.

Interrogative Reasoning in this case parallels deductive reasoning (see [Hintikka et al., 1999]).

Interrogative reasoning with *uncertain* (multiple) sources (possibly conflicting) is investigated in [Genot, 2009].

Yet uncertainty is not the result of choices made by Inquirer and her sources. Rather, it is the result of Nature's initial move: the choice of M and sources' access to M, i.e. *answers*.
But Nature is a *non-strategic player* with a 'flat' preference ordering over outcome.

Sources as Strategic Players

What if the source is a strategic player? The outcome of an inquiry depends on Inquirer's *strategic choices of questions*.

Inquirer's questions reveals that Inquirer:

1. *knows* (or believes) that the question *precondition* (presupposition) holds in M:
2. *does not know* the *answer*;
3. *wants to know* it (possibly to use it in her reasoning).

If 1) and 2) are assumed, a question is equivalent to a *public announcement.* This announcement may influence Source's willingness to answer.

Silver Blaze's Problem

In Holmes' case, the question used is a *yes-no question*:

- it has a *vacuous precondition* – which induces a 'trivial' update;
- Holmes' ignorance does not warrant inference to the state of Nature;
- Holmes' intention to use it is *not* revealed.

Had Holmes asked the question: "Do you own such and such a dress?", *the first two facts would have remained unchanged, but not the third.*

This example poses a *modeling challenge*:

1. Logics of epistemic update do not account for the *information content* Holmes wants to conceal;
2. Classical game theory does not account for the *potential effect of revealing* the information.

Information-seeking and standard Game-theory

The First Game (I)

Let's assume *a standard game theory*.
Let also assume that Holmes wants to reach the same conclusion as in Ex.1, but that he has only two options:

1. to ask the widow about her husband's being unfaithful;
2. to ask about her owning a certain dress.

Both questions are 'yes-no' questions, hence need no prior information to be raised (their presupposition always hold). Let us use the following notational convention:

- ϕ = Straker bought a dress
- ψ = Straker's widow possesses such a dress
- χ = Straker had a mistress

The First Game (II)

P_1 Straker bought a dress: ψ

P_2 If Straker bought a dress, and his widow does not own it, then Straker had a mistress: $(\psi \wedge \neg\phi) \rightarrow \chi)$

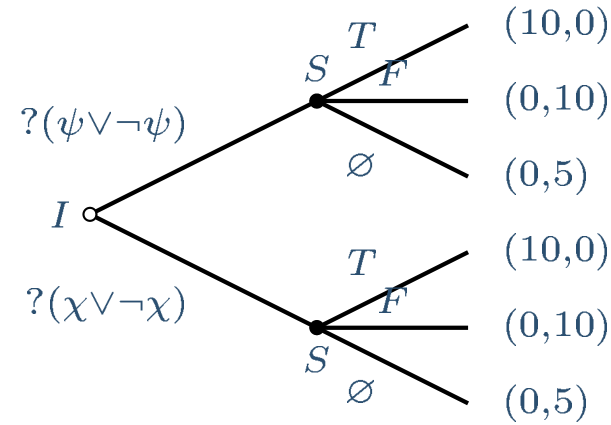
- Holmes (I) knows (and is aware of) that P_1 and P_2 – i.e. can answer the question whether P_1 and P_2 holds ;
- Straker's widow (S) knows that P_2: she would be able to answer the question whther P_2, but has not consider it so far.

Holmes wants to know whether or not Straker had a mistress, but cannot risk asking the question with presupposition $(\chi \vee \neg\chi)$. It could cause S to lie, or be reluctant to answer (plus, she is not a reliable source with respect to this question!)

Game Tree for the First Game

A better strategy is the following:

1. ask the question with presupposition $(\psi \vee \neg\psi)$;
2. if answer $\neg\psi$ is obtained, infer χ from P_1, P_2 and $\neg\psi$.



A Problem with the First Game

Holmes' strategy is meant to prevent the widow from realizing *which game she is playing*.

If S is simply in doubt as to I's intention, she has a *uniform strategy*: **always lie.**

Hence, the first game tree does not model adequately the situation form the short story.

Preventing strategic inference

Unawareness (I)

Holmes' problem is:

- 'disguise' his question about ψ asking another question, for instance whether θ (first), which would seem *irrelevant to the inquiry*.
- choose θ so that the widow has no reason to lie.
- choose θ so that he can ask whether ψ, but with the question seemingly unrelated to his inquiry (no reason to lie either).

After asking his question, and getting the expected (and true) answer $\neg\phi$, Holmes refuses the widow's answer, asserting θ following with the *yes-or-no*– question $?(\psi \vee \neg\psi)$. The rationale for this strategy is as follows: he draws the widow's attention on a fact of least importance, insisting on it, namely θ, and he asks the question he is really interested (about ϕ)in as a follow-up of the first, *as if it were an exam question* meant to show her that she was mistaken about θ.

Unawareness (II)

We must therefore add a third option at the outset, for the first game to be a more faithful representation of the 'game' of Ex.1. Besides $?(\chi \vee \neg\chi)$ and $?(\psi \vee \neg\psi)$, I has the option $?(\theta \vee \neg\theta)$, where I draws S's attention on something completely different from what she might expect, given what she knows about I's interests.

But this question must have another virtue: not only has S's attention to be diverted, but S has also to be unaware of other strategies I could have followed, preventing her from guessing in what game she is. In classical game theory, players' rationality implies that they are all aware of the structure of the game, and such an awareness is common knowledge. Common knowledge of all possible moves in the game is not a reasonable assumption in games of inquiry, precisely because some possible moves need to be hidden until they are performed, to prevent the source from planning in advance her answers.

That is why we must turn to non-classical game-theory in order to provide an adequate model of Holmes' inquiry.

A model of empirical inquiry

Games with unawareness

Following some of the developments presented by Halpern and Rego in [Halpern and Rego, 2006], we propose to introduce the notion of *awareness* – and unawareness – as central to information-seeking games, making them a special case of *extensive games with unawareness*.

An extensive game with unawareness represents what players are aware of at each node of the game, resulting in an *augmented game*. An augmented game shows the way a player views the game at a given stage, that is what she believes to be the true game in that situation.

Thus, the complete description of a game *with unawareness* is constituted by a set of augmented games, one for the modeler and one for each game other players think they might be in at some stage of the game.

Extensive games

Since an augmented game is built after a standard extensive game, let us briefly recall the definition of such a game (we follow the definition of [Osborne and Rubinstein, 1994]): an extensive game G with perfect information is a structure $G = (N, H, Z, P, (U_i)_{i \in N})$ where:

1. $N = \{1, \ldots, n\}$ is a set of players;
2. H is a set of sequences (histories) of the game (whith $Z \subset H$ the set of terminal histories of the game);
3. $P : H \longrightarrow N$ is a function which assigns to each non-terminal history a member of N;
4. $U_i : Z \longrightarrow \mathbb{R}$ is a utility function for each player $i \in N$ which associates a payoff to each terminal history of the game.

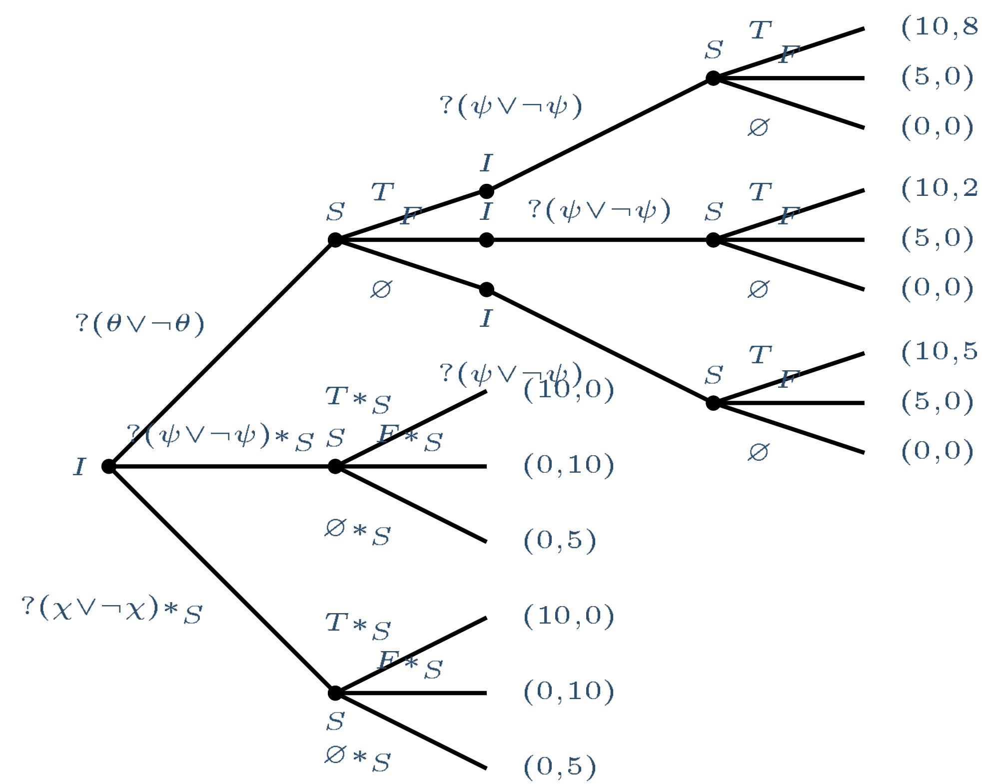
Augmented games

An augmented game G' is based on a game G and augments G by "describing each agent's awareness level at each node, where [a player's] awareness level at a [given node] is essentially the set of runs (complete histories) in G that [the player] is aware of at [that] node." (Halpen and Rego, [Halpern and Rego, 2006]).

What essentially distinguishes an augmented game from a standard extensive game is that at each node, it is not only specified which player's turn it is, but also what is that player's awareness level. Intuitively, this specifies the game the player believes, at that node, to be playing.

The game with unawareness for Ex.1 (I)

Let us now represent the information-seeking game with unawareness, where unawareness of (possible) moves is represented by indexing each move player S is unaware of with $*_S$. This is the description of the game from the modeler's viewpoint.



The game with unawareness for Ex.1 (II)

After I's added move $?(\theta \vee \neg\theta)$, S has, as always, three options,:

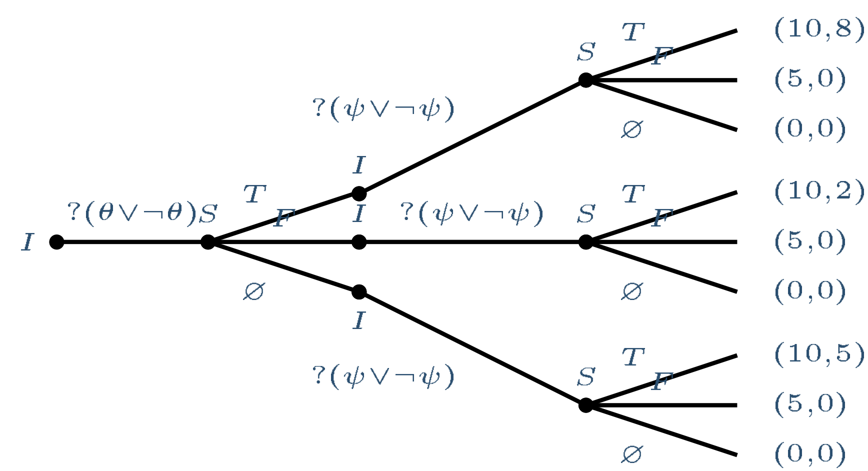
- saying the truth,
- lying,
- saying nothing.

This does not affect I's next move – since Holmes' strategy in Ex.1 can be carried even in case of other answers. Indeed, I feigns to takes answer θ for granted, and follows with a new *yes-or-no*– question $?(\psi \vee \neg\psi)$, as if it were an 'exam' question (stressing answer ψ as expected).

But adding the branch $?(\theta \vee \neg\theta)$ cuts off S's possibility to become aware of other moves available to I at first stage of the game, since asking $?(\theta \vee \neg\theta)$ seems irrelevant to the inquiry.

The game from Source's viewpoint (I)

Asking the question $?(\theta \vee \neg\theta)$ makes S believe she is in another situation, hence playing another game, unrelated to I's inquiry (given what she knows about the latter). The following tree shows what game G^S she believes to be playing, after I's first move at $?(\theta \vee \neg\theta)$.



The game from Source's viewpoint (II)

Because S thinks she's playing the game previously shown, she has no incentive to lie or to refuse to answer.

The *yes-or-no*– question $?(\theta \vee \neg\theta)$ 'signals' indeed (for S) a situation unrelated to the current inquiry (as determined by the background knowledge of S).

The strategy S then follows is only related to this local state (local situation) she believes to be the actual situation. Therefore we can say that, in the augmented game G^S, S follows a *local strategy*.

Unlike in standard extensive games, S is not aware of all possible moves in the game. In games with unawareness, a strategy for a player is a collection of local strategies, one for each augmented game depending of his awareness level.

Conclusion

We have argued that, in some cases, demands for information relative to strategic aspects of reasoning are important enough to be integral part of **models of interaction**.

Inquiry games, or information-seeking games, as developed by Hintikka, are described as games against Nature, hence are degenerate games, game-theoretic representations of decision problems.

Our example shows that inquiry games can be generalized to more than one players (vs. Nature), but also that standard game-theory is ill-suited to model such games.

When inquiry is such that it should be modeled as a *non-cooperative game*, inferences of agents about other agent's demand for information matters as much as inference from their ignorance to state of Nature.

We have then proposed to identify this information as **strategic information**, to treat it through extensive games with unawareness. An interesting open problem is to represent in epistemic languages this type of information, since the dynamic logics for belief and knowledge update which have recently been flourishing are not designed to accommodate this phenomenon.

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