Expressivity and Complexity of Reasoning about Coalitional Interaction

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Motivation

LOGICAL METHODS FOR SOCIAL CONCEPTS (LMSC'09)

MODAL LOGICS FOR COOPERATION

Motivation

LOGICAL METHODS	**FOR**	**SOCIAL CONCEPTS**	**(LMSC'09)**
MODAL LOGICS	**FOR**	**COOPERATION**	

Motivation

LOGICAL METHODS FOR SOCIAL CONCEPTS (LMSC'09)
MODAL LOGICS FOR COOPERATION

Modal Logics for reasoning about coalitional power in MAS

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Motivation

LOGICAL METHODS FOR SOCIAL CONCEPTS (LMSC'09)

MODAL LOGICS FOR COOPERATION

Modal Logics for reasoning about coalitional power in MAS

- **Coalitional power.** $\langle\!\lbrack C \rbrack\!\rangle \varphi$: "*Coalition C can force that* φ"
- Preferences. $\Diamond^{\leq_i}\varphi$ "*Agent i* preferes some state in which φ holds."
  $\varphi \leq_i \psi$: Agent i prefers ψ over ψ
- Actions/Strategies. $[a]\varphi$: "*After any execution of a,* φ *is the case.*"

Evaluation

→ **Expressivity**: GT-, SCT concepts

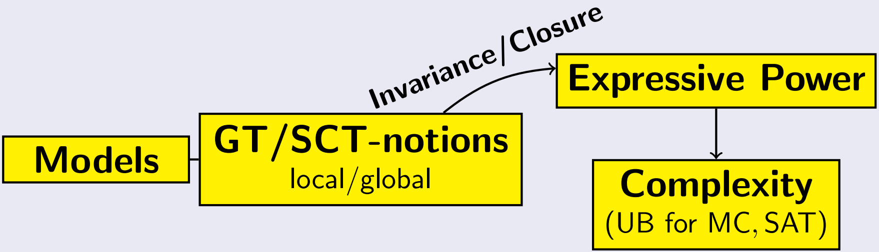
→ **Complexity**: SAT,MC

Aim and Methodology

Aim

How much **expressive power** is needed for talking about **GT/SCT notions** in modal logic, and what is the **complexity**?

Methodology



Outline

The Models

Three ways of modelling coalitional power

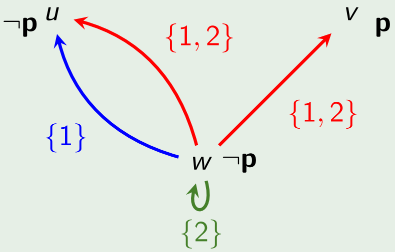
- simplified, generalized versions of existing classes of models
  - avoid additional complexity, focus on complexity required to express certain notions
- different perspectives on cooperation
- Preferences are modelled as TPO over the states
- Three classes of Kripke models for cooperation
  1. $\wp(\mathtt{N}) - \mathtt{LTS}$ (Coalition-labelled transition systems)
     - Coalitional power as primitive
  2. ABC (Action-based coalitional models)
     - Coalitional power arises from individuals' abilities to perform actions
  3. PBC (Power-based coalitional models)
     - Coalitional power arises from the power of subcoalitions
     - Generalization of **NCL** (normal simulation of Pauly's **CL**)

$\wp(\mathtt{N}) - \mathtt{LTS}$ (Coalition-labelled transition systems)

- sequential/turn-based systems

Example

$\mathtt{N} = \{1, 2\}$



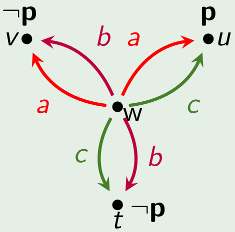
$$M, w \models \langle\!\langle\{1,2\}\rangle\!\rangle\, p \wedge \langle\!\langle\{1,2\}\rangle\!\rangle\, \neg p$$

ABC (Action-based coalitional models)

- coalitional power is made explicit
- power of a coalition arises from the power of its members to perform actions

Example

$\mathtt{N} = \{1, 2\}$
Actions $A = \{a, b, c\}$
$A_1 = \{a, b\}$, $A_2 = \{c\}$



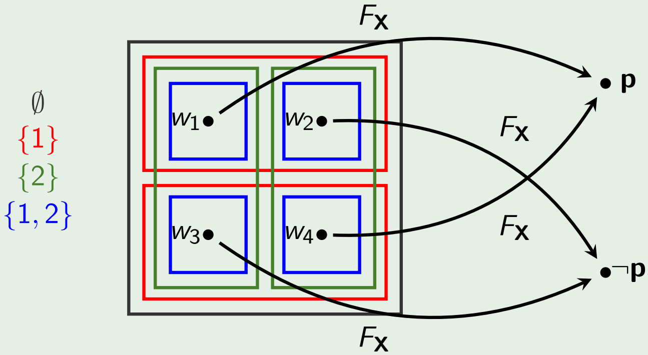
In w, $\{1, 2\}$ can force p because $M, w \models [a \cap c]p$

PBC (Power-based coalitional models)

- focus lies on the structure of coalitional power itself
- power of a coalition to achieve something arises from the power of its subcoalitions

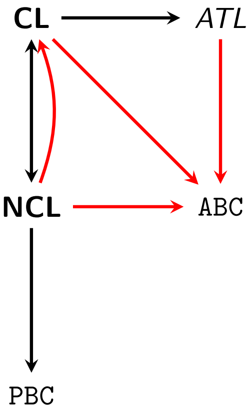
Example

$\mathtt{N} = \{1, 2\}$



In each w_i, $\{1, 2\}$ can force p because $M, w_i \models \langle\emptyset\rangle[\{1, 2\}]\mathbf{X}p$

The Models – The Big Picture



$\mathbf{L_1} \longrightarrow \mathbf{L_2}$ means:

there is a function $\tau : \mathcal{L}_{\mathbf{L_1}} \to \tau : \mathcal{L}_{\mathbf{L_2}}$ and a function $\tau' : \mathbb{M}_{\mathbf{L_1}} \to \mathbb{M}_{\mathbf{L_2}}$such that for all $\varphi \in \mathcal{L}_{\mathbf{L_1}}$ and $M \in \mathbb{M}_{\mathbf{L_1}} : M, w \models \varphi$ iff $\tau'(M, w) \models \tau(\varphi)$.

Outline

The Notions of Interest –Some Examples

Local Notions: Properties of a particular state in a model.

Simple combinations of coalitional power and preferences

- C can guarantee that the next state is one j finds a.l.a.g.
- There is a state all agents in C prefer, but C cannot achieve it.

GT/SCT concepts

- The current state is (strongly) *Nash* stable, i.e. no agent has the power to guarantee that the next state will be one that she strictly prefers to (finds a.l.a.g. as) the current one.
- There is a *strong local dictator*; i.e. there is an agent d such that all coalitions can only achieve that the system moves into a state d finds a.l.a.g. as the current one.
- The current state is (weakly/strongly) *Pareto*-efficient.

The Notions of Interest –Some Examples

Global Notions: Properties of classes of frames.

Restrictions and reasonable properties of the power of coalitions

- Only coalitions containing a majority of $\mathtt{N}$ have nontrivial power.
- Coalition monotonicity: if D is a subset of C then for all sets of states X, if D can force the system to move into X, then so can C.
- Coalitions can achieve only what all its members prefer.

Global GT/SCT concepts

- One agent is a strong local dictator in every state.

The Notions: Some Remarks

- All the notions are expressible in **FOL**.
- Interpretation in the models slightly different in some cases.

Outline

Some Characterization Results

Theorem ([van Benthem, 1983])

A formula of the FO correspondence language with at most one free variable is invariant under bisimulations iff it is equivalent to the standard translation of a ML formula.

Theorem ([Feferman, 1969, Areces et al., 2001])

A formula of the FO correspondence language with at most one free variable is invariant under taking generated submodels iff it is equivalent to the standard translation of a formula of ML+ $\downarrow x.\varphi \mid @_x\varphi$.

Theorem ([Goldblatt and Thomason, 1975])

A FO *definable class of frames is definable in ML iff it is closed under taking* $\mathtt{BMI}, \mathtt{GSF}, \mathtt{DU}$ *and reflects ultrafilter extensions.*

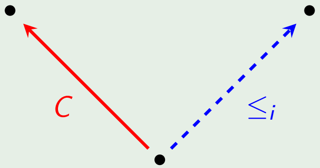
Determining the Required Expressive Power

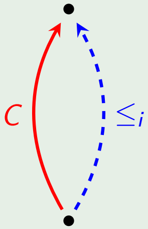
- Use invariance results (closure results) to determine how much expressive power is needed to express each of the local notions (to define the class of frames having the global property).

Example

"Coalition C can achieve a state that agent i finds at least as good."

$\wp(N) - \mathtt{LTS}$:





- In $\wp(N) - \mathtt{LTS}$, not invariant under bisimulation, but invariant under $\cap$-bisimulation.

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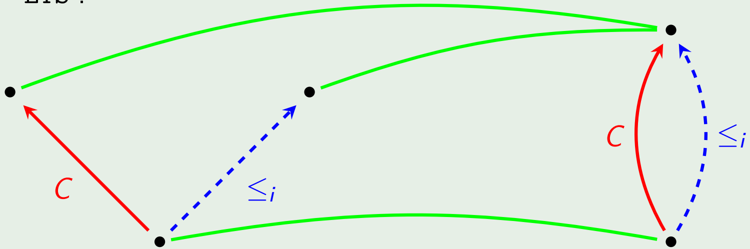

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Determining the Required Expressive Power

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Local Notions

- bisimulation
- $^{-1}$-bisimulation
- $\cap$-bisimulation
- total bisimulation
- $\mathcal{H}$-bisimulation
- $\mathcal{H}(@)$-bisimulation
- $\mathcal{H}(E)$-bisimulation
- bounded morphism
- generated submodels
- disjoint union

Global Notions

- bounded morphic images
- generated subframes
- disjoint union
- reflects generated subframes
- bisimulation systems

Outline

"Coalition can make agent happy." easiest in $\wp(N) - \mathtt{LTS}$

"$\mathtt{C}$ *can guarantee that the next state is one* j *finds a.l.a.g. as the current one.*"

	$\wp(\mathtt{N}) - \mathtt{LTS}$	$\mathtt{ABC}$	$\mathtt{PBC}$
Invariance	$\cap$-Bis.	$\mathtt{GSM}$ and $\mathtt{DU}$	$\mathtt{GSM}$ and $\mathtt{DU}$
Formula	$\langle C \cap \leq_j \rangle \top$	$\bigvee_{\vec{a} \in \vec{C}} (\downarrow x.[\bigcap \vec{a}](\downarrow y.@_x \langle \leq_j \rangle y))$	$\downarrow x.\langle \emptyset \rangle [C] \mathbf{X} \downarrow y.@_x \langle \leq_j \rangle y$
SAT	PSPACE	Π^0_1	Π^0_1
MC	PTIME	PSPACE	PSPACE

Nash vs. strict Nash: opposite results for $\wp(\mathtt{N}) - \mathtt{LTS}$, and $\mathtt{ABC}$&$\mathtt{PBC}$

Nash-stability: No agent can achieve a *strict* improvement by himself

	$\wp(\mathtt{N}) - \mathtt{LTS}$	$\mathtt{ABC}$	$\mathtt{PBC}$
Invariance	GSM and DU	GSM and DU	GSM and DU
Formula	$\bigwedge_{j\in\mathtt{N}} \downarrow x.[j \cap \leq_j]\langle\leq_i\rangle x$	$\bigwedge_{j\in N, a_j \in A_j} \downarrow x.\langle a_j\rangle\langle\leq\rangle x$	$\bigwedge_{j\in N} \downarrow x.[\emptyset]\langle\{j\}\rangle\mathbf{X}\langle\leq\rangle x$
SAT	Π_1^0	EXPTIME	EXPTIME
MC	PSPACE	PSPACE	PSPACE

Strong Nash-stability: No one can achieve an improvement by himself

	$\wp(\mathtt{N}) - \mathtt{LTS}$	$\mathtt{ABC}$	$\mathtt{PBC}$
Invariance	$\cap$-Bis.	GSM and DU	GSM and DU
Formula	$\bigwedge_{j\in\mathtt{N}}[i \cap \leq_j]\bot$	$\neg \bigvee_{j\in N} \bigvee_{a_j\in A_j} \downarrow x.[a_j]\langle\leq^{-1}\rangle x$	$\neg \bigvee_{j\in N} \downarrow x.\langle\emptyset\rangle[\{i\}]\mathbf{X}\langle\leq^{-1}\rangle x$
SAT	PSPACE	Π_1^0	Π_1^0
MC	PTIME	PSPACE	PSPACE

Pareto-Efficiency: Same Results for all Models

Preferences = TPO

	weak P.	Pareto	strong P.
Invariance	GSM and DU	GSM and DU	$\cap$-Bis.
Formula	$\downarrow x.[\bigcap_{j\in N}\leq_j]\bigvee_{j\in N}\langle\leq_i\rangle x$	$\neg\downarrow x.\langle\bigcap_{j\in N}\leq_j\rangle(\bigvee_{j\in N}[\leq_j]\neg x)$	$\perp$
SAT	Π^0_1	Π^0_1	
MC	PSPACE	PSPACE	

Without TPO requirement for preferences.

	weak P.	Pareto	strong P.
Invariance	GSM and DU	GSM and DU	$\cap$-Bis.
Formula	$\downarrow x.[\bigcap_{j\in N}\leq_j]\bigvee_{j\in N}\langle\leq_i\rangle x$	$\neg\downarrow x.\langle\bigcap_{j\in N}\leq_j\rangle(\bigvee_{j\in N}[\leq_j]\neg x)$	$[\bigcap_{j\in N}\leq_j]\perp$
SAT	Π^0_1	Π^0_1	PSPACE
MC	PSPACE	PSPACE	PTIME

$\Rightarrow$ **Talking about strict preferences is complicated, even with** TPO.

Global Notions: "having no power": difficult in $\mathtt{ABC}$

Only Majorities have nontrivial power

	$\wp(\mathtt{N}) - \mathtt{LTS}$	$\mathtt{ABC}$	$\mathtt{PBC}$
Closure	$\mathtt{GSF},\ \mathtt{DU}, \mathtt{BMI}$	$\mathtt{GSF}$	$\mathtt{GSF},\ \mathtt{DU}, \mathtt{BMI}$
Axiom	$\bigwedge_{\mathtt{C}:\lvert\mathtt{C}\rvert<\lvert\mathtt{N}\rvert/2}[\mathtt{C}]\bot$	$\bigwedge_{\mathtt{C}:\lvert\mathtt{C}\rvert<\lvert\mathtt{N}\rvert/2}(\mathtt{E}p \rightarrow \bigwedge_{\vec{a}\in\vec{C}}\langle\bigcap\vec{a}\rangle p)$	$\bigwedge_{\mathtt{C}:\lvert\mathtt{C}\rvert<\lvert\mathtt{N}\rvert/2}([\mathtt{C}]p \leftrightarrow [\emptyset]p)$
SAT	PSPACE	EXPTIME	PSPACE
MC	PTIME	PTIME	PTIME

Outline

Summary

- identified notions for reasoning about cooperation in MAS.
- considered three classes of models for cooperation and clarified their relation to other classes.
- determined required expressive power (via invariance results) for each of the notions and classes of models.
- gave explicit definability results.
- for each class of models, determined upper bounds for SAT and MC (combined complexity) of modal logics being able to express the notions.

Conclusion

- Global notions: not very demanding; most are expressible in ML
- Local notions: more demanding; many notions not BM-invariant
- Choice of primitives not only conceptually important but also has an impact on complexity required to express certain notions
- Whether *weak* or *strong* efficiency notions are "dangerous" w.r.t. complexity, heavily depends on the choice of models.
- Complexity results have to be taken with some caution; they crucially depend on the parameters.
    - Some formulas defining the notions are exponential in the number of agents or actions.

Future Work

- Find lower bounds.
  - For LB on MC (data complexity), use results from computational social choice.
- Determine required complexity for encoding *concrete arguments* from GT and SCT
  $\Rightarrow$ complexity of *actual* reasoning.

Merci!

Thank you!

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