A Dynamic Logic for Multi-agent Partial Knowledge Information Systems

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Abstract

Information systems, providing information about a set of objects regarding a set of attributes, are extended to multi-agent partial knowledge information systems (MPISs). MPISs also account for missing information. A logic which is sound and complete for MPISs is presented along with its dynamic version. The latter accommodates 'information updates'.

1 Introduction

An information or knowledge representation system [14] is a data table that is used to represent information about a set of objects regarding a set of attributes. Formally,

Definition 1 *An* information system *(IS)* $\mathcal{S} := (W, A, \bigcup_{a\in A} Val_a, f)$, *comprises a non-empty set* W *of objects, a non-empty set* A *of attributes, a non-empty set* Val_a *of attribute values for each* $a \in A$, *and* $f : W \times A \to \bigcup_{a\in A} Val_a$ *such that* $f(x, a) \in Val_a$.

In tabular form, $\mathcal{S}$ has columns labelled with attributes from A, and rows with objects from W. Any group of columns, i.e. subset B of A, would induce an equivalence (*indiscernibility*) relation $Ind_{\mathcal{S}}(B)$ on W:

$$x\ Ind_{\mathcal{S}}(B)\ y \text{ if and only if } f(x, a) = f(y, a) \text{ for all } a \in B.$$

So x and y cannot be distinguished using only the information provided by the attributes of the set B.

Given $\mathcal{S}$ and $B \subseteq A$, any concept represented as a subset (say) X of the universe W, is approximated using the attributes from the set B from 'within' and 'outside', by its *lower* and *upper approximations* [14] defined as $\underline{X}_{Ind_{\mathcal{S}}(B)} := \{[x]_{Ind_{\mathcal{S}}(B)} : [x]_{Ind_{\mathcal{S}}(B)} \subseteq X\}$ and $\overline{X}_{Ind_{\mathcal{S}}(B)} := \{[x]_{Ind_{\mathcal{S}}(B)} : [x]_{Ind_{\mathcal{S}}(B)} \cap X \neq \emptyset\}$ respectively. $[x]_{Ind_{\mathcal{S}}(B)}$ denotes the equivalence class of $x (\in W)$ with respect to

$Ind_{\mathcal{S}}(B)$. Any x in $\underline{X}_{Ind_{\mathcal{S}}(B)}$ is termed a *positive* element of X. Elements in the complement of $\overline{X}_{Ind_{\mathcal{S}}(B)}$ are *negative*, while those in $\overline{X}_{Ind_{\mathcal{S}}(B)} \setminus \underline{X}_{Ind_{\mathcal{S}}(B)}$ are the *boundary* elements.

From Definition 1 it is clear that for each object of the domain, we have information about the attribute value(s) of every attribute in A. But in reality, this may not be the case. In this article, we follow Kryszkiewicz [8] to allow a situation when *there is no information available regarding an attribute a for some object x*. In order to accommodate this, we take a new symbol ϵ and write '$f(x, a) = \epsilon$'. Thus we have the following definition.

Definition 2 *A* partial knowledge information system *(PIS) is a tuple* $\mathcal{K} := (W, A, Val \cup \{\epsilon\}, f)$*, where* $W, A, Val := \bigcup_{a \in A} Val_a$ *are as in Definition 1, and* $f : W \times A \rightarrow Val \cup \{\epsilon\}$.

Moreover, we consider a multi-agent scenario: agents come from a set (say) $Ag := \{1, 2, \ldots, n\}$, with a common domain of discourse, attribute set and values, but each with her own value assignment function f_i. Formally, this is represented by a *multi-agent partial knowledge information system* (MPIS).

Definition 3 *A MPIS is a tuple* $\mathcal{K} := (W, A, Val \cup \{\epsilon\}, \{f_i\}_{1 \leq i \leq n})$*, where for each* $i = 1, 2, \ldots, n$*,* $f_i : W \times A \rightarrow Val \cup \{\epsilon\}$.

It may be mentioned that MPIS is actually a special case of a multi-source knowledge representation system [7].
For each $B \subseteq A$ and agent i, one then obtains $Ind_{\mathcal{K}}(i, B)$:

$$x \; Ind_{\mathcal{K}}(i, B) \; y \text{ if and only if } f_i(x, a) = f_i(y, a) \text{ for all } a \in B.$$

Let us consider a MPIS. It is possible that an agent does not have information regarding some attributes for some of the objects, or the agent may have 'wrong' information. Missing information may be provided or correction (or refinement) of information may be made with the help of incoming information from outside the system. On the other hand, an agent may update her information with help from agents inside the system as well: she may borrow from information of another agent. We note that the flow of information or action taken by agents as mentioned above will have effects on the indiscernibility relations and hence on elements in the extensions of concepts. Thus, for instance, it could happen that a positive element of (the extension of) a concept may not remain so after the update of information for an agent.

In this paper, we formally study these kinds of situations. Any discussion on information here would be in terms of attributes and attribute values of the objects. In Section 2, a logic LMPIS is proposed, a fragment (denoted $\mathfrak{L}_f$) of which is a logic for MPISs. $\mathfrak{L}_f$ can express properties of indiscernibility relations corresponding to any finite set of attributes, as well as notions related to dependencies in data and data reduction [14]. LMPIS has attribute and attribute value constants in its language. It should be remarked that there are logics defined for information systems with this feature in the language (cf. [3]), but none in our knowledge, which are sound and complete and which also relate attribute and attribute values with indiscernibility relations. In Section 3, we briefly survey some of the logics for information systems and indiscernibility relations to make a comparison. In Section 4, LMPIS is extended to define a dynamic logic DLMPIS, which is related with LMPIS in the same way as dynamic epistemic logic is related with epistemic logic [19]. In the process, we also obtain a dynamic version $\mathfrak{L}_f^D$ of $\mathfrak{L}_f$. The actions discussed above are expressible in DLMPIS and $\mathfrak{L}_f^D$. Using these logics, one is able to explore how the actions may affect properties of objects. We may remark that a temporal logic was proposed in [4] to study the effect of flow of information on indiscernibility relations. But a multi-agent scenario was not considered there, and the flow of information itself did not appear in the picture. The dynamic logics presented here overcome this limitation. Section 5 concludes the article.

2 The logic LMPIS

The language of LMPIS contains (i) a non-empty finite set $Ag := \{1, 2, \ldots, n\}$ of agents, (ii) a non-empty countable set $\mathcal{AC}$ of attribute constants, (iii) for each $a \in \mathcal{AC}$, a non-empty finite set $\mathcal{VC}_a$ of attribute value constants, (iv) a non-empty countable set PV of propositional variables and (v) a special symbol ϵ. Atomic formulae are the propositional variables p from PV, and *descriptors* (i, a, v), for each $a \in \mathcal{AC}$, $v \in \mathcal{VC}_a \cup \{\epsilon\}$ and $i \in Ag$.
Using the Boolean logical connectives $\neg$ (negation) and $\wedge$ (conjunction) and unary modal connectives $[(i, B)]$ for each $B \subseteq \mathcal{AC}$ and $i \in Ag$, well-formed formulae (wffs) of $LMPIS$ are then defined recursively as:

$$(i, a, v) \mid p \mid \neg\alpha \mid \alpha \wedge \beta \mid [(i, B)]\alpha.$$

Let $\mathcal{D}$ denote the set of all descriptors, and $\mathfrak{L}$ the set of all LMPIS-wffs.

2.1 Semantics

The semantics of LMPIS is based on the notion of a *MPIS-structure*.

Definition 4 *A* MPIS-structure *is a tuple* $\mathfrak{F} := (W, \{R(i, B)\}_{i \in Ag,\ B \subseteq \mathcal{AC}}, m)$, *where* $R(i, B) \subseteq W \times W$ *and* $m : \mathcal{D} \to 2^W$ *satisfying the following.*

(S1) *For each* $a \in \mathcal{AC}$ *and* $i \in Ag$, $\bigcup\{m(i, a, v) : v \in \mathcal{VC}_a \cup \{\epsilon\}\} = W$.

(S2) *For each* $a \in \mathcal{AC}$, $m(i, a, v) \cap m(i, a, v') = \emptyset$, *for* $v \neq v'$.

(S3) *For each* $B \subseteq \mathcal{AC}$, $R(i, B)$ *is an equivalence relation.*

(S4) $R(i, \emptyset) = W \times W$.

(S5) $R(i, B) \subseteq R(i, C)$ *for* $C \subseteq B \subseteq \mathcal{AC}$.

(S6) *For* $B \subseteq \mathcal{AC}$ *and* $b \in \mathcal{AC}$, $R(i, B) \cap R(i, b) \subseteq R(i, B \cup \{b\})$.

(S7) *For* $b \in \mathcal{AC}$, $(w, w') \in R(i, b)$ *if and only if there exists* $v \in \mathcal{VC}_b \cup \{\epsilon\}$ *such that* $w, w' \in m(i, b, v)$.

Conditions (S1) and (S2) correspond to the fact that each object takes for each attribute, exactly one value (which could be ϵ). (S6) and (S7) relate the attribute, attribute value pairs with the indiscernibility relations.

Given a MPIS $\mathcal{K} := (W, \mathcal{AC}, \bigcup_{a\in\mathcal{AC}} \mathcal{VC}_a \cup \{\epsilon\}, \{f_i\}_{i\in Ag})$, we get the MPIS-structure $\mathfrak{F} := (W, \{Ind_{\mathcal{K}}(i,B)\}_{i\in Ag,\ B\subseteq\mathcal{AC}}, m^{\mathcal{K}})$, where $m^{\mathcal{K}}(i,a,v) := \{w \in W : f_i(w,a) = v\}$. In the line of [15], let us call it the *standard MPIS-structure generated by $\mathcal{K}$. The class of all standard MPIS-structures is thus identifiable with the class of all MPISs.*

Note that there may be a MPIS-structure $\mathfrak{F} := (W, \{R(i,B)\}_{i\in Ag, B\subseteq\mathcal{AC}}, m)$ which does not satisfy the equality $R(i,B) = \bigcap_{b\in B} R(i,b)$. Such a MPIS-structure cannot be standard and hence all MPIS-structures are not standard. But for any $\mathfrak{F}$, we can determine $\mathfrak{F}' := (W, \{R'(i,B)\}_{i\in Ag,\ B\subseteq\mathcal{AC}}, m)$, a standard MPIS-structure, such that $R(i,B) = R'(i,B)$ for every $i \in Ag$ and finite subset B of $\mathcal{AC}$. In fact, $\mathfrak{F}'$ is generated by the MPIS $\mathcal{K} := (W, \mathcal{AC}, \bigcup_{a\in\mathcal{AC}} \mathcal{VC}_a \cup \{\epsilon\}, \{f_i\}_{i\in Ag})$, where $f_i(x,a) = v$ if and only if $x \in m(i,a,v)$. Then $Ind_{\mathcal{K}}(i,B) = R(i,B) = R'(i,B)$ for all $i \in Ag$ and finite B.

Definition 5 *A* MPIS model *is a tuple* $\mathfrak{M} := (\mathfrak{F}, V)$ *where* $\mathfrak{F}$ *is a MPIS-structure and* $V : PV \to 2^W$.
$\mathfrak{M}$ *is* standard*, if* $\mathfrak{F}$ *is a standard MPIS-structure.*

Let us now define *satisfiability* of a LMPIS-wff α in a MPIS model $\mathfrak{M}$ at an object w of the domain W, denoted as $\mathfrak{M}, w \models \alpha$. We only give the cases for atomic and modal wffs.

Definition 6 $\mathfrak{M}, w \models (i,a,v)$ *if and only if* $w \in m(i,a,v)$*, for* $(i,a,v) \in \mathcal{D}$.
$\mathfrak{M}, w \models p$*, if and only if* $w \in V(p)$*, for* $p \in PV$.
$\mathfrak{M}, w \models [(i,B)]\alpha$*, if and only if for all* w' *in* W *with* $(w,w') \in R(i,B)$*,*
$\mathfrak{M}, w' \models \alpha$.

Let $\mathfrak{L}_f$ denote the set of all LMPIS-wffs which do not involve any modal operator $[(i,B)]$, where B is an infinite subset of $\mathcal{AC}$. One can prove the following easily with the help of the earlier observations about standard MPIS structures.

Proposition 1 *Any* $\alpha \in \mathfrak{L}_f$ *is satisfiable in the class of all MPIS models, if and only if* α *is satisfiable in the class of all standard MPIS models.*

2.2 Axiomatization of LMPIS

We now present an axiomatization for LMPIS. Note that for each i, the modal operator $[(i,\emptyset)]$ is interpreted as the global modal operator. Let $F \subseteq \mathcal{AC}$ and B, C be *infinite* subsets of $\mathcal{AC}$. Apart from the axioms of classical propositional logic, we have

Ax1. $[(i,F)](\alpha \to \beta) \to ([(i,F)]\alpha \to [(i,F)]\beta)$;
Ax2. $\alpha \to \langle(i,B)\rangle\alpha$;
Ax3. $\alpha \to [(i,B)]\langle(i,B)\rangle\alpha$;

Ax4. $\langle(i,B)\rangle\langle(i,B)\rangle\alpha \to \langle(i,B)\rangle\alpha$;
Ax5. $[(i,C)]\alpha \to [(i,B)]\alpha$ for $C \subseteq B$;
Ax6. $[(i,\emptyset)]\alpha \to [(j,F)]\alpha$;
Ax7. $(i,a,v) \to \neg(i,a,v')$, for $v \neq v'$;
Ax8. $\bigvee_{v\in\mathcal{VC}_a}(i,a,v)$;
Ax9. $(i,a,v) \to [(i,\{a\})](i,a,v)$;
Ax10. $((i,b,v) \wedge [(i,F\cup\{b\})]\alpha) \to [(i,F)]((i,b,v) \to \alpha)$;

Rules of inference:

$$N. \quad \frac{\alpha}{[(i,F)]\alpha} \qquad MP. \quad \frac{\begin{array}{c}\alpha \\ \alpha \to \beta\end{array}}{\beta}$$

The notion of theoremhood is defined in the standard way. We will write $\vdash \alpha$, if α is a theorem of the above deductive system. Observe that for finite B, it is not necessary to write the axioms Ax2-Ax4, as these can be deduced as theorems. One can easily prove the following.

Theorem 1 (Soundness) *If $\vdash \alpha$, then α is valid in the class of all MPIS models.*

Note that the modal operator $[(i,B)]$ is very similar to the distributed knowledge operator of epistemic logic. The relation corresponding to $[(i,B)]$ is given by $\bigcap_{b\in B} R(i,b)$. Completeness theorem of epistemic logic with distributed knowledge operator is proved, e.g. in [5, 17, 1]. But our approach here is different, as LMPIS has the extra feature of descriptors. This feature, in fact, makes the completeness proof much simpler. The canonical model $\mathfrak{M}^C := (W^C, \{R^C(i,B)\}_{i\in Ag, B\subseteq\mathcal{AC}}, m^C, V^C)$ is defined in a standard way over the collection of all maximally consistent sets of $\mathfrak{L}$-wffs, with $m^C(i,a,v) := \{w \in W^C : (i,a,v) \in w\}$. It can be proved that $\mathfrak{M}^C$ has all the properties of a MPIS-structure except (S4) and (S7). But these can be incorporated by considering the generated sub-model of $\mathfrak{M}^C$, using $R^C(i,\emptyset)$. This gives

Theorem 2 (Completeness) *If α is valid in the class of all MPIS models, then $\vdash \alpha$.*

Due to Proposition 1, completeness of $\mathfrak{L}_f$ with respect to the class of all standard MPIS models is also obtained. Thus $\mathfrak{L}_f$ serves as a logic for MPISs, which can express properties of indiscernibility relations corresponding to finite sets of attributes.

We note that $\mathfrak{L}_f$ can also be proved to be decidable. Let us also remark here that a trivial modification of LMPIS will lead to a logic for information systems (cf. Definition 1).

3 LMPIS in perspective

The salient features of LMPIS are as follows.

1. The semantics of LMPIS is based on structures of the form $\mathfrak{F} := (W, \{R(i,B)\}_{i\in Ag,\ B\subseteq \mathcal{AC}}, m)$.
2. The language of LMPIS includes attribute and attribute value constants.
3. The semantics of LMPIS is defined such that attribute, attribute value pairs are connected with the relations, just as in the case of information systems.
4. There is a sound and complete deductive system for LMPIS.

Let us briefly survey logics with one or more of these features.

Recall from Section 1 that given an information system $\mathcal{S}$ and a set B of attributes, we get the equivalence relation $Ind_{\mathcal{S}}(B)$. On the other hand, given an equivalence relation R on a domain U, one can define an information system $\mathcal{S}$ with domain U and a set B of attributes such that $Ind_{\mathcal{S}}(B) = R$. From this correspondence, it would seem that the modal system $S5$ would serve as a logic for information systems and for studying the indiscernibility relation. But Orłowska, in [12], observed that when we say two objects are indiscernible, the indiscernibility is meant not in the absolute sense, but with respect to a set of attributes. Thus, whenever we mention indiscernibility of objects, the concerned attribute set should be brought into the picture. A structure with *relative accessibility* relations was thus proposed for the study of indiscernibility relations. These are of the form $(W, \{R_B\}_{B\subseteq\mathcal{A}})$, called *information structure*, where W is a non-empty set, $\mathcal{A}$ is a non-empty set of parameters or attributes and for each $B \subseteq \mathcal{A}$, R_B is an equivalence relation satisfying

$$R_\emptyset = W \times W, \text{ and } R_{B\cup C} = R_B \cap R_C.$$

Orłowska cited the axiomatization of a logic with semantics based on information structures as an open problem. Later, Balbiani [1] gave a complete axiomatization of the set of wffs valid in every information structure, using the technique of *copying* introduced by Vakarelov [16]. However, we note that the language of Balbiani's logic does not contains attribute, attribute value constants.

Let us consider the question whether the information structure is suitable for studying information systems? If we are only interested in indiscernibility relations, then the answer appears to be yes. But, in reality, an information system is more than an information structure. The former actually provides information about what value an object takes for an attribute, and this is used to *generate* the indiscernibility relations, which is not the case in an information structure. Thus it seems that in order to define a logic for information systems, one must have attribute, attribute value pairs explicitly in the syntax.

Several logics are defined with this last-mentioned feature in their language [13, 10, 11, 9] (cf. [3]). Although the language of the logic NIL [13] contains descriptors (a, v), it does not connect it with the indiscernibility relations. The logic DIL presented in [9] does not have modal operators for indiscernibility or any other relations induced by information systems. So it can only express changes in attribute values of the objects with time, but not (changes in) set approximations. For the class of logics with attribute expressions defined in [10, 11], we do not know of complete axiomatizations.

4 Dynamic logic for MPISs

The presence of attribute, attribute value pairs in the language of LMPIS enables us to express the flow of information in terms of attribute and attribute values. In this section, we define a dynamic logic for MPISs. The alphabet of DLMPIS is that of LMPIS with the added symbols "," and ";". In order to define the wffs of DLMPIS, we specify what is meant by *information.*

Definition 7 *The set Inf of* information *is the smallest set such that*

- $(\phi, P, a, v), (\phi, P, j, a) \in Inf$*, where* $\phi \in \mathfrak{L},\ a \in \mathcal{AC}, v \in \mathcal{VC}_a$ *and* $P \subseteq Ag$*,*
- *if* $\sigma, \sigma' \in Inf$*, then* $\sigma;\sigma',\ \sigma \vee \sigma' \in Inf$*.*

An information of the form (ϕ, i, a, v) or (ϕ, i, j, a) will be called an *atomic information.* (ϕ, P, a, v) signifies that information is obtained by the agents $i \in P$ according to which the objects represented by ϕ take the value v for the attribute a. On the other hand, (ϕ, P, j, a) signifies that the agents belonging to the set P replace the information about the objects represented by ϕ regarding the attribute a with the information the agent j has about these objects regarding the same attribute a. Thus $((i, a, \epsilon), i, j, a)$ says that the agent i adopts the information regarding attribute a that agent j has about any object lying in the interpretation of (i, a, ϵ) (i.e. an object for which i has no information for attribute a). $\sigma \vee \sigma'$ says that we have got the information that either σ is the case or σ'. $\sigma;\sigma'$ means that first we get the information σ and then σ'.

The set $\mathfrak{L}^D$ of wffs of DLMPIS is obtained by extending the formation rules of LMPIS-wffs with the clause:

$$\text{if } \sigma \in Inf,\ \alpha \in \mathfrak{L}^D \text{ then } [\sigma]\alpha \in \mathfrak{L}^D.$$

Let $\mathfrak{L}^D_f$ denote the set of all DLMPIS-wffs which do not involve any modal operator $[(i, B)]$, where B is infinite. This fragment of DLMPIS gives the dynamic logic for MPISs.

4.1 Semantics

We first define the notion of *update* $\mathfrak{M}^I$ of a (standard) MPIS model $\mathfrak{M} := (W, \{R(i, B)\}_{i \in Ag,\ B \subseteq \mathcal{AC}}, m, V)$, given an atomic information I of form (ϕ, i, a, v) or (ϕ, i, j, a).
For $\phi \in \mathfrak{L}$, let $[\![\phi]\!]_{\mathfrak{M}}$ denote the set $\{w \in W : \mathfrak{M}, w \models \phi\}$.

Definition 8 *Let* $I := (\phi, i, a, v)$*.* $\mathfrak{M}^I := (W, \{R^I(P, B)\}_{P \subseteq Ag,\ B \subseteq \mathcal{AC}}, m^I, V)$*, where*

- m^I *is given as follows:*

- $m^I(j,b,u) = m(j,b,u)$ *for* $j \neq i$ *or* $b \neq a$,
- $m^I(i,a,v') := m(i,a,v') \setminus [\![\phi]\!]_{\mathfrak{M}}$ *for* $v \neq v'$,
- $m^I(i,a,v) := m(i,a,v) \cup [\![\phi]\!]_{\mathfrak{M}}$.

- $R^I(j,B), B \subseteq \mathcal{AC}$, *are defined as follows.*
  - *For* $a \notin B$ *or* $j \neq i$, $R^I(j,B) = R(j,B)$.
  - *If* $a \in B$, *then* $(x,y) \in R^I(i,B)$ *if and only if any of the following holds:*
    * $x,y \notin [\![\phi]\!]_{\mathfrak{M}}$ *and* $(x,y) \in R(i,B)$,
    * $x \in [\![\phi]\!]_{\mathfrak{M}}$, $y \notin [\![\phi]\!]_{\mathfrak{M}}$, $y \in m(i,a,v)$ *and* $(x,y) \in R(i, B \setminus \{a\})$,
    * $x \notin [\![\phi]\!]_{\mathfrak{M}}$, $y \in [\![\phi]\!]_{\mathfrak{M}}$, $x \in m(i,a,v)$ *and* $(x,y) \in R(i, B \setminus \{a\})$,
    * $x,y \in [\![\phi]\!]_{\mathfrak{M}}$ *and* $(x,y) \in R(i, B \setminus \{a\})$.

Definition 9 *Let* $I := (\phi, i, j, a)$. $\mathfrak{M}^I := (W, \{R^I(P,B)\}_{P \subseteq Ag,\ B \subseteq \mathcal{AC}}, m^I, V)$, *where*

- m^I *is given as follows:*
  - $m^I(k,b,v) = m(k,b,v)$ *for* $k \neq i$, *or* $b \neq a$,
  - $m^I(i,a,v) = (m(j,a,v) \cap [\![\phi]\!]_{\mathfrak{M}}) \cup (m(i,a,v) \cap [\![\neg\phi]\!]_{\mathfrak{M}})$.

- $R^I(k,B), B \subseteq \mathcal{AC}$, *are defined as follows.*
  - *For* $a \notin B$ *or* $k \neq i$, $R^I(k,B) = R(k,B)$.
  - *If* $a \in B$, *then* $(x,y) \in R^I(i,B)$ *if and only if any of the following holds:*
    * $x,y \notin [\![\phi]\!]_{\mathfrak{M}}$ *and* $(x,y) \in R(i,B)$,
    * $x \in [\![\phi]\!]_{\mathfrak{M}}$, $y \notin [\![\phi]\!]_{\mathfrak{M}}$, $(x,y) \in R(i, B \setminus \{a\})$ *and* $x \in m(j,a,v)$, $y \in m(i,a,v)$ *for some* v,
    * $x \notin [\![\phi]\!]_{\mathfrak{M}}$, $y \in [\![\phi]\!]_{\mathfrak{M}}$, $(x,y) \in R(i, B \setminus \{a\})$ *and* $x \in m(i,a,v)$, $y \in m(j,a,v)$ *for some* v,
    * $x,y \in [\![\phi]\!]_{\mathfrak{M}}$ *and* $(x,y) \in R(j,\{a\}) \cap R(i, B \setminus \{a\})$.

Note that in either case, $R^I(i,\emptyset) = R(i,\emptyset)$. It can be shown that $\mathfrak{M}^I$ is a (standard) MPIS model.

Each information σ induces a relation $\mathfrak{R}_\sigma$ on the set of all MPIS models:

- for atomic information $I := (\phi,i,a,v)$ or $I := (\phi,i,j,a)$, $\mathfrak{M}\mathfrak{R}_I\mathfrak{M}'$ if and only if $\mathfrak{M}' = \mathfrak{M}^I$,
- for $I := (\phi,P,a,v)$, $\mathfrak{R}_I := \mathfrak{R}_{I_1};\mathfrak{R}_{I_2};\ \cdots\ ;\mathfrak{R}_{I_m}$, where $P := \{i_1, i_2, \ldots, i_m\}$ and $I_k := (\phi, i_k, a, v)$,
- for $I := (\phi,P,j,a)$, $\mathfrak{R}_I := \mathfrak{R}_{I_1};\mathfrak{R}_{I_2};\ \cdots\ ;\mathfrak{R}_{I_m}$, where $P := \{i_1, i_2, \ldots, i_m\}$ and $I_k := (\phi, i_k, j, a)$,

- $\mathfrak{R}_{\sigma;\sigma'} := \mathfrak{R}_\sigma; \mathfrak{R}'_\sigma$,
- $\mathfrak{R}_{\sigma\vee\sigma'} := \mathfrak{R}_\sigma \cup \mathfrak{R}'_\sigma$.

$R_1; R_2$ denotes the composition of the two relations R_1 and R_2. For $\mathfrak{R}_{I_i}$ and $\mathfrak{R}_{I_j}$, observe that $\mathfrak{R}_{I_i}; \mathfrak{R}_{I_j} = \mathfrak{R}_{I_j}; \mathfrak{R}_{I_i}$.

So $\mathfrak{R}_I$, where I is an atomic information, behaves like a function, i.e. for each $\mathfrak{M}$, there exists a unique $\mathfrak{M}'$ such that $\mathfrak{M}\mathfrak{R}_I\mathfrak{M}'$. Further, we could have information σ which is non-deterministic, i.e. update of a MPIS-model with this information could result in more than one MPIS-model.

Satisfiability of the wff $\alpha \in \mathfrak{L}^D$ in a model $\mathfrak{M}$ at the world w is defined by extending Definition 6:

$$\mathfrak{M}, w \models [\sigma]\alpha \text{ if and only if } \mathfrak{M}', w \models \alpha, \text{ for all } \mathfrak{M}' \text{ such that } \mathfrak{M}\mathfrak{R}_\sigma\mathfrak{M}'.$$

Let us pause to make a comparison with dynamic epistemic logic. In public announcement logic [19], the updated model is obtained from a model by restricting it to some subset of the domain. More complex epistemic actions, considered e.g. in [6, 2], may result in the refinement of accessibility relations while the domain of the model remains unchanged, and they may even result in the enlargement of the domain of the model. Contrary to these, in the case of DLMPIS, the domain of the model remains unchanged whatever information is provided, and relations can change in any manner. It could happen that two objects not related earlier become related after the update.

In dynamic epistemic logic, usually an action or flow of information does not change the value of the atomic propositions. However, in [18], an action called 'public assignment' expressed as $[p := \phi]$, is considered which *only* affects the atomic information p. The update of the epistemic model $\mathfrak{M} := (U, \{R_n\}_{n\in Ag}, V)$ with respect to the public assignment $[p := \phi]$ gives the model $(U, \{R_n\}_{n\in Ag}, V')$, where $V'(q) = V(q)$ for $q \neq p$ and $V'(p) = [\![\phi]\!]_{\mathfrak{M}}$. Note that relations remain unchanged in this case.

In the case of DLMPIS, we have two kinds of atomic wffs: the propositional variables and descriptors. Although we have the validity of $[\sigma]p \leftrightarrow p$, we may have information σ and descriptor (i, a, v) such that the wff $[\sigma](i, a, v) \leftrightarrow (i, a, v)$ is not valid. This shows that atomic facts may change here. We also note that the update due to public assignment is different from the one due to the actions of the kind (ϕ, i, a, v). In fact, in the latter case, not only is the assignment to the atomic wff (i, a, v) affected, assignments to the other atomic wffs as well as the relations may also get affected. This is because relations are connected with the attribute, attribute value pairs. Another difference is that the value $m^I(i, a, v)$ of the atomic wff (i, a, v) *contains* the extension $[\![\phi]\!]_{\mathfrak{M}}$ of ϕ in $\mathfrak{M}$, and is not necessarily equal to it.

4.2 Reduction Axioms

A deductive system for DLMPIS can be given by taking the axiom schema and inference rules of LMPIS along with the following set of reduction axioms.

Ax11. $[I]p \leftrightarrow p$.

Ax12. $[I]\neg\alpha \leftrightarrow \neg[I]\alpha$.

Ax13. $[I](\alpha \rightarrow \beta) \leftrightarrow ([I]\alpha \rightarrow [I]\beta)$.

Ax14(a). $[(\phi, i, a, v)](k, b, v') \leftrightarrow (k, b, v')$, for $k \neq i$, or $b \neq a$.

Ax14(b). $[(\phi, i, a, v)](i, a, v') \leftrightarrow (\neg\phi \wedge (i, a, v')),\ v' \in (\mathcal{VC} \cup \{\epsilon\}) \setminus \{v\}$.

Ax14(c). $[(\phi, i, a, v)](i, a, v) \leftrightarrow (\neg\phi \rightarrow (i, a, v))$.

Ax15(a). $[(\phi, i, a, v)][(k, B)]\alpha \leftrightarrow [(k, B)]\alpha$, where $a \notin B$ or $i \neq k$.

Ax15(b). For $a \in B$,

$$\begin{aligned}
[(\phi, i, a, v)][(i, B)]\alpha \leftrightarrow\ & ((\phi \rightarrow [(i, B \setminus \{a\})](\phi \rightarrow [(\phi, i, a, v)]\alpha)) \\
\wedge\ & (\phi \rightarrow [(i, B \setminus \{a\})](\neg\phi \wedge (i, a, v) \rightarrow [(\phi, i, a, v)]\alpha)) \\
\wedge\ & ((\neg\phi \wedge (i, a, v)) \rightarrow [(i, B \setminus \{a\})](\phi \rightarrow [(\phi, i, a, v)]\alpha)) \\
\wedge\ & (\neg\phi \rightarrow [(i, B)](\neg\phi \rightarrow [(\phi, i, a, v)]\alpha)).
\end{aligned}$$

Ax16(a). $[(\phi, i, j, a)](k, b, v) \leftrightarrow (k, b, v)$, for $k \neq i$, or $b \neq a$.

Ax16(b). $[(\phi, i, j, a)](i, a, v) \leftrightarrow [((j, a, v) \wedge \phi) \vee ((i, a, v) \wedge \neg\phi)]$.

Ax17(a). $[(\phi, i, j, a)][(k, B)]\alpha \leftrightarrow [(k, B)]\alpha$, where $a \notin B$ or $i \neq k$.

Ax17(b). For $a \in B$,

$$\begin{aligned}
& [(\phi, i, j, a)][(i, B)]\alpha \leftrightarrow [(\neg\phi \rightarrow [(i, B)](\neg\phi \rightarrow [(\phi, i, j, a)]\phi)) \\
\wedge\ & (\phi \rightarrow \bigwedge_{v \in \mathcal{VC}_a} ((j, a, v) \rightarrow [(i, B \setminus \{a\})](((i, a, v) \wedge \neg\phi) \rightarrow [(\phi, i, j, a)]\phi))) \\
\wedge\ & (\neg\phi \rightarrow \bigwedge_{v \in \mathcal{VC}_a} ((i, a, v) \rightarrow [(i, B \setminus \{a\})](((j, a, v) \wedge \phi) \rightarrow [(\phi, i, j, a)]\phi))) \\
\wedge\ & (\phi \rightarrow \bigwedge_{v \in \mathcal{VC}_a} ((j, a, v) \rightarrow [(i, B \setminus \{a\})]((j, a, v) \wedge \phi) \rightarrow [(\phi, i, j, a)]\phi))].
\end{aligned}$$

18(a). For $P = \{i_1, i_2, \ldots, i_m\}$,
$[(\phi, P, a, v)]\alpha \leftrightarrow [I_1][I_2] \cdots [I_m]\alpha$, where $I_k := (\phi, i_k, a, v)$.

18(b). For $P = \{i_1, i_2, \ldots, i_m\}$,
$[(\phi, P, j, a)]\alpha \leftrightarrow [I_1][I_2] \cdots [I_m]\alpha$, where $I_k := (\phi, i_k, j, a)$.

Ax19. $[\sigma; \sigma']\alpha \leftrightarrow [\sigma][\sigma']\alpha$.

Ax20. $[\sigma \vee \sigma']\alpha \leftrightarrow [\sigma]\alpha \wedge [\sigma']\alpha$.

Rules of inference:

$$N(I).\quad \frac{\alpha}{[I]\alpha}$$

5 Conclusions

Multi-agent partial knowledge information systems (MPISs) are defined, and a logic for these structures is presented along with its dynamic version.

We have considered knowledge operators of the kind $[(i, B)]$, $i \in Ag$, in the logic LMPIS. One may think of enriching the language with the distributed knowledge operator $[(P, B)]$, $P \subseteq Ag$. The corresponding relation will have the property $R(P, B) = \bigcap_{i \in P} \bigcap_{b \in B} R(i, b)$. It is easy to extend the axiom schema of LMPIS to this case, but completeness is an open question. It may be remarked that due to the presence of descriptors in the language, the technique of copying employed by Balbiani to prove the completeness of epistemic logic with distributed knowledge will not work.

The work in this paper is a first step towards a formal study of dynamic aspects of information systems and indiscernibility relations. There are many issues which need further work. For instance, we have restricted our study to consider finite sets of attribute values Val_a for each attribute a. Although in practical problems we usually consider finite sets, one may check the consequences of removing the restriction. Similarly, one would like to extend this work to incomplete information systems where an object-attribute pair is mapped to a *set of* attribute values. This represents uncertainty, in the sense that we know some possible attribute values that an object may take for an attribute, but do not know exactly which one. In this kind of situation one could get information which reduces or removes this uncertainty. Moreover, in this case, relations other than the indiscernibility, such as inclusion or similarity, may be considered.

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