Epistemic change and factual change

Barteld Kooi
Department of Philosophy
University of Groningen
B.P.Kooi@rug.nl
www.philos.rug.nl/~barteld

Overview

- dynamic epistemic languages
- semantics
- expressive power
- belief revision
- conclusion

Dynamic epistemic languages

- p
- $\neg\varphi$
- $\varphi \wedge \psi$
- $[a]\varphi$

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- $[B^*]\varphi$ $\quad C$
- $[(B; ?\varphi)^*]\psi$

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- $[(B; ?\varphi)^*]\psi$ $\quad R$

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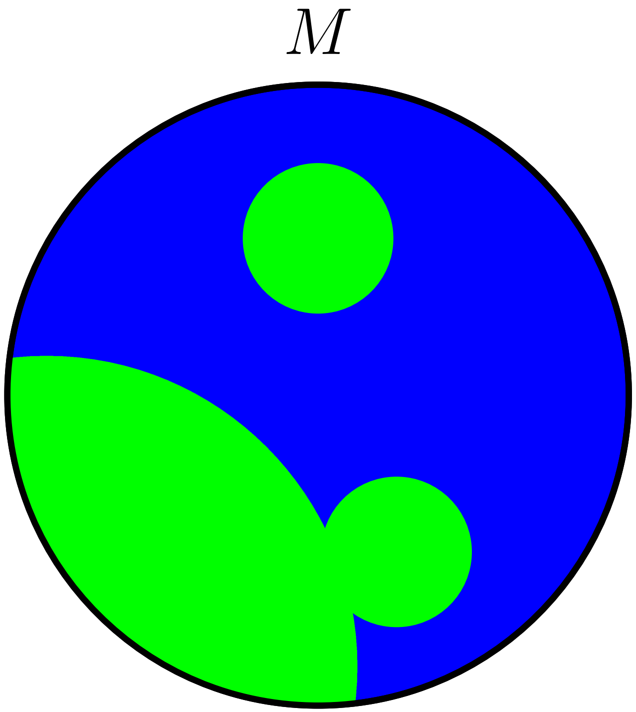
- $[B^*]\varphi$ $\quad C$
- $[(B; ?\varphi)^*]\psi$ $\quad R$

$\mathscr{L}_{ASR}$

Public announcements: semantics



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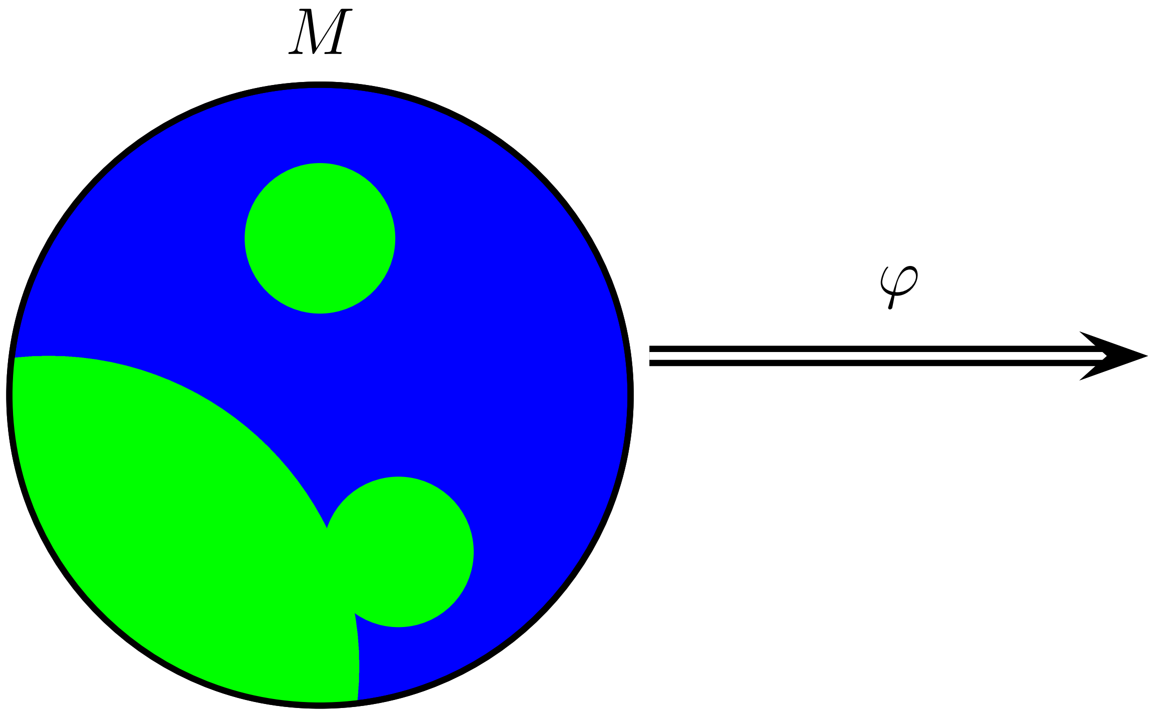

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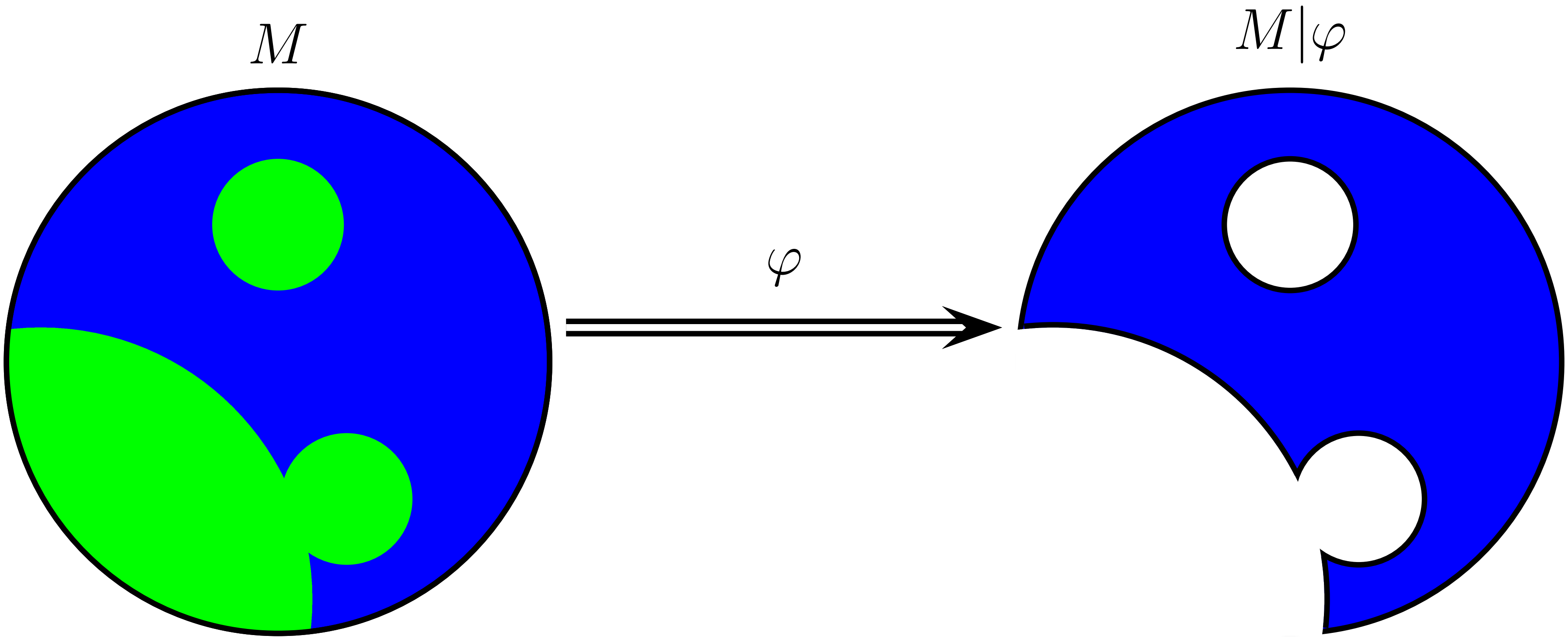

Substitution: semantics

A substitution is a partial function $\sigma : P \rightharpoonup \mathscr{L}$ with a finite domain.

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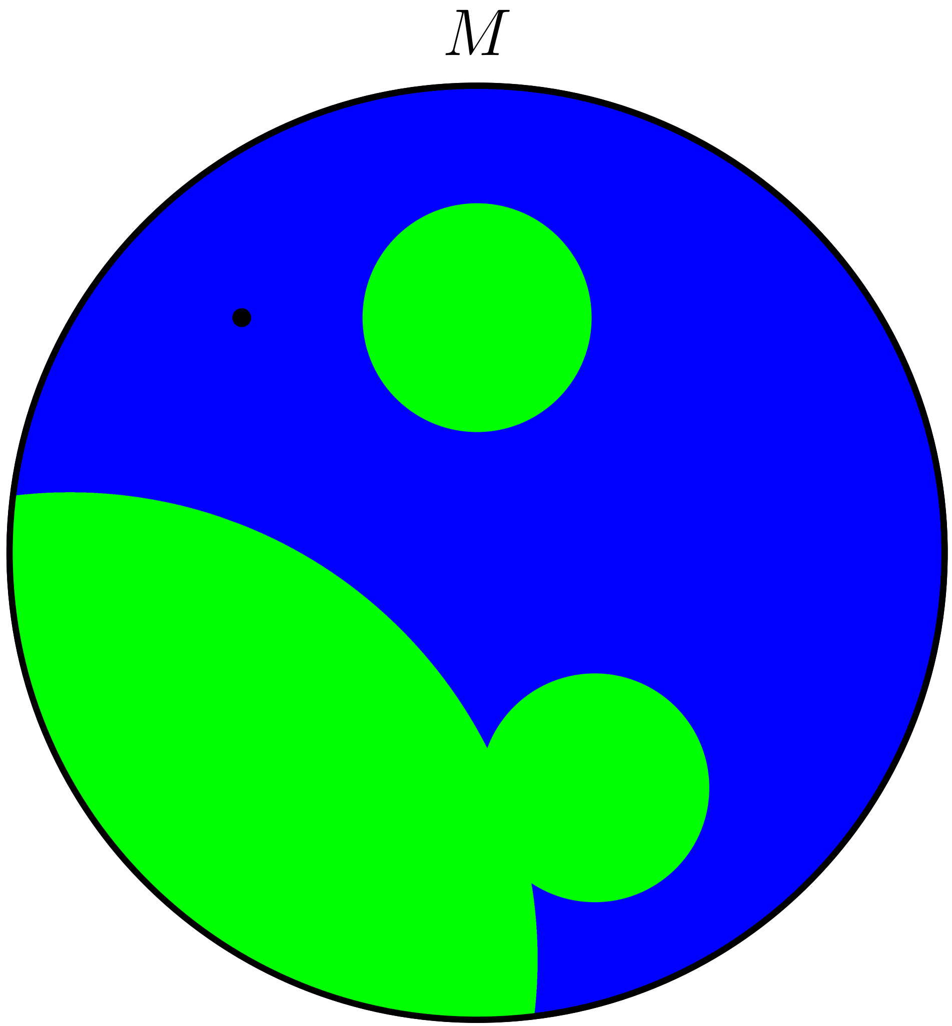
$$(M, w) \models [\sigma]\varphi \quad \text{iff} \quad (M^\sigma, w) \models \varphi$$

where $M^\sigma = (W, R, V^\sigma)$:

$$V^\sigma(p) = \begin{cases} [\![\varphi]\!] & \text{if } p \in \mathsf{dom}(\sigma) \\ V(p) & \text{otherwise} \end{cases}$$

Relativize common knowledge: semantics

Consider $[(B; ?\varphi)^*]\psi$.



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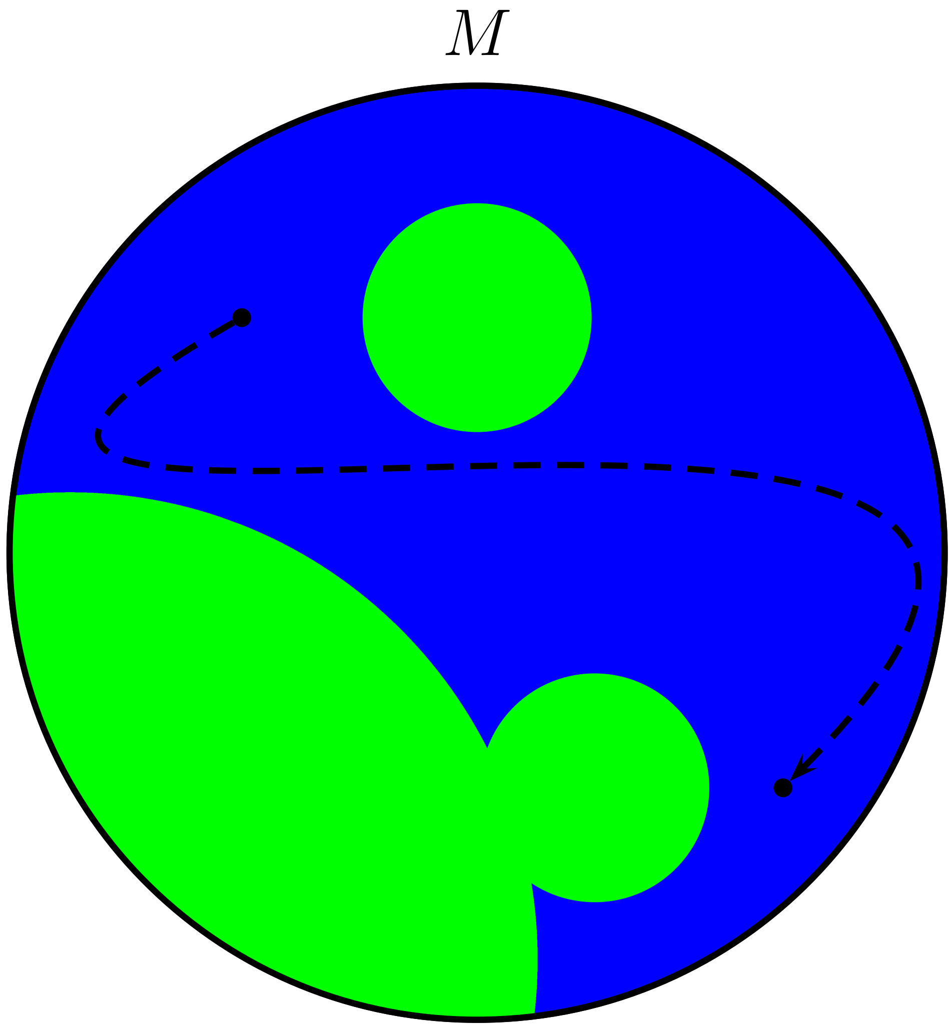

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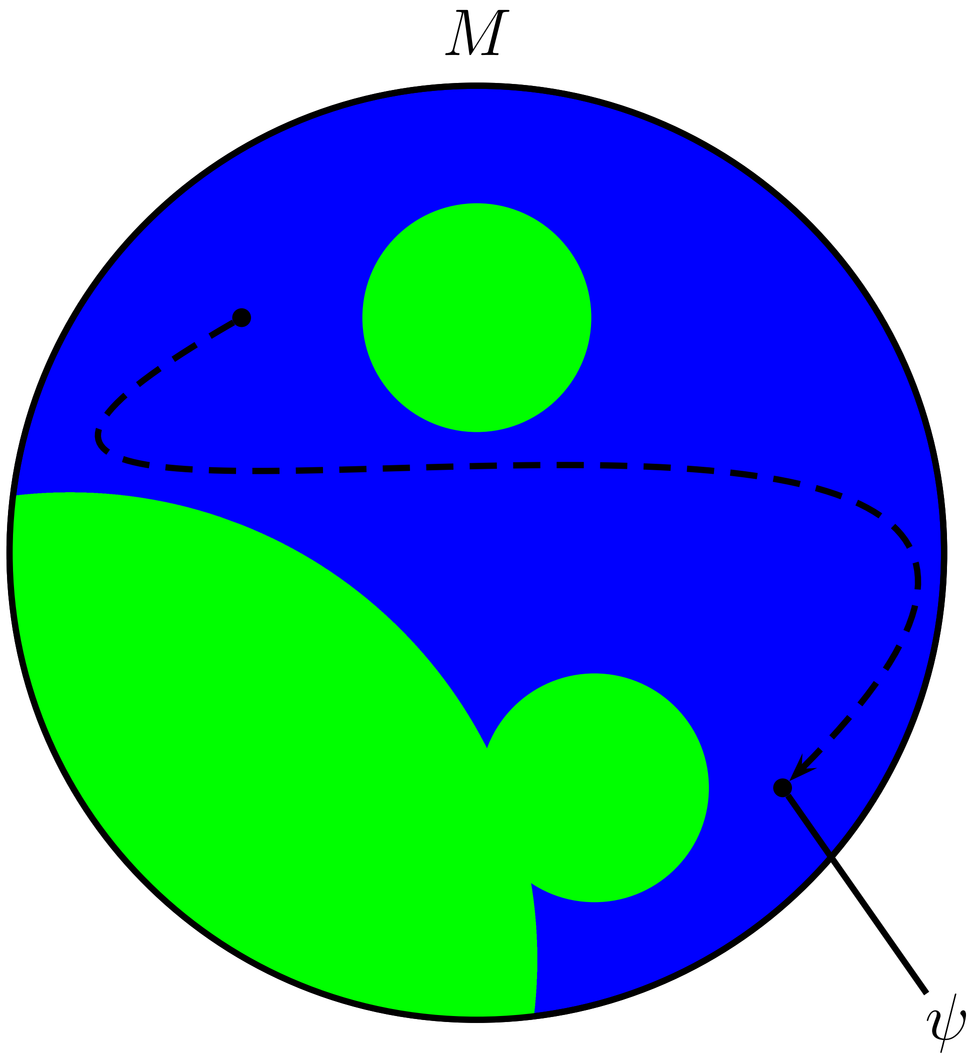

Reduction axioms

$$[\varphi]p \leftrightarrow (\varphi \rightarrow p)$$

$$[\varphi]\neg\psi \leftrightarrow (\varphi \rightarrow \neg[\varphi]\psi)$$

$$[\varphi](\psi \wedge \chi) \leftrightarrow ([\varphi]\psi \wedge [\varphi]\chi)$$

$$[\varphi][a]\psi \leftrightarrow (\varphi \rightarrow [a][\varphi]\psi)$$

$$[\varphi][\psi]\chi \leftrightarrow [\varphi \wedge [\varphi]\psi]\chi$$

$$[\varphi][(B;?\psi)^*]\chi \leftrightarrow [(B;?(\varphi \wedge [\varphi]\psi))^*][\varphi]\chi$$

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$$[\varphi][a]\psi \leftrightarrow (\varphi \rightarrow [a][\varphi]\psi)$$

$$[\varphi][\psi]\chi \leftrightarrow [\varphi \wedge [\varphi]\psi]\chi$$

$$[\varphi][(B;?\psi)^*]\chi \leftrightarrow [(B;?(\varphi \wedge [\varphi]\psi))^*][\varphi]\chi$$

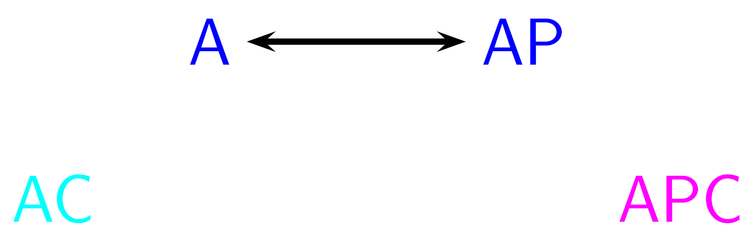
(Baltag, Moss, Solecki): there is no reduction axiom for

$$[\varphi][B^*]\psi$$

Expressive power

A $\longleftrightarrow$ AP

Expressive power



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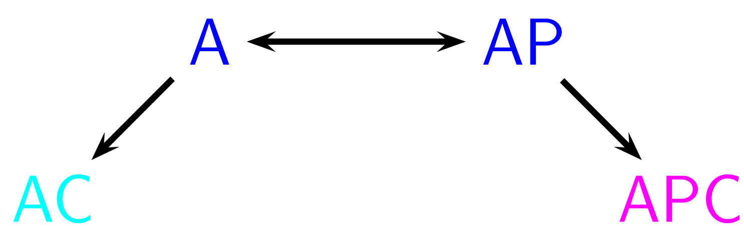

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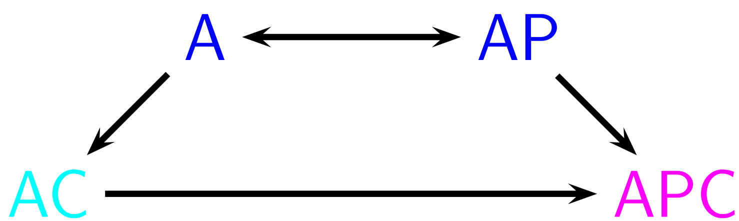

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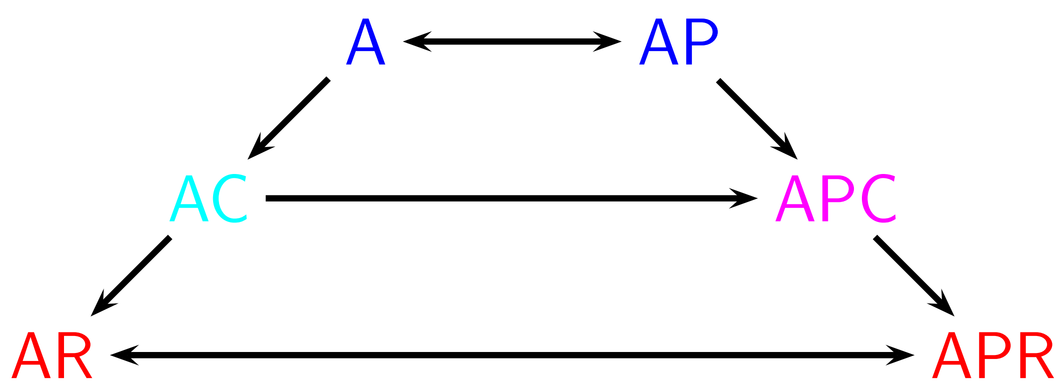
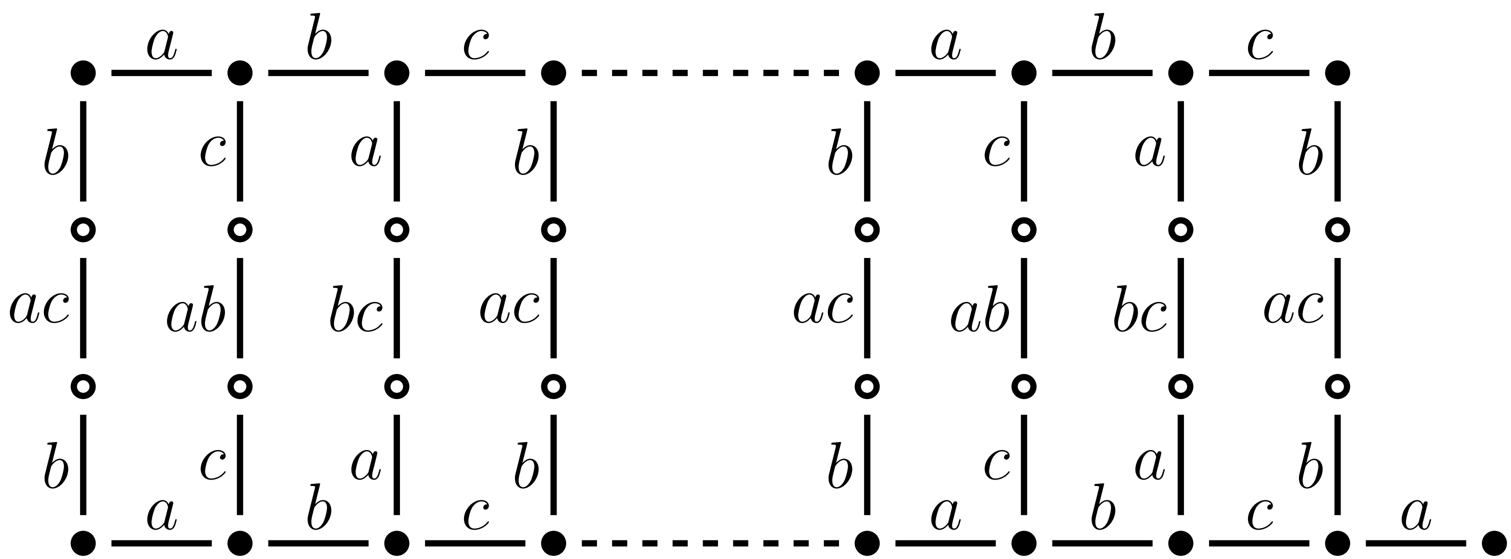


Top or bottom?



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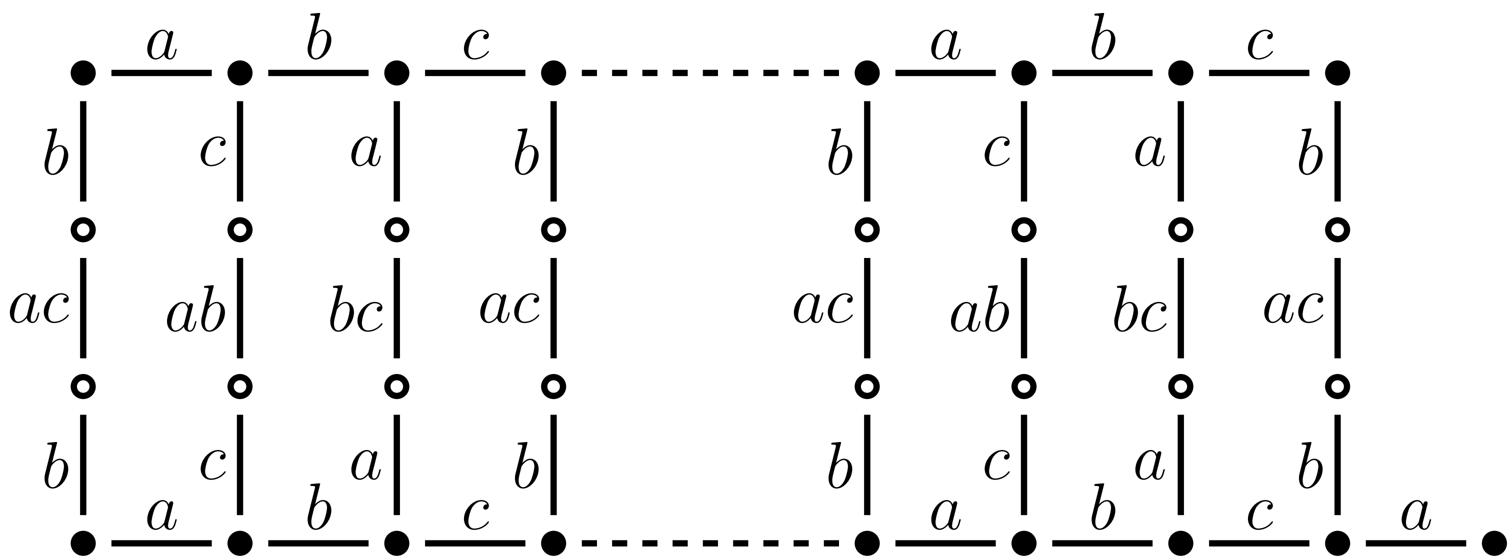


$$[(abc;?p)^*]\neg[abc]p$$

Reduction axioms

$$[\sigma]p \leftrightarrow \sigma(p)$$

$$[\sigma]\neg\psi \leftrightarrow \neg[\sigma]\psi$$

$$[\sigma](\psi \wedge \chi) \leftrightarrow ([\sigma]\psi \wedge [\sigma]\chi)$$

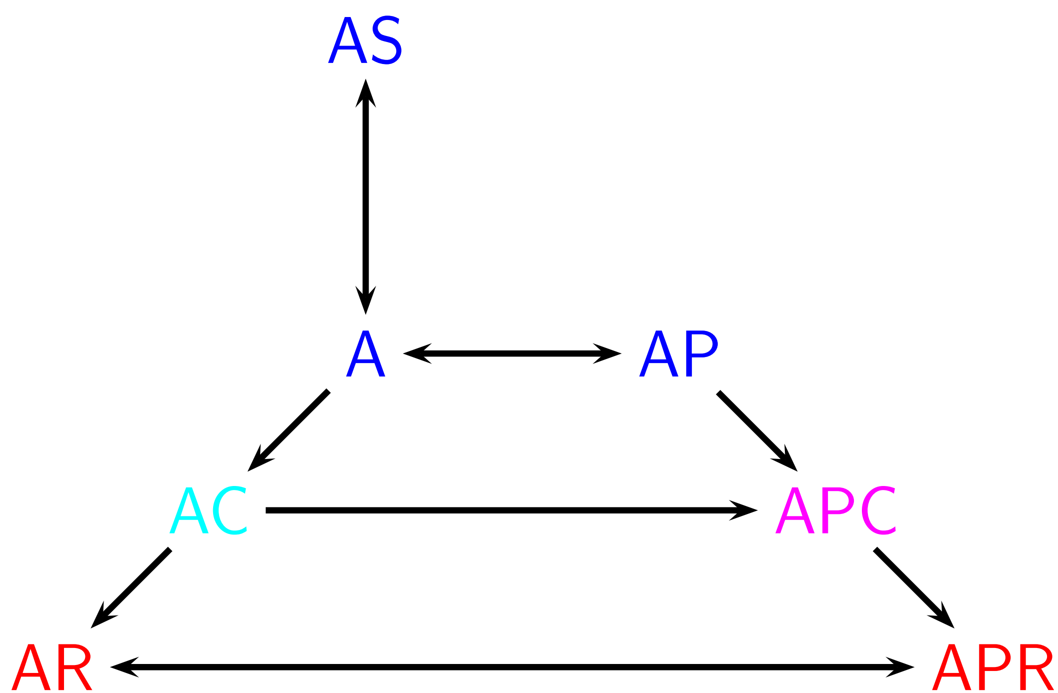
$$[\sigma][a]\psi \leftrightarrow [a][\sigma]\psi$$

$$[\sigma][B^*]\varphi \leftrightarrow [B^*][\sigma]\varphi$$

$$[\sigma][(B;?\varphi)^*]\psi \leftrightarrow [(B;?[\sigma]\varphi)^*][\sigma]\psi$$

$$[\sigma][\tau]\varphi \leftrightarrow [\sigma(\tau)]\varphi$$

Expressive power



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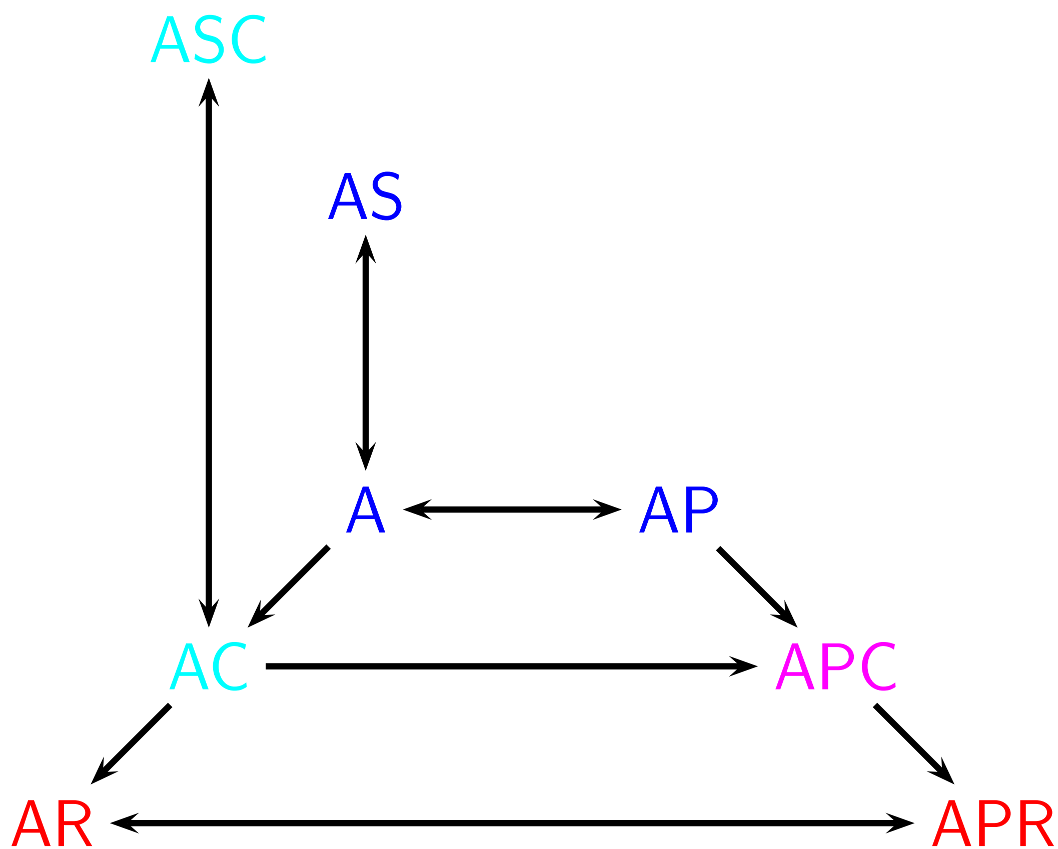

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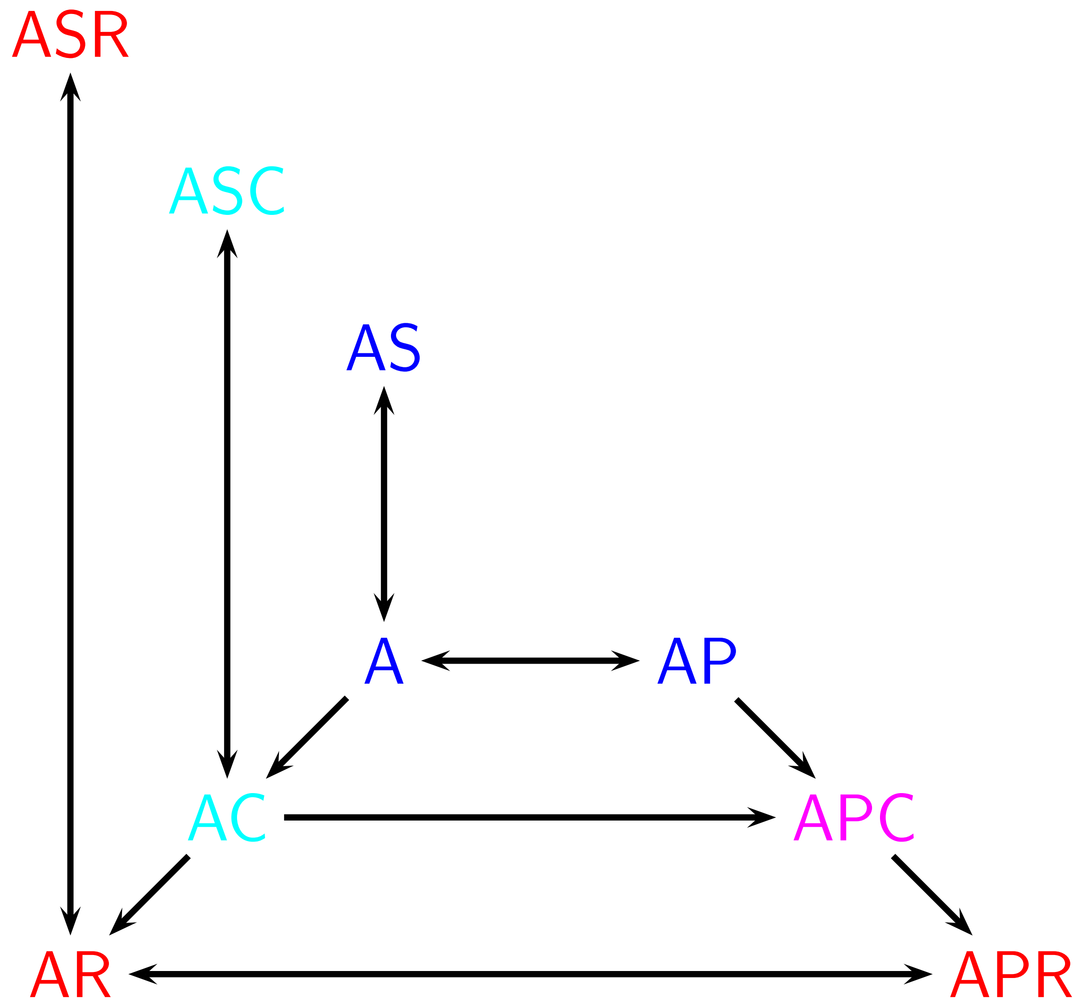

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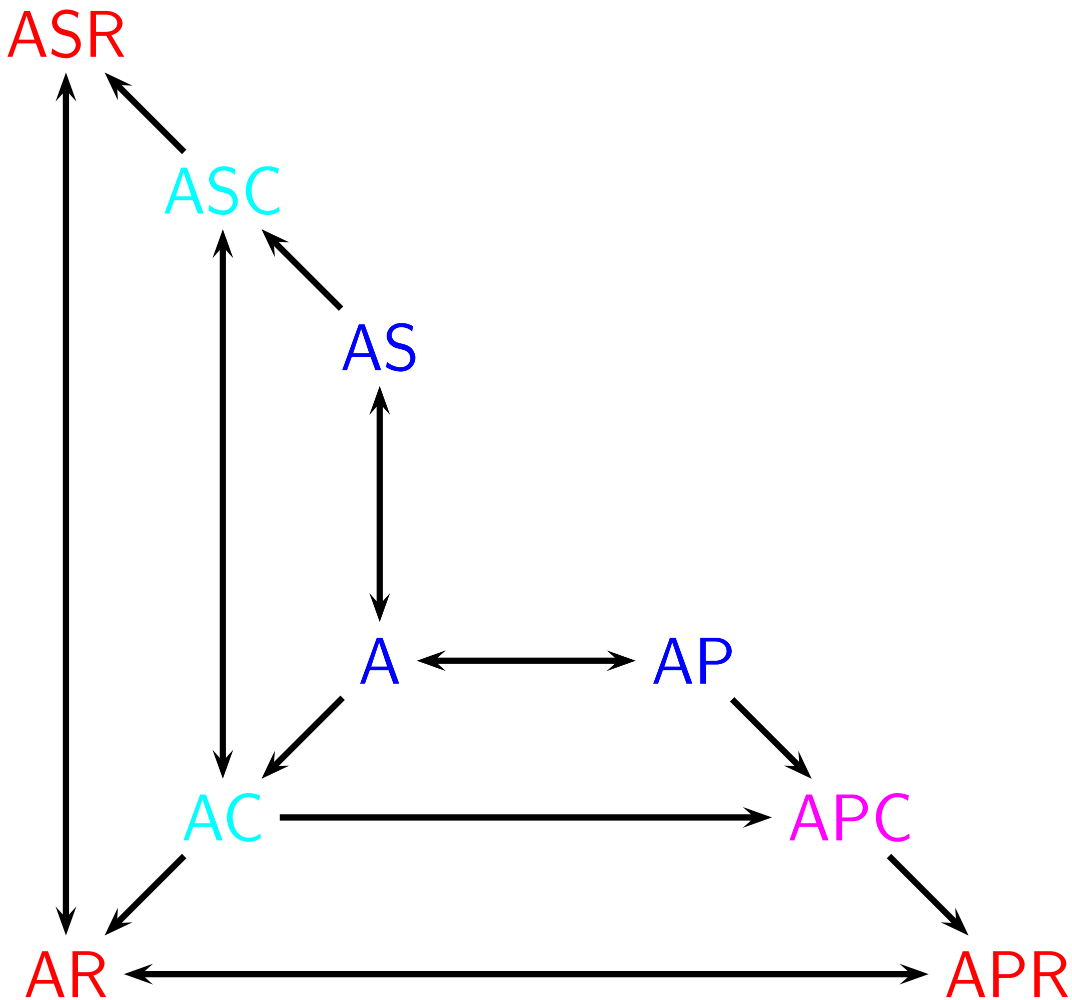

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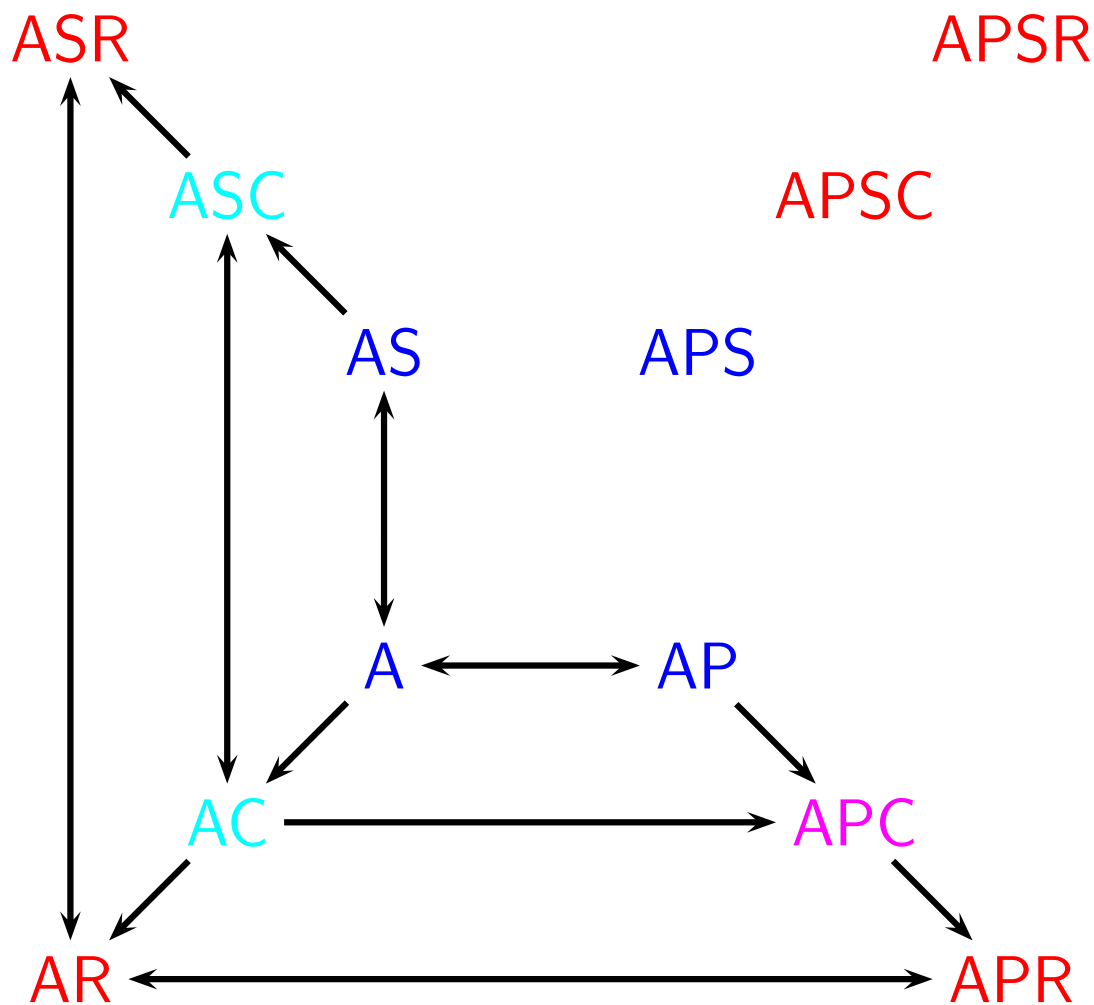

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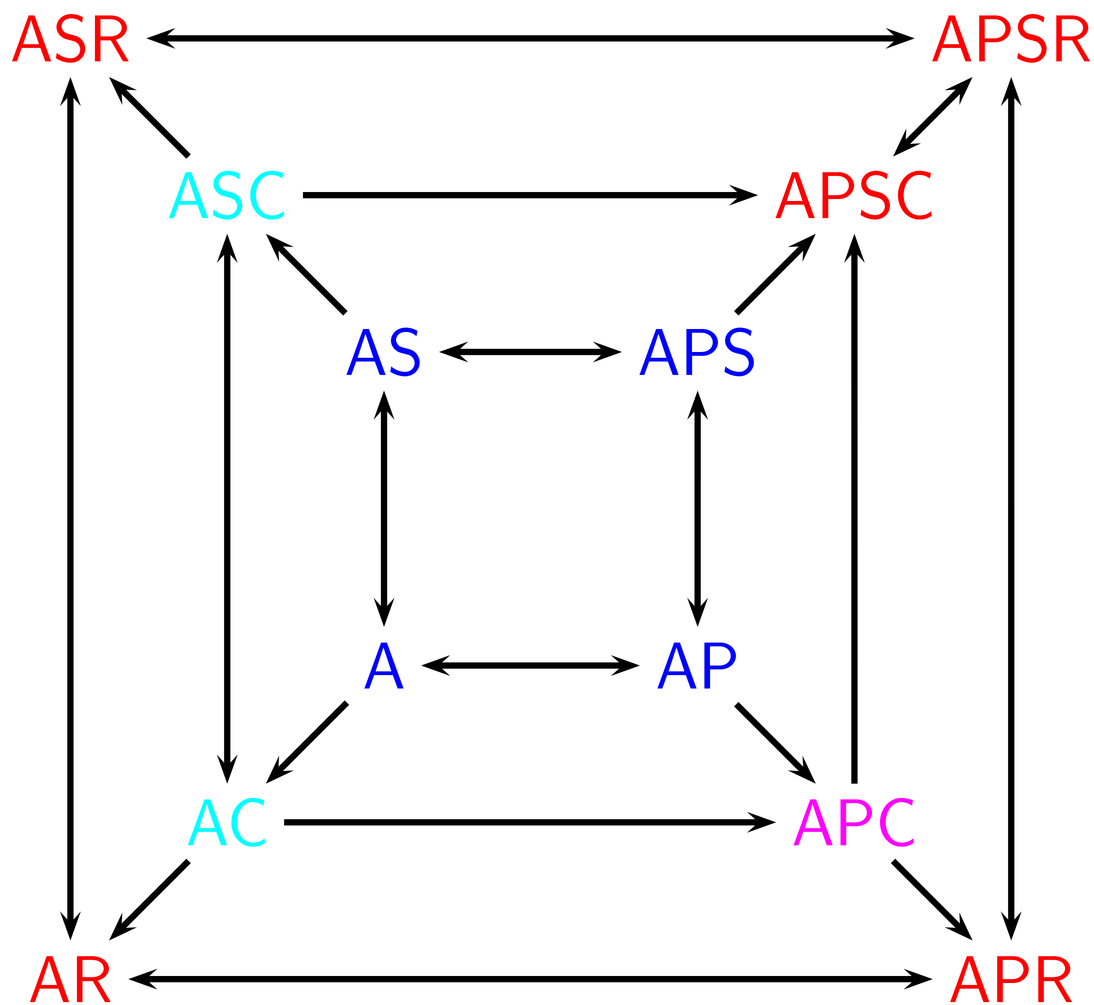

public announcements and substitutions

$$[\varphi][\sigma]\psi$$
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Translation axiom

$$[(B;?\varphi]\psi \leftrightarrow (\psi \wedge [B][p := \psi][\varphi][B^*]p)$$

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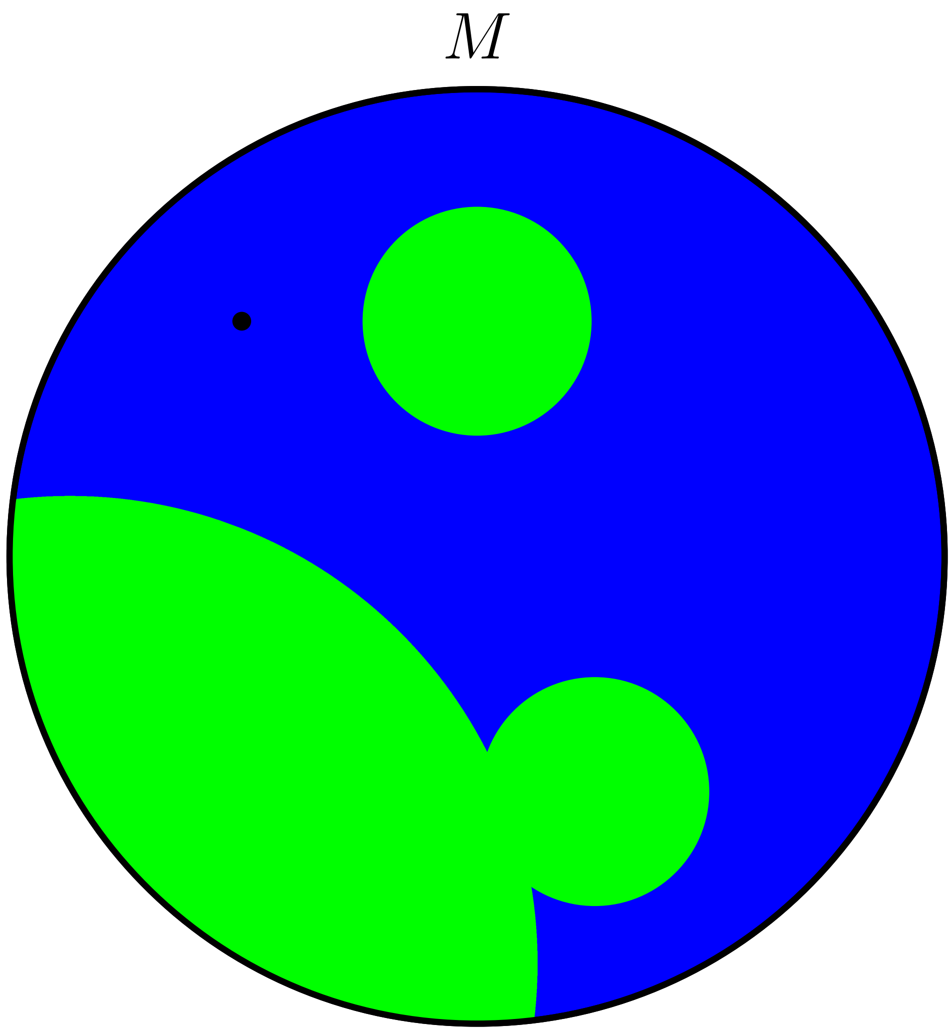

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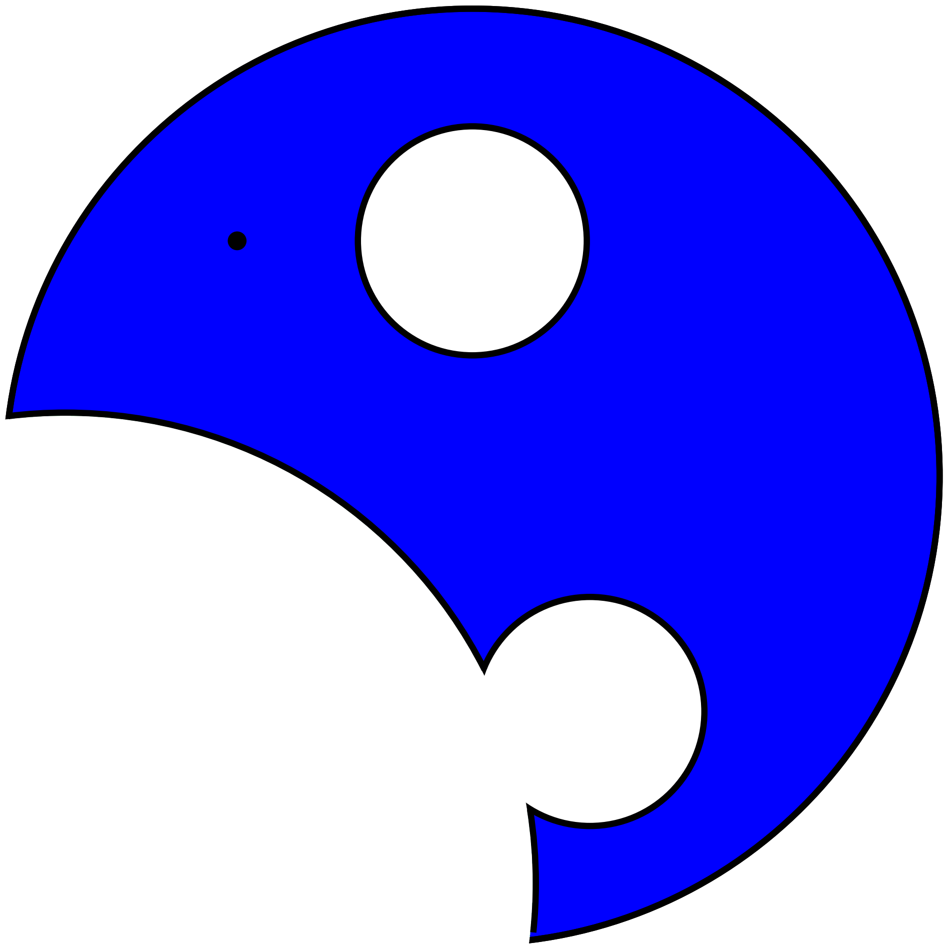
$M|\varphi$



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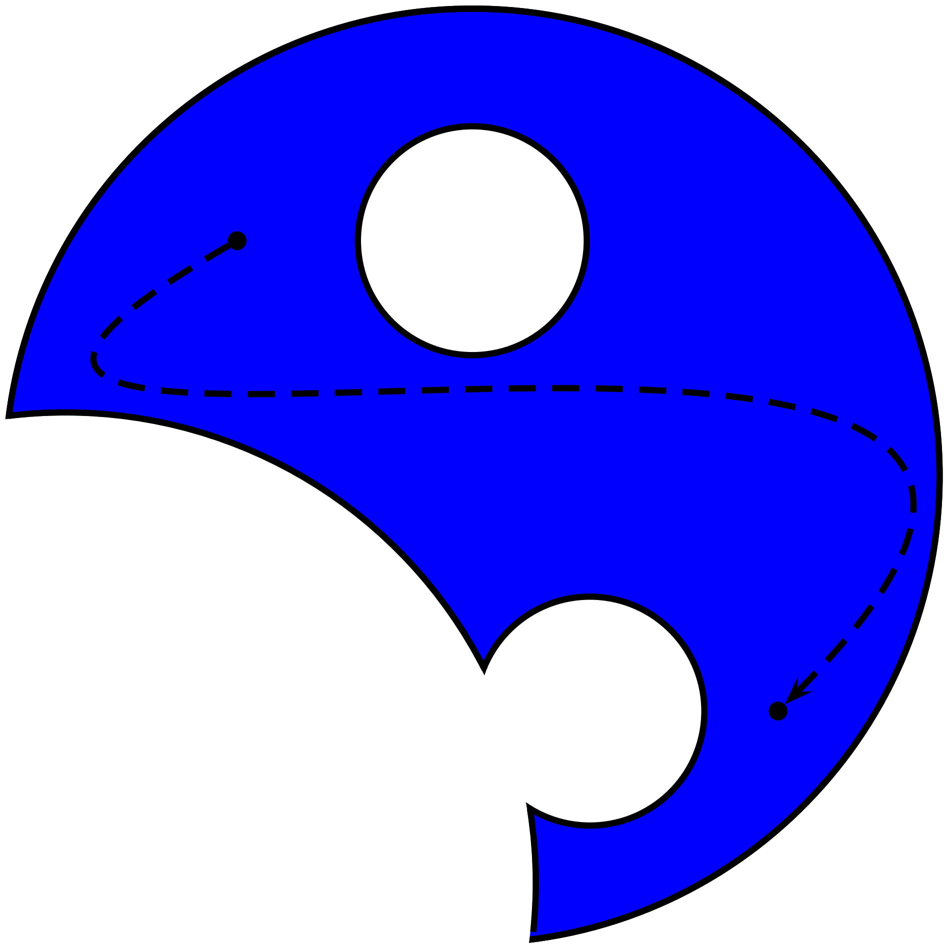

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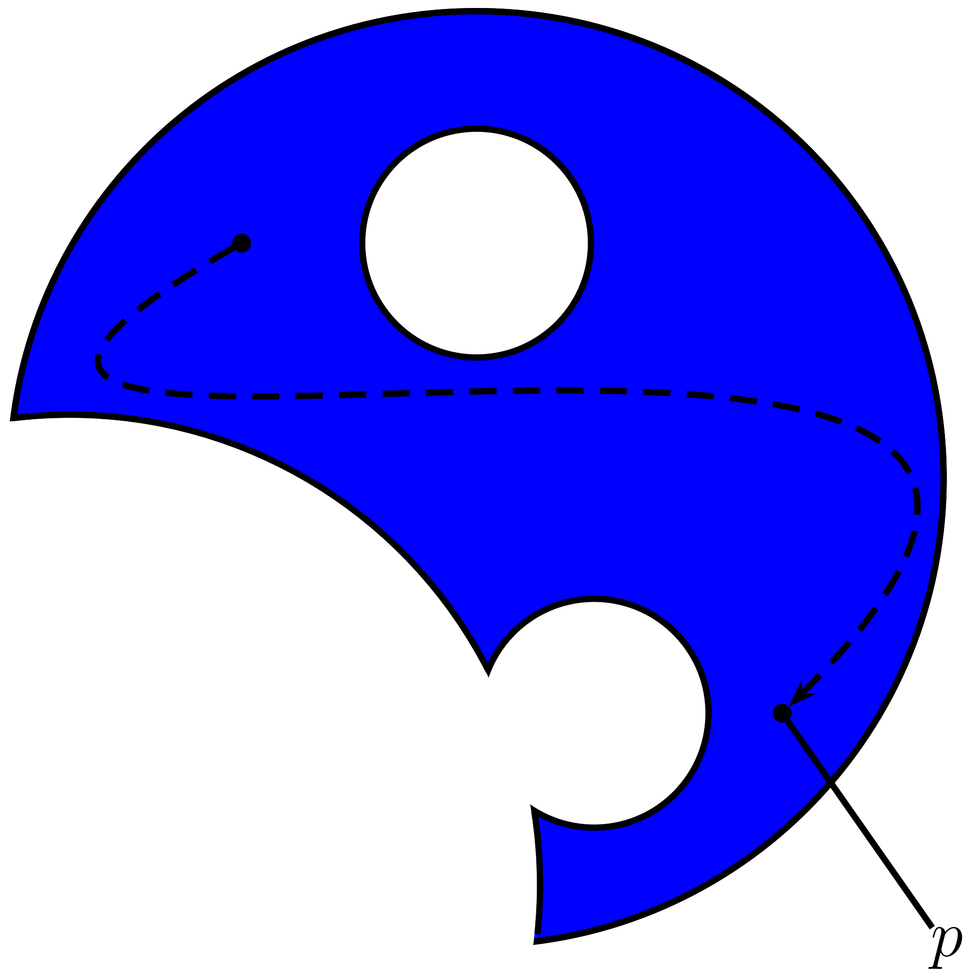

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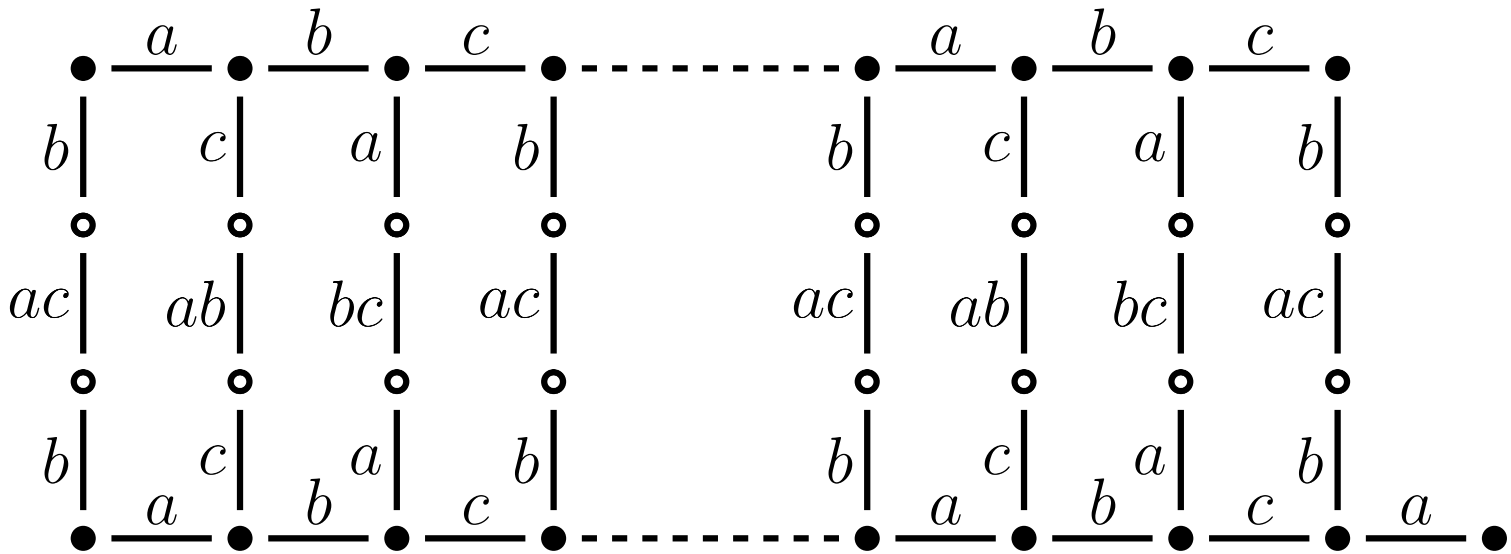
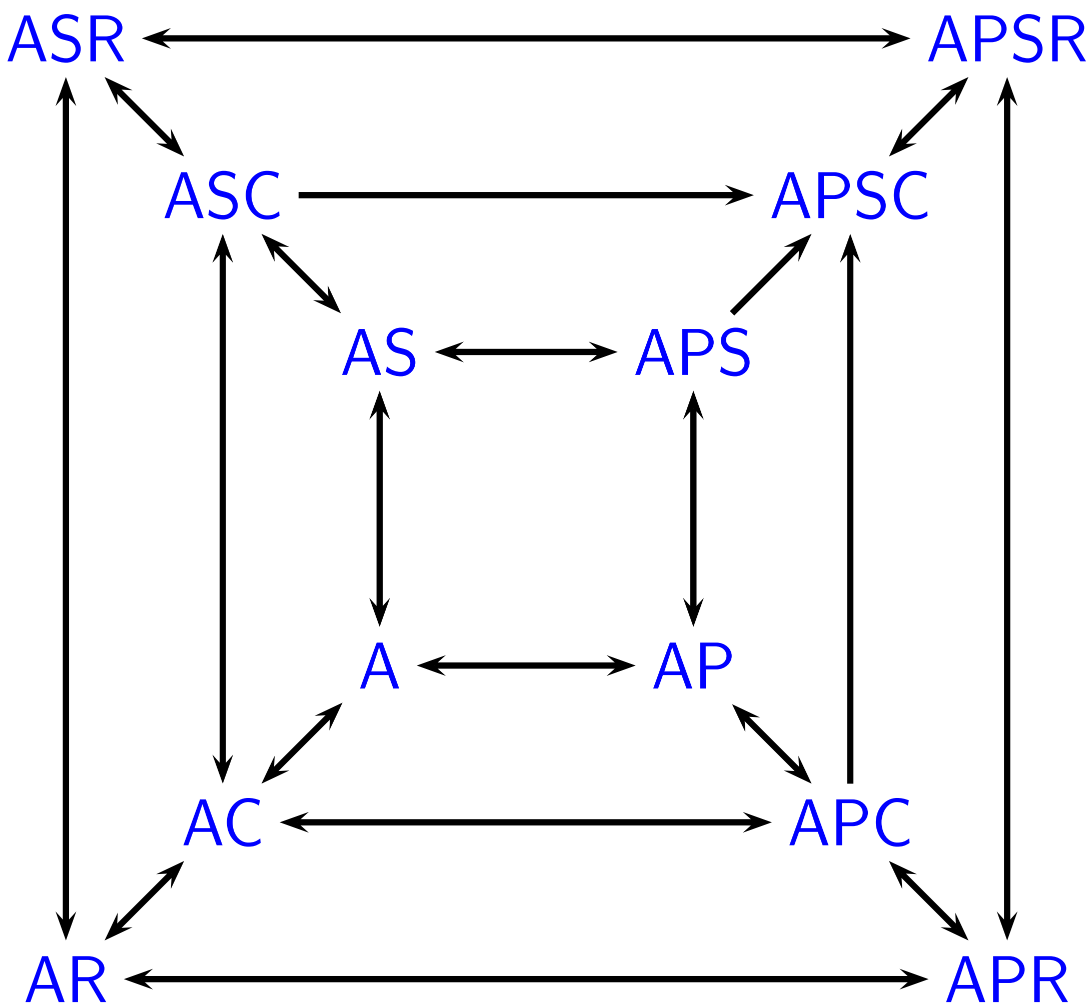


$$[q := [abc]p][p][(abc)^*]\neg q$$

Single agent S5



Expansion and update

$$[p]\varphi$$

$$[p := \top]\varphi$$

What about contraction?

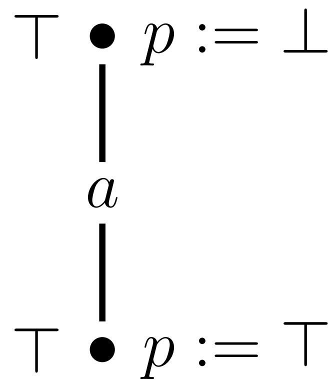
$$[p := \bot]\varphi$$

Update models

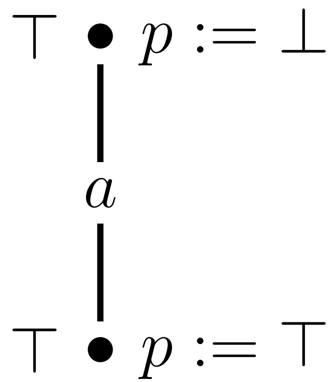
An update model for a finite set of agents $\mathcal{A}$ with a language $\mathscr{L}$ is a quadruple $\mathsf{U} = (\mathsf{E}, \mathsf{R}, \mathsf{pre}, \sigma)$ where

- $\mathsf{E} = \{\mathsf{e}_0, \ldots, \mathsf{e}_{n-1}\}$ is a finite non-empty set of events,
- $\mathsf{R} : \mathcal{A} \to \wp(\mathsf{E}^2)$ assigns an accessibility relation $\mathsf{R}(a)$ to each agent $a \in \mathcal{A}$.
- $\mathsf{pre} : \mathsf{E} \to \mathscr{L}$ assigns a precondition to each event,
- $\sigma : \mathsf{E} \to \Sigma_{\mathscr{L}}$ assigns a $\mathscr{L}$ substitution to each event.

A model for contraction



A model for contraction



contract(p)

The Levi identity

$$K \star \varphi = (K - \neg\varphi) + \varphi$$

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$$[\mathsf{revise}(p)]\varphi \leftrightarrow [\mathsf{contract}(p)][p]\varphi$$

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$$[\mathsf{revise}(p)]\varphi \leftrightarrow [\mathsf{contract}(p)][p]\varphi$$

$$[\mathsf{revise}(p)]\varphi \leftrightarrow [p := \top]\varphi$$

Conclusion and questions

- It is worthwhile to study factual change with dynamic epistemic logic.
- Does substitution add expressive power to the logic of epistemic actions?
- How de we model more difficult contractions? (For instance contraction of knowledge formulas?)
- When do revision and update coincide?