From Onions to Broccoli: Generalizing Lewis's counterfactual logic

Workshop on Belief Revision and Dynamic Logic
ESSLLI 2005

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Goal

- Relational belief revision. Proposal by Lindström and Rabinowicz.

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- Generalization of Segerberg's Onion semantics for belief revision (DDL).

Outline

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- Interlude: Minimal Relational Logic (MRL, [Me]).

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- Interlude: Minimal Relational Logic (MRL, [Me]).
- Broccoli Logic = Minimal conditional Logic (MCL, [Veltman, Burgess]).

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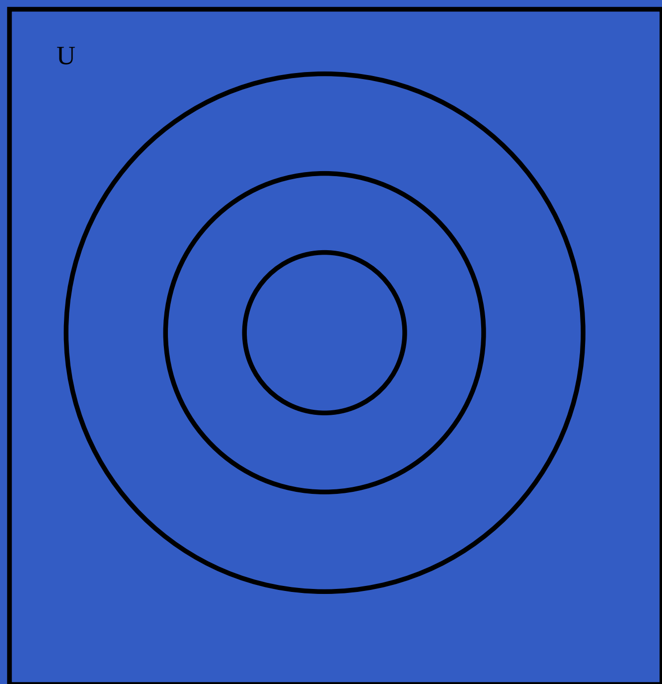

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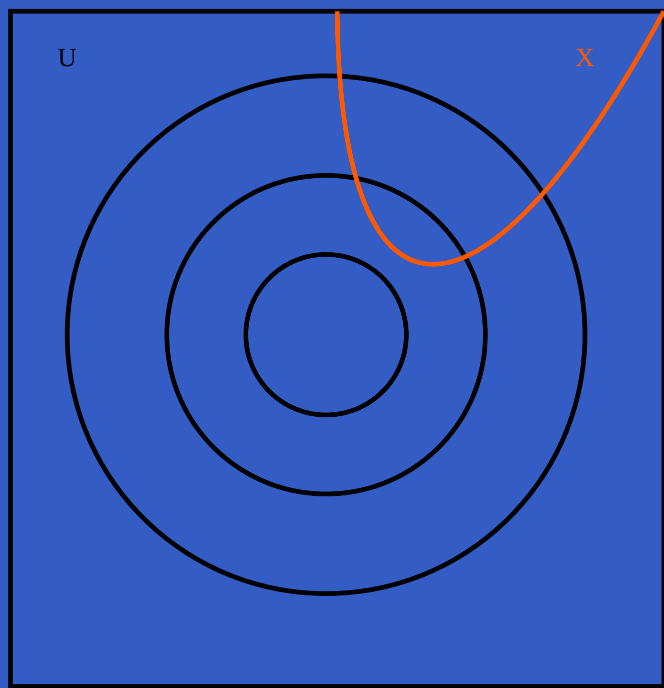

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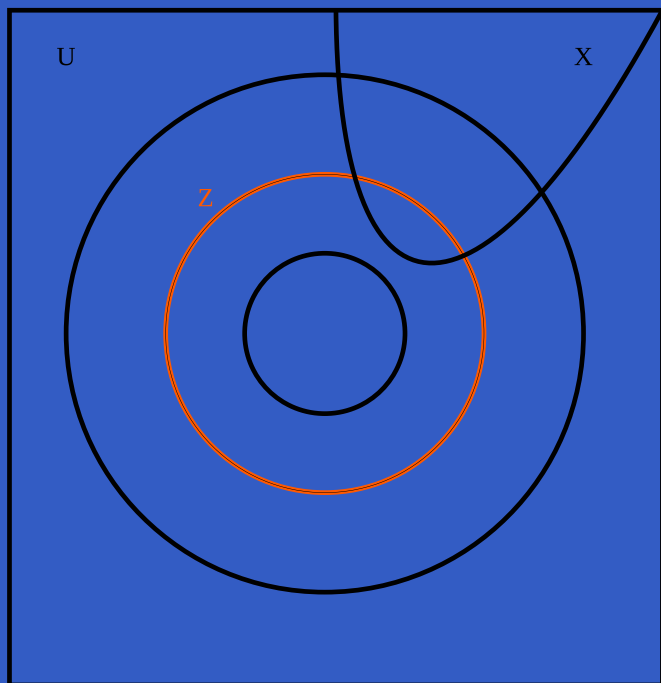

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There are two ways to generalize the limit condition.

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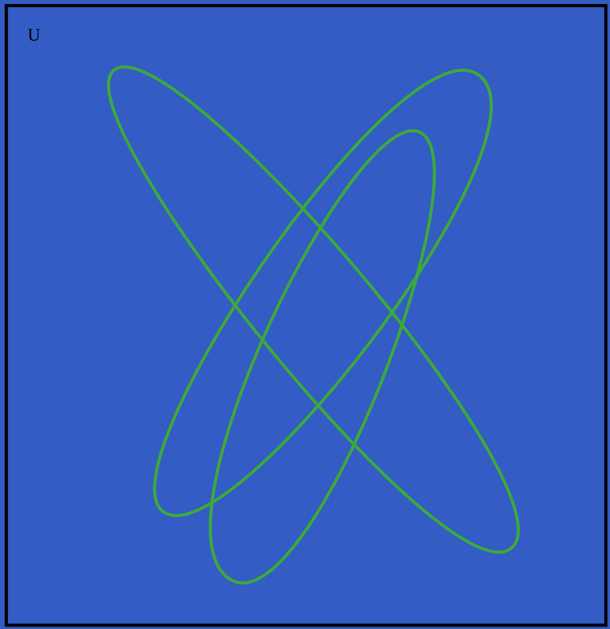

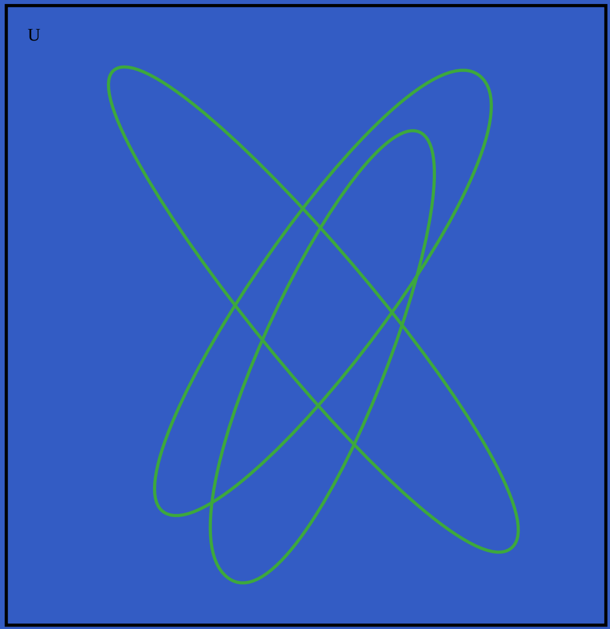

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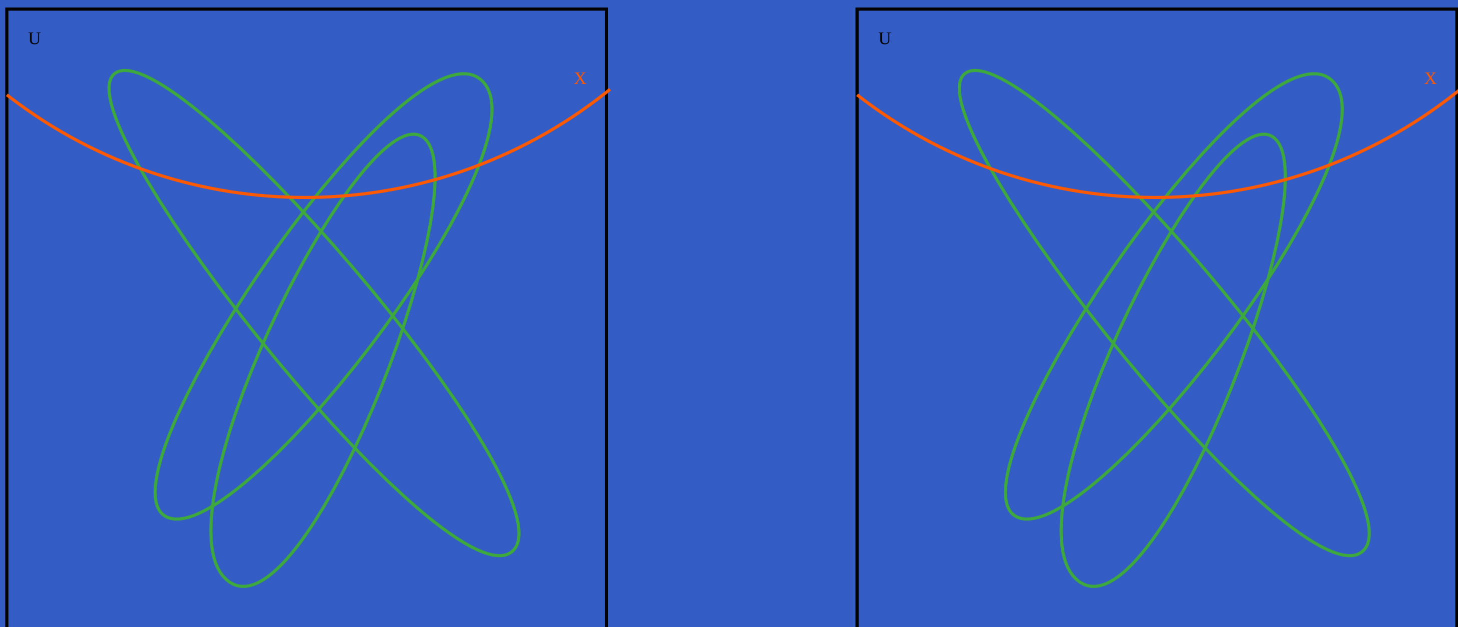

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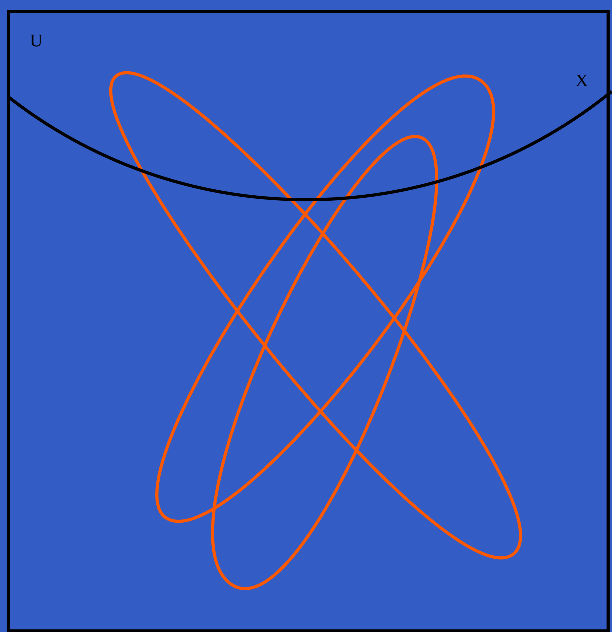

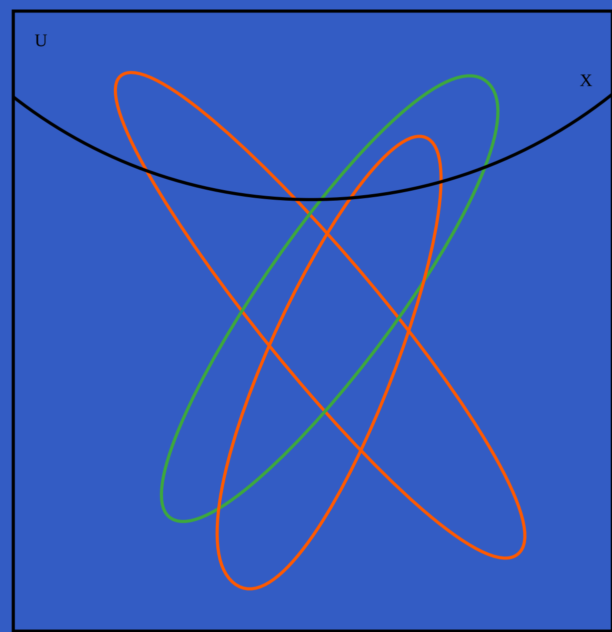

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- We use the notation $Z\mu(\mathcal{B} \bullet X)$ to express that Z is a minimal element of the broccoli flower $\mathcal{B}$ intersecting X.
- Let $|\varphi| = \{u : \mathfrak{M}, u \vDash \varphi\}$. We call $|\varphi|$ the *associated proposition* to φ.

Language

$$\mathcal{L} = p \mid \neg\varphi \mid \varphi \vee \psi \mid [\varphi]\psi \mid [\varphi\rangle\psi$$

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- V is a valuation assigning sets of worlds to propositions.

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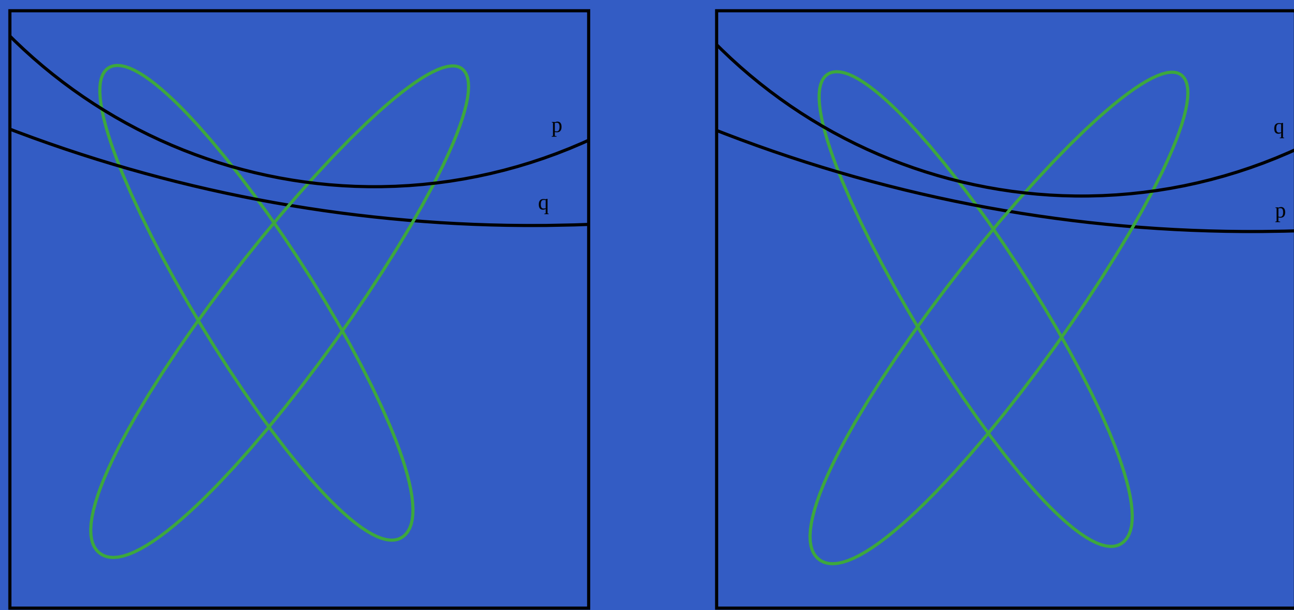
In pictures

Broccoli semantics of the counterfactual operators $[p]q$ and $[p\rangle q$.



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- $R = \{R_{|\varphi|} : \varphi$ is a formula, $R_{|\varphi|} \subseteq U \times \mathcal{P}(U)\}$.

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MRL Proof System

RULES

1. Modus Ponens.
2. Necessitation for $[\varphi]$ and $[\varphi\rangle$.
3. If φ and φ' are formulas differing only in φ having an occurrence of θ in one place where φ' has an occurrence of θ', and if $\theta \equiv \theta'$ is a theorem, then $\varphi \equiv \varphi'$ is also a theorem.

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MRL Proof System

AXIOMS

1. Classical tautologies
2. $\langle\varphi\rangle\psi \equiv \neg[\varphi]\neg\psi$
3. $\langle\varphi]\psi \equiv \neg[\varphi\rangle\neg\psi$
4. $[\varphi](\psi \rightarrow \theta) \rightarrow ([\varphi]\psi \rightarrow [\varphi]\theta)$
5. $\langle\varphi]\psi \rightarrow \langle\varphi](\psi \vee \theta)$
6. $[\varphi]\psi \wedge \langle\varphi]\theta \rightarrow \langle\varphi](\psi \wedge \theta)$
7. $\neg\langle\varphi]\top \rightarrow [\varphi]\psi$

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Two ways to proceed

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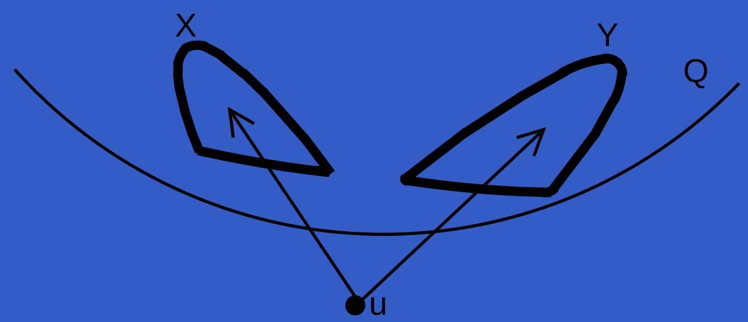

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   DIFFICULT!

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1. Add restrictions on the relation $R_{|\varphi|}$, or introduce so-called selection functions, in order to get minimal sets returned under revision by φ from broccoli flowers.
2. Show that broccoli logic is equivalent to a well-know conditional logic, viz., the *minimal conditional logic* of Veltman and Burgess.

MCL Language

$$\mathcal{L} = p \mid \neg\varphi \mid \varphi \vee \psi \mid [\varphi]\psi$$

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- U and V are as above;
- $\leq_x$ is a reflexive and transitive ternary relation.
  $y \leq_x z$ should be read as:

"According to world x, world y is no farther away than world z."

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Some more notation

$W_u = \{y : \exists z, y \leq_u z\}$ is the *zone of entertainability* for world $u \in U$.

Intuitively, worlds outside the zone of entertainability for u are worlds so far away that their distance from the real world is not appreciable.

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MCL Truth-definition

- $\mathfrak{M}, u \vDash [\varphi]\psi$, iff:

  $\forall y \in W_u \cap V(\varphi), \exists z \in W_u \cap V(\varphi)[z \leq_u y \& \forall w \in W_u \cap V(\varphi)(w \leq_u z \rightarrow w \in V(\psi)]$

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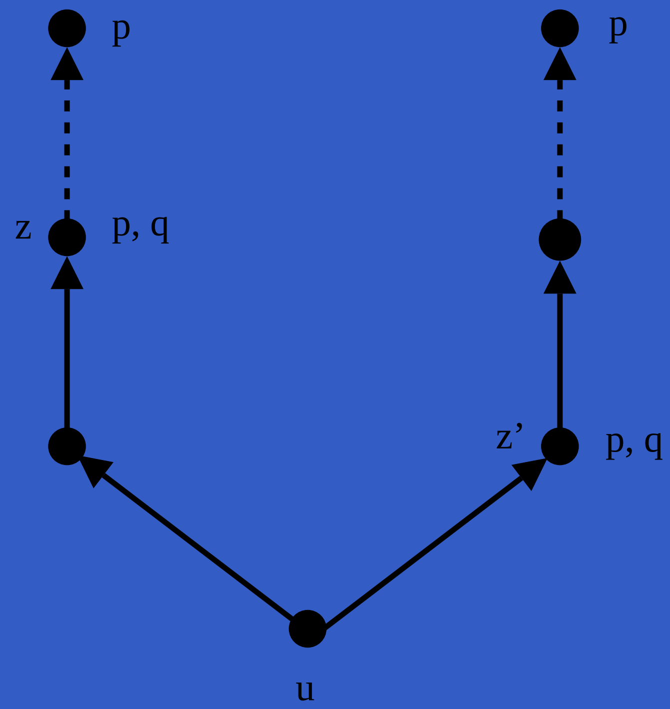
- No minimality condition! But we only need finite models. The semantic condition reduces to:

  $\forall y \in W_u \cap V(\varphi), \exists z \in W_u \cap V(\varphi)[z \leq_u y \& \forall w <_u z, w \notin V(\varphi) \& z \in V(\psi)].$

In picture

$\mathfrak{M}, u \models [p]q$:



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6. $[\varphi]\psi \wedge [\theta]\psi \rightarrow [\varphi \vee \theta]\psi$

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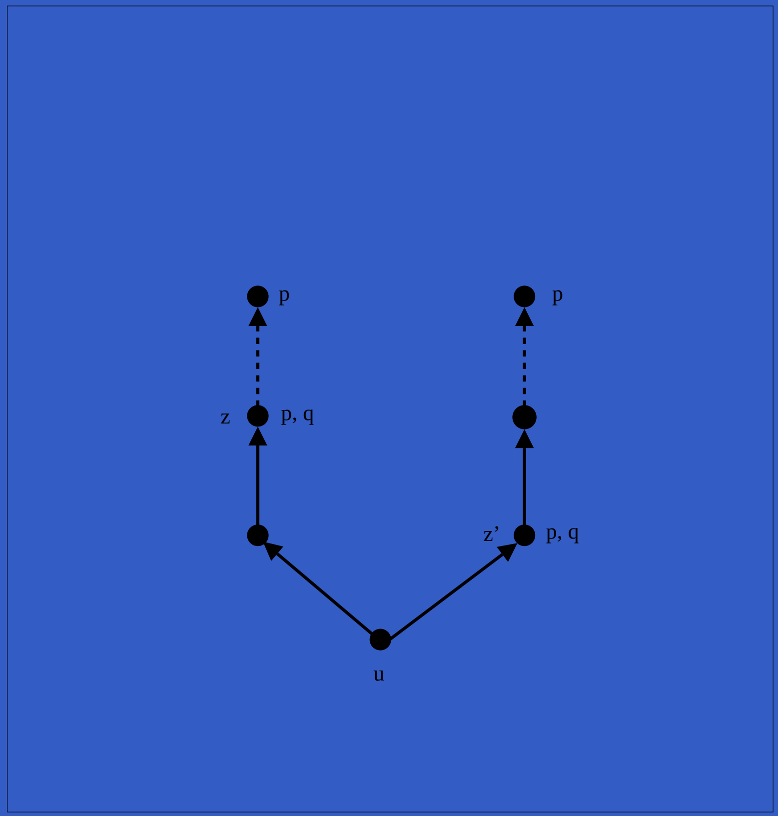
Main Theorem

BROCCOLI LOGIC = MINIMAL CONDITIONAL LOGIC

Key Idea – Induced Broccoli Models



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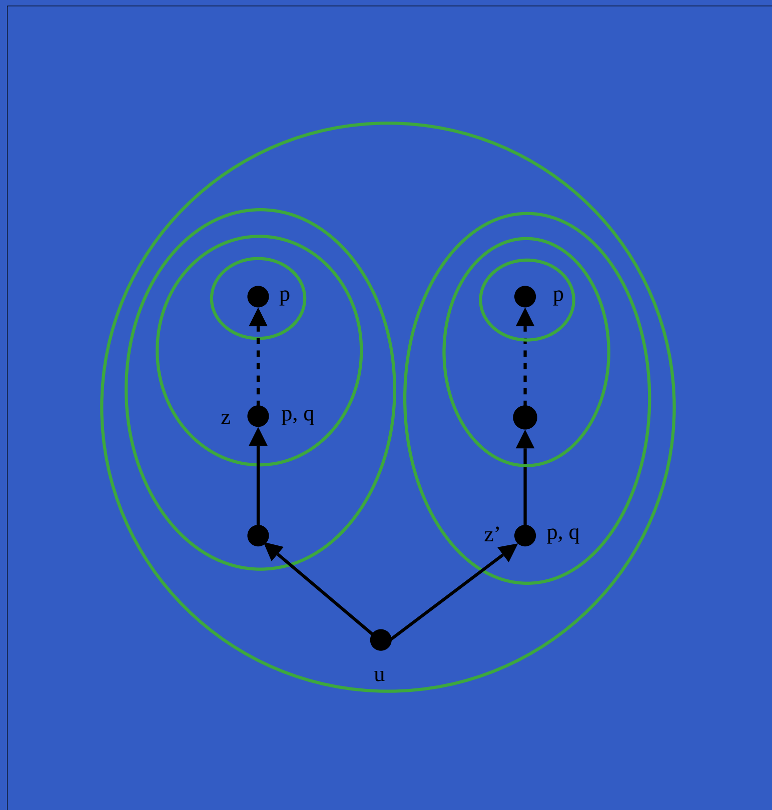

Conclusion

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- Open question: What is the complete minimal logic of $[\varphi]\psi$ and $[\varphi\rangle\psi$ over the Burgess-Veltman models?

Lesson

As so often over the past years, we see that logics of belief revision are largely conditional logics.