Communication Strategies in Games

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Communication strategies in games

Setting: A game of imperfect information, in which you and/or your opponent are not completely informed about all aspects of the game.

- What should you say?
- What should you *not* say?
- Should you believe the other players?
- Is it always good to get information?
- Etc.

For example

- *Bargaining:* say enough to make a deal, say little enough to make a good deal
- *Selling:* communicate your prices to your customers, not to your competitors.
- *Selling information:* communicate enough to show that you have the information, but not so much that you have nothing to sell anymore
- *Love:* tell your object of desire those things about yourself that make you desirable, not those that make you undesirable.

More specifically

General Idea: use *dynamic epistemic logic* to study information change in *games of imperfect information*

- Model games of imperfect information as Kripke models
- Use Dynamic Epistemic Logic to model the effect of communication.
- So: Communicative acts change the game
- The question “What should you say” becomes “With what act can I change the game to my advantage”

Running Example

- Three closed envelopes
- Ann can choose any envelope she wants
- Envelope 1 is empty
- Envelope 2 contains 3 euro for Ann, 3 for Bill
- Envelope 3 contains 6 euro for Bill
- Bill knows this
- Ann does not know which envelope contains what

What should Ann do? What should Bill say?

What should Ann do?

- She has no idea which envelope has money in it for her.
- So, she chooses at random
- Ann can expect 1 euro (on the average)
- Bill can expect 3 euro (on the average)

What should Bill say?

- Lie!
- Bill: ”Envelope 3 contains 3 euro for Ann”
- Ann (if she believes him) chooses envelope 3.
- Ann expects 3, but gets 0.
- Bill gets 6.

But if Bill can only tell the truth?

- Bill: “Envelope 2 contains 3 euro for Ann”
- This does not help Bill (he still expects 3 euro)
- Better to say *less*
- Bill: “Envelope 1 is empty”
- Ann (if she believes him) will choose randomly between envelopes 2 and 3
- Good for both: Bill expects $4\frac{1}{2}$, Ann expects $1\frac{1}{2}$

How to model this type of situation

- *Game Theory* to model games
- *Epistemic Semantics* to model lack of information
- *Dynamic Epistemic Semantics* to model information change

Game Theory

A strategic game (of perfect information): $(N, (S_i)_{i \in N}, \pi)$

- N is a set of *players*
- S_i is a set of *strategies* for player i
- π tells us what the players get for each combination of strategies

Our game

Bill's choices $\backslash$ Ann's choices	env. 1	env. 2	env. 3
(Bill has no choice)	$0 \backslash 0$	$3 \backslash 3$	$6 \backslash 0$

Epistemic Semantics

- Information of each agent represented by a *set of possibilities* (those that are consistent with her information).
- Typical representation: A Kripke model $(W, (\rightarrow_a)_{a \in N}, w)$ (for a set of agents N)
- W is a set of possible worlds
- w is the actual world
- $w \rightarrow_a v$ means: a thinks that v is possible in w

Game Theory + Epistemic Semantics

- Games of imperfect information: players do not know exactly which game they are playing.
- *A Game of imperfect information is a Kripke model in which possible worlds are games.*
- Difference with the standard model of such games: in our case, players may have *false* information as well
- (We really need *probabilistic* models...)

What Ann believes

These are Ann’s ’possible worlds’. In each of them, Bill knows what’s going on.

Bill \ Ann	env. 1	env. 2	env. 3
-	$0 \backslash 0$	$3 \backslash 3$	$6 \backslash 0$

Bill \ Ann	env. 1	env. 2	env. 3
-	$0 \backslash 0$	$6 \backslash 0$	$3 \backslash 3$

Bill \ Ann	env. 1	env. 2	env. 3
-	$3 \backslash 3$	$0 \backslash 0$	$6 \backslash 0$

Bill \ Ann	env. 1	env. 2	env. 3
-	$3 \backslash 3$	$6 \backslash 0$	$0 \backslash 0$

Bill \ Ann	env. 1	env. 2	env. 3
-	$6 \backslash 0$	$3 \backslash 3$	$0 \backslash 0$

Bill \ Ann	env. 1	env. 2	env. 3
-	$6 \backslash 0$	$0 \backslash 0$	$3 \backslash 3$

Dynamic Epistemic Semantics

- Study of ‘epistemic actions’ that change the information of agents: private and public communication, observation, lying, suspicion, etc
- Change of information is modeled as change in a Kripke model
- An epistemic action is an operation on Kripke models
- Plaza (1989), Gerbrandy (1997/8), Baltag, Moss, Solecki (1998), van Ditmarsch (2000), ...
- *We can (now) simply plug in dynamic epistemic semantics into game theory*

Public announcements

- Particularly simple case of an epistemic action: *public announcement*
- **definition:**
  Kw is a model. The model $K'w'$ that results after a public announcement of ϕ is obtained by removing all arrows pointing to worlds where ϕ is false.
- We write $Kw[\![\phi]\!]$

For example

After a public announcement of ‘the first envelope is empty’, these are Ann’s possibilities:

Bill \ Ann	env. 1	env. 2	env. 3
-	$0 \setminus 0$	$3 \setminus 3$	$6 \setminus 0$

Bill \ Ann	env. 1	env. 2	env. 3
-	$0 \setminus 0$	$6 \setminus 0$	$3 \setminus 3$

The value of communication

- What should you say? That which has the highest value.
- **intuition:** The value of an announcement is the difference in expected utility before and after the announcement.
- **definition:** The value of a public announcement with a sentence ϕ in a game of imperfect information G for a player i is:

  The difference in expected utility for i in G and that in $G[\![\phi]\!]$
- (can easily be generalized to cover *all* epistemic actions)
- But: *What is expected utility?*

Expected Utility

- Expected utility for player i in a game of imperfect information is what i gets when:

  1. he plays his strategy, and
  2. the other players do what i expects them to do.

- No *universal* notion. Depends on: properties of i, properties that i ascribes to the other players.
- Our example is easy: the outcome does not depend on Bill.
- A reasonable choice in our case: Ann checks for each choice of envelope what she can expect (given what she knows). She chooses randomly among those envelopes with the highest payoff.

The value of “Envelope 1 is empty”

After learning that envelope 1 is empty, Ann has two possibilities:

Bill \ Ann	env. 1	env. 2	env. 3
-	$0 \setminus 0$	$3 \setminus 3$	$6 \setminus 0$

Bill \ Ann	env. 1	env. 2	env. 3
-	$0 \setminus 0$	$6 \setminus 0$	$3 \setminus 3$

- Before his announcement, Bill could expect 3
- Now, Bill expects $(3+6)/2$ = $4\frac{1}{2}$.
- The value (to Bill) of “envelope 1 is empty” is $1\frac{1}{2}$

Another option for Bill

- Bill to Ann “The last envelope is not empty”
- This is true.
- If Ann believes him, she retains 4 possibilities:

Bill $\setminus$ Ann	env. 1	env. 2	env. 3
-	$0 \setminus 0$	$3 \setminus 3$	$6 \setminus 0$

Bill $\setminus$ Ann	env. 1	env. 2	env. 3
-	$0 \setminus 0$	$6 \setminus 0$	$3 \setminus 3$

Bill $\setminus$ Ann	env. 1	env. 2	env. 3
-	$3 \setminus 3$	$0 \setminus 0$	$6 \setminus 0$

Bill $\setminus$ Ann	env. 1	env. 2	env. 3
-	$6 \setminus 0$	$0 \setminus 0$	$3 \setminus 3$

- Now, Ann’s dominating strategy is “Choose envelope 3”

- Bad for Ann, good for Bill. Ann gets nothing, Bill gets 6. *This is just as good as lying.*
- (Moral: it can be bad to get true information)

Conclusions

- Sketched a formal framework for studying communication in games based on existing tools: Game Theory + Dynamic Epistemic Semantics
- Provides Game Theory with a theory of epistemic actions.
- Provides epistemic logic with a *pragmatics*

Future work

Specific ‘pragmatic’ questions:

- What should you believe? What is credible?
- What kind of ‘Gricean’ implicatures does our ‘theory’ imply

General questions such as:

- Is it always valuable to communicate in cooperative games?
- Is it always costly to communicate in zero-sum games?
- (This is a series of questions, with answers depending on properties of players and on properties of games, many of which have been studied in a different setting in game theory)

Another still open question:

- A theory of *Probabilistic* Dynamic Epistemic Semantics

Credibility

- Ann does not trust Bill to speak the truth.
- She does not believe him if he says that envelope 3 contains 3 euro for her. (And rightly so)
- Symmetrically, she does not believe him when he says (truthfully) that the second envelope has 3 for her.
- But if Bill says that envelope 1 is empty, she should believe him. Why? *he has no conceivable reason to lie*
- Bill saying that envelope 1 is empty is *credible*

Credibility

(Aumann, Farrell, Robert van Rooy)

- The speaker has a reason to say that ϕ iff (he knows that) ϕ has a positive value
- The speaker has a reason to lie iff (he knows that) ϕ has a positive value for him, but ϕ is false.
- ϕ is *self-signaling* iff the speaker has no conceivable reason to lie about ϕ
- ϕ is *optimal* if there is no ψ with a higher value

Credibility

When should the hearer believe the speaker?

- **intuition**: ϕ is *credible* iff

  1. ϕ is self-signaling: the speaker has no conceivable reason to lie about ϕ

  [2. The speaker has a reason to say that ϕ]

- **definition:** An announcement ϕ of player i to player j is *credible* to player j in game G IFF it holds that in all games that j considers possible:

  1. if ϕ has a positive value for i, then ϕ is true.

  [2. if ϕ is true, then ϕ has a positive value for i]

What should Ann believe

- Should Ann believe Bill if what he says is credible?
- Credible = garanteed to be true, so yes.
- But Bill will try to mislead Ann to make the wrong choice — using the truth, if he can
- For example, “Envelope 3 is not empty” is credible
- Need something stronger: Bill has no incentives, when saying what he says, to mislead Ann