Gábor Alberti (U. Pécs):[1]
alberti.g@invitel.hu, albi@btk.pte.hu

Generalized Truth-Conditional Interpretation in ReAL Interpretation System

❶ Aim:

to define a *generalized static interpretation* in a DRT-based complex IS, called ReALIS (→●App.2), instead of the intensional semantics (→●App.1) proposed in the current *standard DRT*, because

- this latter system is *oversimplified* (→ I-IV in ●App.1);
- it is *necessary* to generalize the classical way of interpretation (REPRESENTATION ~ MODEL) →❸
- this generalization can *fruitfully* be applied to numerous phenomena →❹

❷ Theoretical background

- **current DRT** (Kamp et al. 2004; cf. Kamp 1981, Karttunen 1976)
  - a *dynamic* interpretation system:

  $[\text{sentence}]_{dyn}$: IS→IS "context change potential" vs. $[\text{sentence}]_{sta}$ = True/False (see Montague)
  - a *representational(ist)* theory (DRSs: partial information on "worlds" vs. common logical content of families of total possible worlds (e.g. Groenendijk et al. 1996)
- **Segmented DRT** (Asher and Lascarides 2003):
  - *rhetorical relations* between eventualities → anaphora resolution and (changes in) temporal / local structures (from s. to s.)
- **LDRT** (Alberti 2000):
  - *lifelong* (i's IS is a gigantic DRS-like str. built from birth to death)
- **ReALIS** (Alberti 2004, 2005a-c, Kleiber 2005, Alberti and Ohnmacht 2005, Farkas and Alberti 2005): REciprocal And Lifelong Interpretation System:
  - *reciprocal* (is' knowledge on each other: "I know that you know that I know..." → truth-conditionally heterogeneous ISs, assigned to anaphorically coherent texts (see also Zeevat 2005)

Capturing the real („ReAL") context of interpretation: allowing for the fact that it is a *temporal* process taking place among *people* (see Benz 2000), which is „REciprocal" and („And") „Lifelong" (hence, „ReAL").

Definition: A quadruple $\mathfrak{R} = \langle \mathbf{W}^{\Omega}_{\mathbf{o}}, \mathbf{W}^{\Lambda}, \mathbf{Dyn}^{\Delta}, \mathbf{Tru}^{\Theta} \rangle$ is a *ReALIS framework* if...

- $\mathbf{W_o = \langle U, T, I, M, \Omega \rangle}$: *tensed human world*, or the *oracle's / external / real world;* components:
  universe of entities,
  temporal entities ($T \subset U$),
  interpreters ($I \subset U$),
  impulses (for $m \in M$, $m : \{T, U\} \cup \mathbf{N} \to U$)
  external relations ($\Omega = \cup_{k \in \mathbf{N}} \Omega^k$; and for $\omega \in \Omega^k$, $[\omega] \subseteq T \times U^k$).
- $\mathbf{W(i,t) = W^i_t = \langle U^i_t, Con^i_t, Ide^i_t, Acc^{\Lambda i}_t, \kappa^i_t, \alpha^i_t \rangle}$: W is a partial function whose values are *information states = (momentary) internal worlds*; W(i,t) is a sextuplet: interpreter *i's internal world / information state at moment t*. Components:
  universe of referents,
  relation of conditions rows,
  identity relation,
  labeled accessability relation → "reciprocal" (with equa. classes called ***worldlets***),
  cursor ("i's attention concentrated on a certain event / time / location / etc."),
  anchor (a partial function from the internal *referents* to the external *entities*).
- $\mathbf{Dyn : M \times W \to W}$ is a partial function: *dynamic interpretation*.
- $\mathbf{Tru(m) = W[m(0), \Theta]}$: *generalized truth evaluation* or *static interpretation* of impulse $m \in M$; i.e. a set of statements on (the body m(0) of) a linguistic impulse performed by speaker m(U) at moment m(T) and perceived by m(1), m(2),..., $m(N_m)$ →❹.

[1] I am grateful to Hungarian National Scientific Research Fund (OTKA T038386) for their contribution to all my costs in connection with this conference of IPrA at Riva del Garda.

❸ Why is it *necessary* to generalize the classical interpretation (REPR ~ MODEL)?

First step of generalization: $REPR_1$ ~ MODEL, $REPR_2$ ~ MODEL, ..., $REPR_N$ ~ MODEL

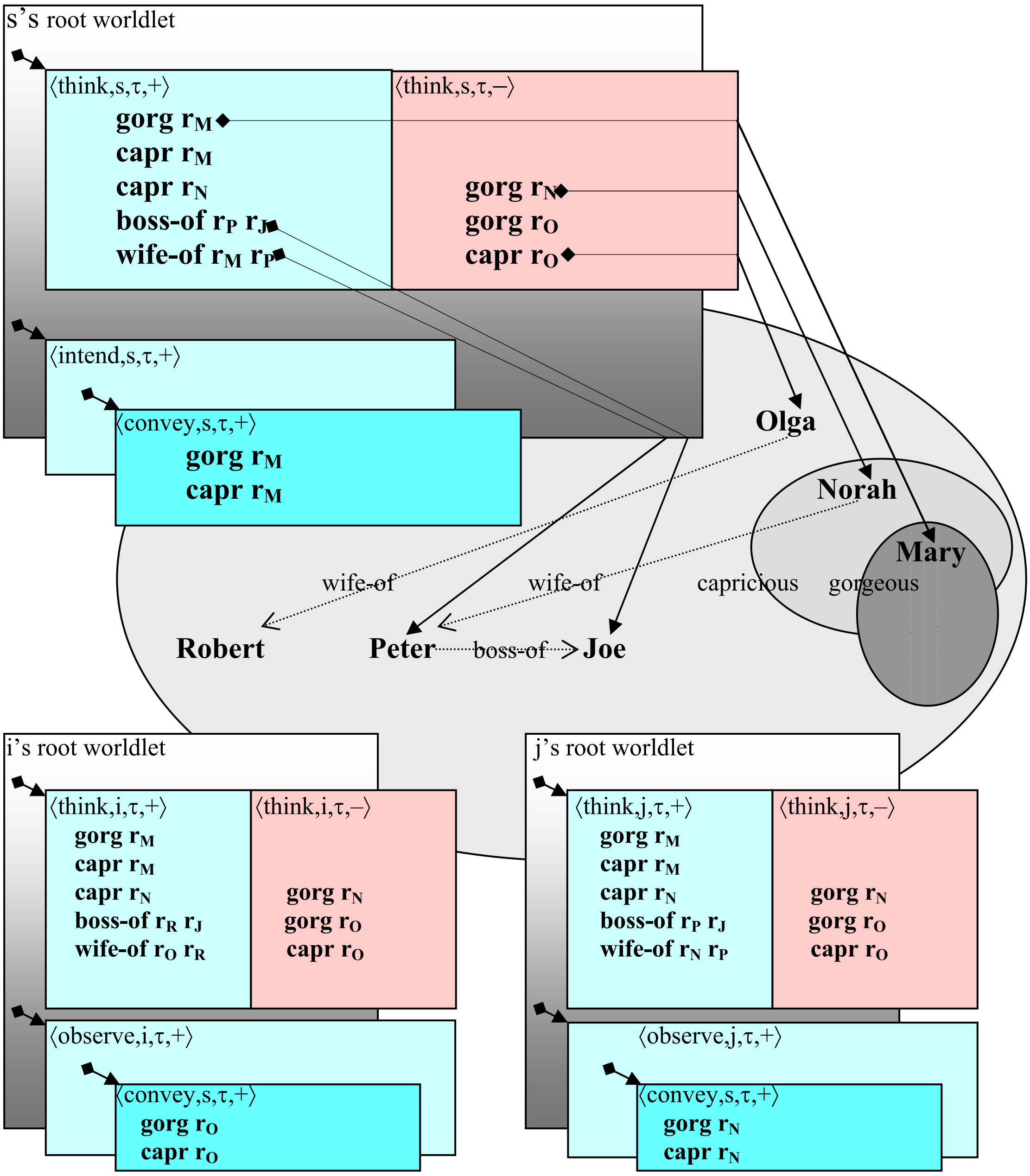
(1) **Which (or rather, *whose*) representation is to be evaluated?**

a. Speaker s (Joe): “My boss’s wife is gorgeous. But she’s capricious.”
intended meaning: *Mary is gorgeous and capricious.*

b. Relations in the external world:

⟨t, Mary⟩ ∈ [gorgeous]	⟨t, Mary⟩ ∈ [capricious]
⟨t, Norah⟩ ∉ [gorgeous]	⟨t, Norah ⟩ ∈ [capricious]
⟨t, Olga⟩ ∉ [gorgeous]	⟨t, Olga⟩ ∉ [capricious]
⟨t, Peter, Joe⟩ ∈ [boss-of]	⟨t, Robert, Joe⟩ ∉ [boss-of]
⟨t, Norah, Peter⟩ ∈ [wife-of]	⟨t, Olga, Robert⟩ ∈ [wife-of]

c. A simplified repr. of a relevant fragment of the speaker’s (s) and two interpreters’ (i, j) inf. states:[2]



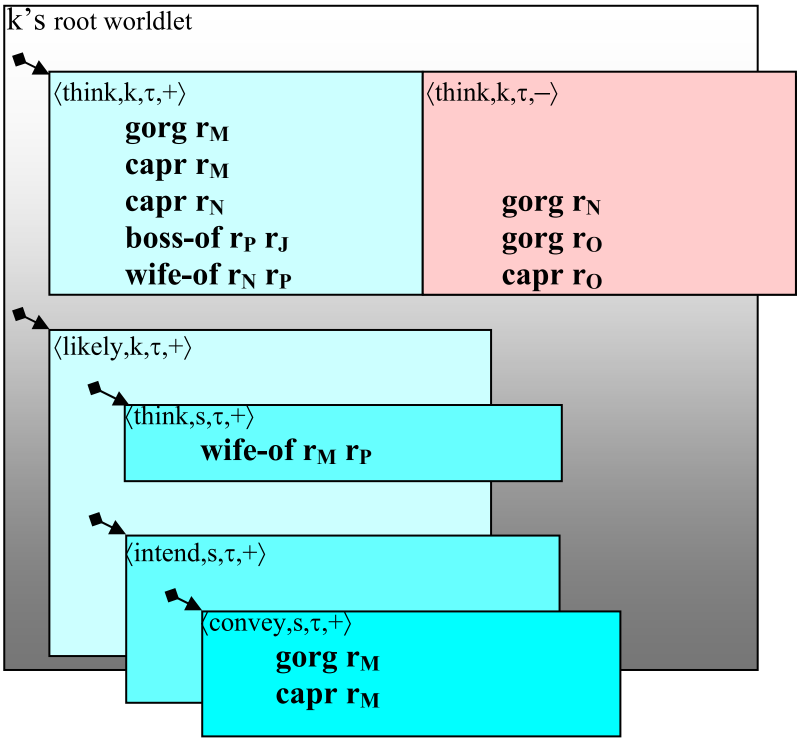
d. Interpreter i: “Both statements are false.”
i’s interpretation: *Olga is neither gorgeous nor capricious.*

e. Int. j: “The first statement is false, and the second one is true.”
j’s interpretation: *Norah is not gorgeous, but she’s capricious.*

---

[2] Interlocutors s, i, j and k are assumed to agree upon the characterization of all the three women.

f. A simplified representation of a relevant fragment of a third interpreter's (k) information state:

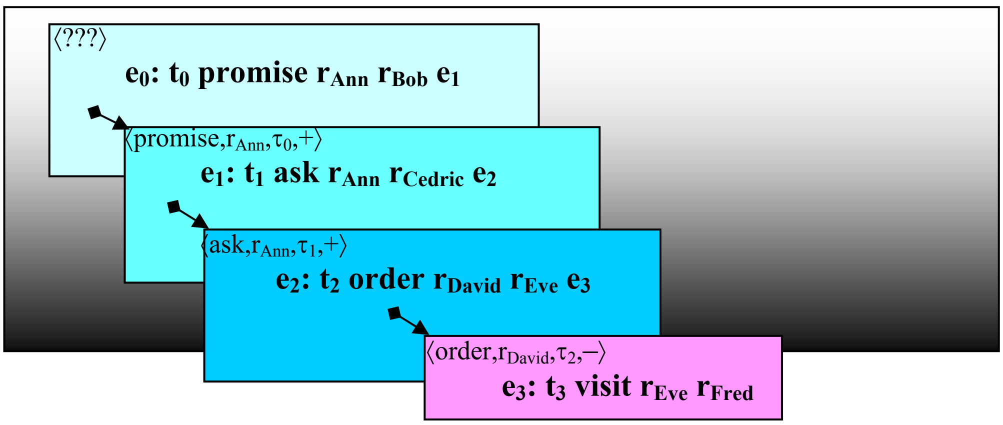


g. Interpreter k: "We are likely to agree upon your characterization of the lady in question. But you've meant *Mary*, haven't you? Peter's wife is called *Norah*, but they have been living apart for three years. Mary is only his new mistress."

❹ Why is it *profitable* to generalize the classical interpretation (REPR ~ MODEL)?

Generalization: $\text{REPR}_n \sim \text{REPR}_q$, n = 1,2,...,N, q = 0,1,...,N where REPR_0 = MODEL (of the world)

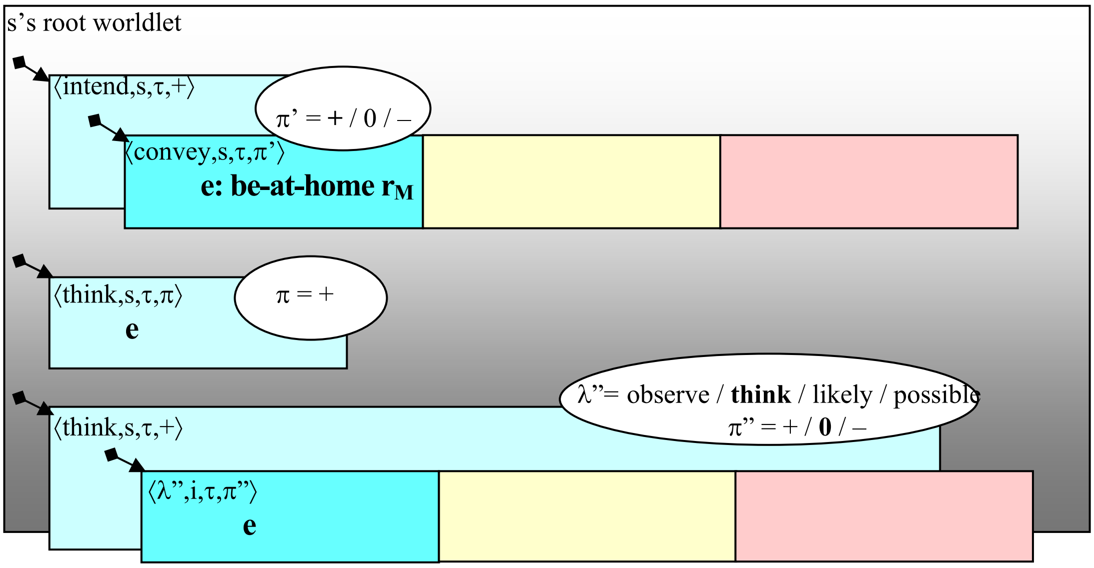
(2) **Representation / evaluation of non-truth-evaluable clauses (promises, questions, instructions)**
Ann promised Bob to ask Cedric whether David had ordered Eve not to visit Fred.



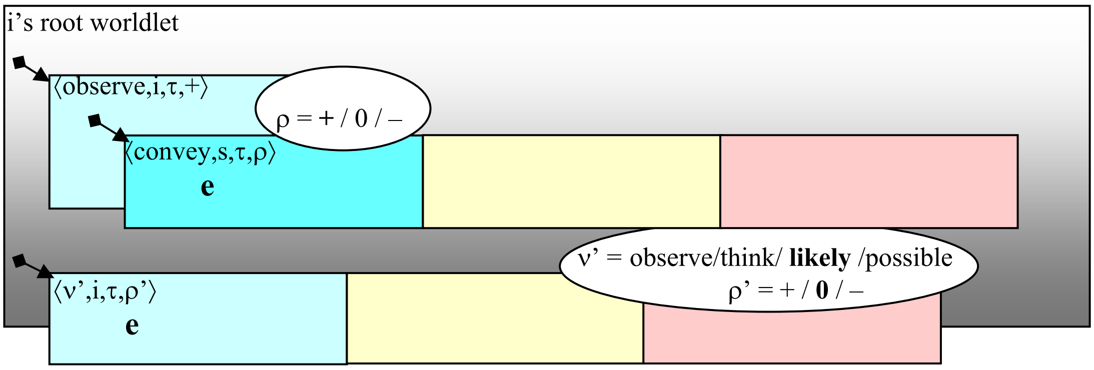
1. ⟨τ''', Ann, Bob, $u_{promise}$⟩ ∈ [MAKE] (τ''' ∈ T, Bob ∈ I, MAKE ∈ Rel)
   assertion (the whole declarative sentence): *true / false*
2. ⟨τ'', Ann, Cedric, $u_{question}$⟩ ∈ [ASK] (τ'' ∈ T, Ann, C. ∈ I, ASK ∈ Rel)
   promise (e.g. Ann's one above): *kept / broken* (also a *binary* evaluation!)
3. ⟨τ', David, Eve, u_{order}⟩ ∈ [GIVE] (τ' ∈ T, David ∈ I, GIVE ∈ Rel)
   question (e.g. Bob's one above): *answered / ignored or ...*
4. ⟨τ, Eve, Fred⟩ ∈ [VISIT] (where τ ∈ T, Eve ∈ I, Fred ∈ I, VISIT ∈ Rel)
   order (e.g. David's one above): *followed / ignored or refused*

(3) **Extending static interpretation to pairs of internal worldlets**

a. A schematic representation of the relevant fragment of the speaker's knowledge:



b. A schematic representation of the relevant fragment of the interpreter's knowledge:



c. The *ideal* case: $\pi' = +$, $\lambda'' =$ think, $\pi''= 0$, $\rho = +$, $\nu' =$ likely, $\rho'= 0$.[3]

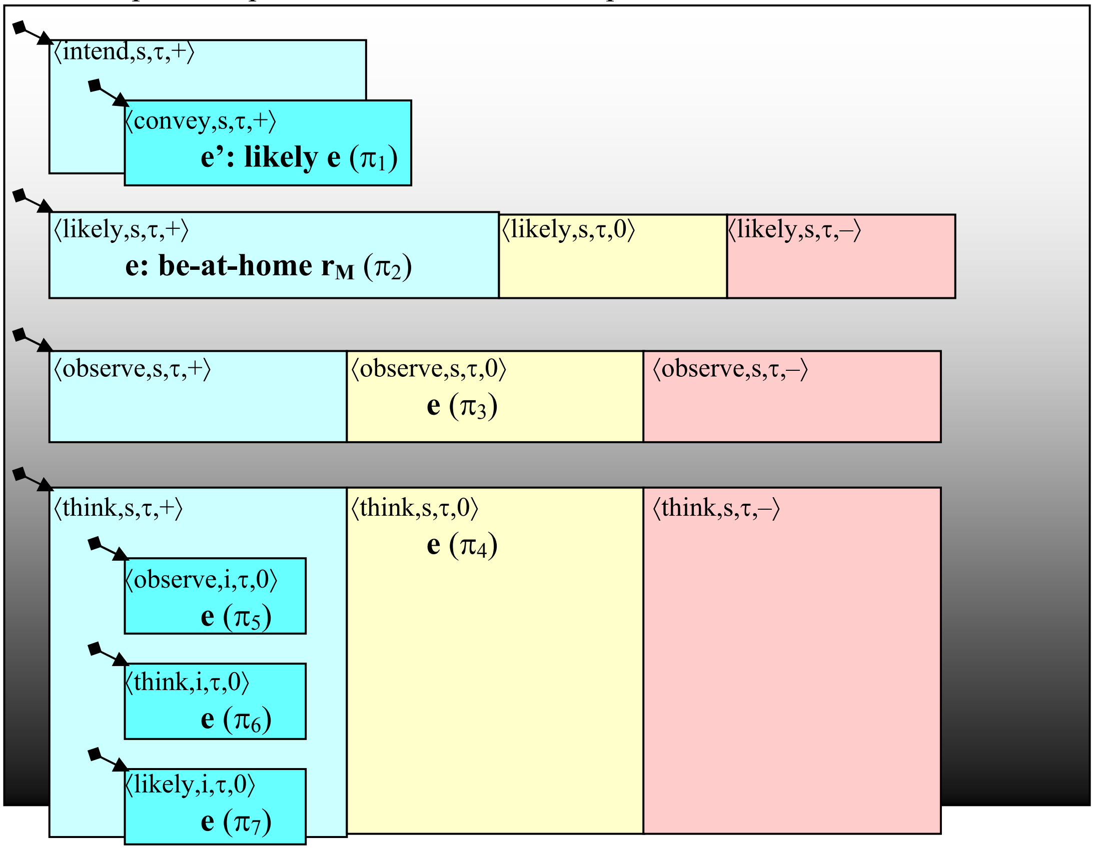
d. $\pi' = 0 \neq \pi$	s intends to *suppress the truth* that Mary is at home
e. $\pi' = - \neq \pi$	s intends to tell a *lie*
f. $\pi'' = + = \pi'$, $\lambda'' =$ observe	s intends to *mislead* i as to his/her information on i's knowledge, perhaps in order to *fish out* i's information or to come to know i's reaction
g. $\rho = 0 \neq \pi'$	i *cannot perceive* s's message
h. $\rho = - \neq \pi'$	i radically *misunderstands* s's message
h'. $\rho \neq \pi' = -$, but $\rho = \pi$	s has *blurted out* the fact that Mary is at home in spite of his/her intention
i. $\rho' =+ \neq \pi''=0$, $\nu' =$ observe	It is proper for i to respond to s's *mistaken assumption* (see the "ideal case" above), say, in this way: "Mary is *sure* to be at home! We are together."
j. $\rho'= - \neq \pi''=0$, $\nu' =$ likely	It is proper for i to respond to s's *mistaken assumption* (see the "ideal case" above), say, in this way: "Oh, really!"

[3] The value of parameter π is supposed to be fixed now. Its positive value expresses the speaker's thought that a person called Mary is at home. What can be regarded as an ideal case of communication, say, in the pragmatic sense formulated by the Gricean (1975) maxims or similar systems (Horn 1996, Ginzburg 1996), the values of parameters π' and ρ are also positive (2)c, expressing that the speaker intends to tell the truth ($\pi' = +$), or more precisely, what (s)he sincerely thinks to be the truth, and exactly that is what the interpreter observes, without any shade of misunderstanding ($\rho = +$); further, in order to be relevant, the speaker should think that the interpreter has no knowledge on Mary's momentary whereabouts ($\langle\lambda'',\pi''\rangle = \langle$think,0$\rangle$), which is true in the ideal case ($\langle\nu',\rho'\rangle = \langle$think,0$\rangle$ or, perhaps even rather, $\langle$likely,0$\rangle$).

(4) **Evaluation of propositions in modal scopes / deviations from the ideal speaker's polarity values**

a. Mary is likely to be at home.

b. A simplified representation of the relevant part of the information state of the "ideal" speaker:



c. It is not true in the *external world* that Mary is at home. → s's guess has *not come true*.

d. $\pi_1 = 0$ → s has *blurted out* the secret that Mary is likely to be at home (and, hence, the interpreter has come to know the truth in spite of the speaker's intention to *suppress* it)

e. $\pi_1 = –$ s has *blurted out* the secret that Mary is likely to be at home (and, hence, i has come to know the truth in spite of s's definite intention to *mislead* him (i))

f. $\pi_2 = –$ s has *told a lie*.

g. $\pi_2 = 0$ s has *told a bluff*.

h. $\pi_3 = +$ s has intended to convey the information concerning Mary's being at home, but (s)he has intended to keep it in secret that (s)he has observed this fact in person; which is a kind of *misleading* behavior.

i. $\pi_3 = –$ s has intended to *mislead* i quite brutally.[4]

j. $\pi_4 = +$ s has been *too careful*. This behavior may serve as a polite conveyance of an unpleasant fact.

k. $\pi_4 = –$ s has intended to *mislead* i.

l. $\pi_5 \neq 0$ s has intended to *mislead* i as to his/her information on i's knowledge, perhaps in order to *fish out* i's information.[5]

m. $\pi_6 = +$ s has been a bit *negligent*. (S)he should have said: "I *also* think that it is at least likely that Mary is at home".

n. $\pi_6 = –$ s has been a bit *negligent*. (S)he should have said: "I *do* think that Mary is likely to be at home".

o. $\pi_7 = +$ s has been a bit *negligent*. s should have said: "I *also* think that Mary is likely to be at home".

---

[4] It is obvious that values of the parameters π_i are not entirely independent of each other. When a deviant value of a certain parameter π_i is being concentrated on, it is assumed that other values have been reconciled with the given value but at the cost of a *minimal* change of the ideal distribution of values.

[5] A similar purpose may also be assumed in the cases $\pi_6 \neq 0$ and $\pi_7 \neq 0$.

❺ Conclusion

- The classical way of interpretation (REPRESENTATION ~ MODEL) can be generalized in ReALIS →❸
- This generalization can *fruitfully* be applied to numerous phenomena →(2)-(4)❹:[6]

₪ clauses which traditionally cannot be *truth-evaluated* (promises, questions, instructions etc.) can also be represented and associated with some *binary* external evaluation

₪ static interpretation can be extended to pairs of *internal* worldlets, providing a complex pragmatic characterization of a sentence (discourse) as an impulse emitted by a speaker and perceived by interpreters

₪ even propositions in *modal* scopes can undergo this generalized static interpretation

Thus the spirit of my approach precisely accords with that of Ginzburg (1996:412), for instance: "the aim is to show that, ... [several phenomena that have] previously been relegated to "pragmatics" can now be provided with a more principled semantic account", i.e. "to take an initial step towards integrating semantics and communication."

●References

Alberti, Gábor 2000: Lifelong DRSs, *Gothenburg Papers in Computational Linguistics* 00–5, 13–20.

Alberti, Gábor 2004: ReAL Interpretation System, *Proc. 8th Symp. on Logic and Language,* U. Debrecen, 1–12.

Alberti, Gábor 2005a: The Proportion Problem and the Problem of Indistinguishable Participants, *Szklarska Poreba Workshop* 6, Poland, 28 February 2005, 1700-17 40.

Alberti, Gábor 2005b: *ReALIS*, doct. diss. subm. to Hung.Acad.Sc; http://lingua.btk.pte.hu/gelexi.asp.

Alberti, Gábor 2005c: Accessible Referents in "Opaque" Belief Contexts, in Hans van Ditmarsch and Andreas Herzig eds.: *Proc. 9th ESSLLI* Belief Revision and Dynamic Logic *Workshop*.

Alberti, Gábor – Magdolna Ohnmacht 2005: Aspect and Eventuality Structure in Hungarian, talk at Seventh International Conference on the Structure of Hungarian, Veszprém, 29-31 May 2005.

Asher, Nicholas – Alex Lascarides 2003: *Logics of Conversation*, Cambridge Univ. Press.

Benz, Anton 2000: Chains and the Common Ground, in M. Poesio – D. Traum eds.: *GötaLog 2000 — Gothenburg Papers in Computational Linguistics* 00–5, 181–184.

Dowty, David R. – R. E. Wall – S. Peters 1981: *Introduction to Montague Semantics,* D. Reidel, Dordrecht.

Farkas, Judit – Gábor Alberti 2005: Analyzing the English, Hungarian and Finnish Progressive in a Representational Dynamic Semantics, *TCSP 2005*, Shanghai.

Ginzburg, Jonathan 1996: Interrogatives: Questions, Facts and Dialogue, in Lappin (1996), 385–422.

Grice, H. Paul 1975: Logic and Conversation, in P. Cole – J. L. Morgan eds.: *Syntax and Semantics Volume 3: Speech Acts,* New York: Academic Press, 41–58.

Groenendijk, Jeroen – M. Stokhof – F. Veltman 1996: Coreference and Modality, in Lappin (1996), 179–213.

Horn, Laurence R. 1996: Presupposition and Implicature, in Lappin (1996), 299–319.

Kamp, Hans 1981: A theory of truth and semantic representation, in Groenendijk – Janssen – Stokhof eds.: *Formal Methods in the Study of Language,* Amsterdam, Mathematical Centre.

Kamp, Hans – Josef van Genabith – Uwe Reyle 2004: Discourse Representation Theory, ms. to appear in *Handbook of Philosophical Logic*; source: http://www.ims.uni-stuttgart.de/~hans.

Karttunen, Lauri 1976: Discourse Referents, in J. D. McCawley ed.: *Notes from the Linguistic Underground, Syntax and Semantics* 7, Academic Press, New York, 363–385.

Keenan, Edward L. 1996: The Semantics of Determiners, in Lappin (1996), 41–63.

Kleiber, Judit 2005: Across World(let)s in a Representationist Interpretation System, in Paul Egré and Laura Alonso i Alemany eds.: *Proc. 9th ESSLLI Student Session*.

Lappin, Shalom ed. 1996: *The Handbook of Contemporary Semantic Theory*, Oxford: Blackwell.

Zeevat, Henk 2005: Overlaying Contexts of Interpretation, in Maier, Bary, Huitink eds.: *Sinn und Bedeutung 9*.

[6] The spirit of my approach precisely accords with that of Ginzburg (1996:412), for instance: "the aim is to show that, ... [several phenomena that have] previously been relegated to "pragmatics" can now be provided with a more principled semantic account", i.e. "to take an initial step towards integrating semantics and communication."

●**Appendix 1.** The intensional semantics for DRSs as is defined in Kamp et al. (2004: 3.2)

"To avoid certain notorious difficulties with existence and the denotation of names, we base the intensional model theory for the simple DRS language ... on models where

[I] all worlds come with the same universe (set of individuals) and where

[II] names denote once and for all (each name N denotes the same individual in every world of the model). Relations, however, are interpreted relative to particular worlds. We further assume that

[III] the accessibility relation between possible worlds is the universal relation (i.e. each world is accessible to itself and to each other world)."

(5) **The intensional semantics for DRSs as is defined in Kamp et al. (2004: 3.2)**

a. An *intensional model* is defined as a triple $\langle W_\mu, U_\mu, \Im_\mu\rangle$ as follows:
   (i) W_μ is a set of possible worlds
   (ii) U_μ is a non-empty set
   (iii) a. for names, $\Im_\mu$: Name $\rightarrow \{\{d\} : d \in U_\mu\}$ (cf. II above)
   b. for n-ary relations, $\Im_\mu$: $Rel^n \rightarrow (W_\mu \rightarrow \wp(U^n))$

b. Verifying embeddings are defined globally, i.e. for some $X \subseteq$ Ref, a verifying embedding g is defined as $g : X \rightarrow U_\mu$.. An intensional semantics for DRSs and DRS conditions of L can now be defined as follows:
   (i) $\langle g,h\rangle \models_{\mu,w} \langle U,Con\rangle$ iff $g \subseteq_U h$ and for all $\gamma \in$ Con: $h \models_{\mu,w} \gamma$
   (ii) $g \models_{\mu,w} x_i = x_j$ iff $g(x_i) = g(x_j)$
   (iii) $g \models_{\mu,w} N(x)$ iff $\{g(x)\} = \Im(N)$
   (iv) $g \models_{\mu,w} P(x_1,...,x_n)$ iff $\langle g(x_1),...,g(x_n)\rangle \in \Im(P)(w)\rangle$
   (v) $g \models_{\mu,w} \neg K$ iff there does not exist an h such that $\langle g,h\rangle \models_{\mu,w} K$
   (vi) $g \models_{\mu,w} K_1 \vee K_2$ iff there is some h such that $\langle g,h\rangle \models_{\mu,w} K_1$ or $\langle g,h\rangle \models_{\mu,w} K_2$
   (vii) $g \models_{\mu,w} K_1 \Rightarrow K_2$ iff for all m such that $\langle g,m\rangle \models_{\mu,w} K_1$, there exist a k such that $\langle m,k\rangle \models_{\mu,w} K_2$

c. A proper DRS K is true in μ at a world w ($\models_{\mu,w}$ K) iff
   [IV] there exists an embedding h of U_K such that $\langle \Lambda,h\rangle \models_{\mu,w}$ K [where Λ denotes the empty embedding].

●**Appendix 2.** Static interpretation in ReALIS (see Alberti 2005b)

(6) **Definition of eventuality-based static interpretation in ReALIS**

We define connections that belong to the partial external world $Tru^\Theta(m) = W[m(0), \Theta]$ for some linguistic impulse m on the basis of an eventuality e of the universe of a momentary internal world in the range of $Dyn^\Delta(m)$, as defined in ...; eventuality e is required to belong to (an element of) a difference eventuality-matrix $W^i_{t'} -_\alpha W^i_t$ where $Dyn^\Delta(m) : W^i_t \mapsto W^i_{t'}$.

a. *Elementary external evaluation*:
   1. Suppose eventuality e is such that every element of the universe of its eventuality-matrix E (i.e. $E = \{e\}[\alpha]^i_t*$) which is not an eventuality referent belongs to the root worldlet.
   2. Suppose, further, the (momentary) anchor α has an identity-preserving ($\langle x,y\rangle \in Ide^i_t* \Rightarrow \alpha'x = \alpha'y$) extension α' that satisfies the formula below for an arbitrary row f of E:
   f: p t r_1 ... r_k:
   $\langle \alpha't, \alpha'r_1, ..., \alpha'r_k\rangle \in [\alpha'p]$.
   3. In this case we say that α' *verifies* E, and (hence) eventuality e is *true* in the external world *according to anchor* α (*true*$_\alpha$).
   4. If there is no extension α' of anchor α that verifies E, we say that e is *false* in the external world *according to* α (*false*$_\alpha$).

b. Elementary internal evaluation
   1. Suppose we have an eventuality referent e with E as its eventuality-matrix ($E = \{e\}[\alpha]^i_t*$), and another eventuality referent e' (of possibly another universe), whose eventuality-matrix is denoted by E' ($E' = \{e'\}[\alpha]^{i'}_{t'}*$).
   2. Suppose, further, there is a mapping $\varphi : E \rightarrow E'$ which preserves all the three relations of internal worlds and the anchor,
   i.e. for arbitrary referents x and y and an arbitrary reality label ν,
   i. $\langle x, y\rangle \in Con \Rightarrow \langle \varphi x, \varphi y\rangle \in Con$;
   ii. $\langle x, y\rangle \in Ide* \Rightarrow \langle \varphi x, \varphi y\rangle \in Ide*$;$\langle x, y\rangle \notin Ide* \Rightarrow \langle \varphi x, \varphi y\rangle \notin Ide*$;
   iii. $x <_\nu y \Rightarrow \varphi x <_{\varphi\nu} \varphi y$; and $x =_{\nu'} y$ for some $\nu' \Rightarrow \varphi x =_{\nu''} \varphi y$ for some ν''

iv. $\alpha^{i}{}_{t}x = \alpha^{i'}{}_{t'}\varphi x$ (incl. value gaps)

iv'. *weak* version: $\alpha^{i}{}_{t}x \neq \alpha^{i'}{}_{t'}\varphi x$ is excluded.

3. In this case we can say that

φ *(weakly) maps* E *onto* E',

φ *(weakly) maps* E *into* worldlet $w^{i'}{}_{t'}$[e'], i.e. the worldlet of e',

e is *(weakly) true / undecided / false* in the worldlet mentioned above if e' can be found in the positive / neutral / negative segment of its worldlet, respectively.

4. If a certain (non-empty) subset γ of the four-element set of requirements in (2i-iv) above is satisfied, we can say that φ *(weakly) γ-maps* E onto E'.

c. External evaluation of a modal eventuality

1. Suppose p is a modal predicate of an eventuality in $U^{i}{}_{t}$, as shown below:

e: t'' p r e'

2. Suppose, further, r is anchored to an interpreter j (i.e. $\alpha r = j \in I$), who has an eventuality f (with F as its eventuality-matrix and) with ν as its reality label:

$\nu = \langle \alpha p, j', t', + \rangle$,

where $\alpha j' = j$, and $\alpha t'' = \alpha t' \leq t$.

3. Suppose, finally, that there is a function φ which (weakly) maps the eventuality-matrix E' of e' onto F.

4. In this case we can say that e is *(weakly) true according to anchor* α; otherwise, e is *false according to* α.

d. *External evaluation of a Boolean eventuality*:

1. Suppose the predicate referent of the eventuality e below in $U^{i}{}_{t}$ is a Boolean predicate in the sense that B is anchored to nothing, but a k-ary *truth-function* $[B] : \Pi^k \to \Pi$ belongs to it:

e: t' B $e_1 \ldots e_k$ (for some $k \geq 1$).

2. Then e is *π (true / false) according to anchor* α, if

$[B] : \langle [e_1], \ldots, [e_k] \rangle \mapsto \pi \in \Pi$,

where, for each j, $[e_j]$ expresses the fact that e_j is *true / false according to* α.

e. External evaluation of generalized quantification

1. Suppose the predicate referent of the eventuality e below in $U^{i}{}_{t}$ is a *(polyadic) generalized quantifier* Q in the sense that Q is anchored to nothing, but an *evaluation function* [Q] (of at most 3k+1 arity) belongs to it:

e: t' Q $f_1 \ldots f_k$ g (for some $k \geq 1$),

where f_j and g are eventualities, for $1 \leq j \leq k$, with F_j and G as its eventuality-matrices.

2. We should define, for an eventuality-matrix X that has a relative root worldlet y and a worldlet w such that $w <_{Acc} y$ (i.e. w precedes y according to the accessibility-ordering), an *unpacked copy* $u_w(X)$ of X *in* w as follows: an appropriate copy $u_w(X)$ should be an eventuality-matrix with w as its relative root worldlet such that there is a function φ mapping X onto $u_w(X)$ (in the sense defined in (6)b above). Instead of $u_v(X)$, where v is the real worldlet, we can write: u(F).

3. Three kinds of families of *extended anchors* are defined below, where

F and G are eventuality-matrices,

u(F) is an unpacked copy of F (in the real worldlet),

⟨F,G⟩ is the double matrix of F and G,

u(⟨F,G⟩) is an unpacked copy of the double matrix (in the real worldlet):

i. *Anchors extended according to* F:

$\alpha[F] := \{\alpha' \supseteq \alpha : \alpha' \text{ verifies } u(F)\}$

ii. *Anchors extended according to* F *further-extendable according to* ⟨F,G⟩:

$\alpha[F+G] := \{\alpha'' \in \alpha[F] : u(\langle F,G \rangle) \text{ is true}_{\alpha''}\}$

iii. *Anchors extended acc. to* F *non-further-extendable acc. to* ⟨F,G⟩:

$\alpha[F-G] := \{\alpha'' \in \alpha[F] : u(\langle F,G \rangle) \text{ is false}_{\alpha''}\}$

4. [Q] is a function, defined in the lexical characterization of Q, that assigns a *truth value* to a sequence of the following families of extended anchors:

$\langle \alpha[F_1], \ldots \alpha[F_k], \alpha[G], \alpha[F_1+G], \ldots, \alpha[F_k+G], \alpha[F_1-G], \ldots, \alpha[F_k-G] \rangle$;

e can be called *true / false according to anchor* α depending on this value.

5. We can obtain some usual categories of binary generalized quantifiers as follows (see Keenan 1996):

i. Q is a *generalized existential* quantifier if the value of [Q] only depends on $\alpha[F_1+G]$.

ii. Q is a *generalized universal* quantifier if the value of [Q] only depends on $\alpha[F_1-G]$.

iii. Q is a *cardinal* quantifier if the value of [Q] only depends on the cardinalities of the families of extended anchors mentioned in 4. above.