Discussion on

Uncertainty handling in Logic Programing

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Uncertainty / Fuzziness

- uncertainty

  due to incomplete information or randomness on Boolean events

  truth-degrees $\in \{0, 1\}$

  can be evaluated in a quantitative / qualitative way

  uncertainty measures on possible worlds

  uncertainty degrees $\in [0, 1]$ (usually)

  various models: probabilistic, possibilistic, belief functions, etc.

- fuzziness

  partial satisfaction of gradual properties

  truth-degrees $\in [0, 1]$ (usually)

  full compositional laws for compound formulas

Logic Programming and Uncertainty

A variety of logic programming languages handling different uncertainty and fuzzy models. One can classify them by:

- Uncertainty / fuzzy model chosen:

  - probabilistic l.p.
  - possibilistic l.p.
  - belief l.p.
  - fuzzy (choices of aggregation operations)

- Annotation-based / implication-based rules

  annotated rule: $A : \mu \leftarrow B_1 : \mu_1 \wedge \ldots \wedge B_n : \mu_n$

  (a interpretation makes true or false each basic annotated fact)

  weighted implication: $(A \leftarrow B_1 \wedge \ldots \wedge B_n, \mu)$

  (mv-valued interpretation of facts / rules)

Logic Programming and Uncertainty

- definite programs: no negation involved
  fix point semantics (minimal models)

- normal programs: negation by failure in the body of the rules
  links to non-monotonic reasoning: *not* $A = A$ is not believed, $\neg A$ is consistent
  answer set semantics (stable models): minimal models of program reducts (Gelfond-Lifschitz reduction)

- extended programs: negation by failure + classical negation
  answer set semantics: coherent stable models

- disjunctive programs
  disjunctions in the head of rules
  qualitative form of uncertainty

Annotated logic programming languages

- Generalized Annotated Programs GAP (Kifer-Subrahmanian, 89)
- Probabilistic logic programs PLP (Ng-Subrahmanian, 92)
  Hybrid Probabilistic logic programs (Dekhtyar-Subrahmanian, 97)
  (Saad 06)
- Action probabilistic programs (Khuller et al., 07), (Simari et al., SUM 2010)
- Extended fuzzy logic programs (Saad, SUM 2009)
  Disjunctive Extended fuzzy logic programs (Saad, SUM 2010)

Conditional / Implication -based approaches

- Conditional probability-based logic programs (Lukasiewicz, 2001)

  rules: $(A \leftarrow B, [\alpha, \beta])$

  interpretations: $Pr : 2^{HB} \rightarrow [0, 1]$ probability function

  $Pr \models (A \leftarrow B, \alpha)$ iff $Pr(A \mid B) \in [\alpha, \beta]$

  inference: linear optimization techniques

- Possibilistic logic programs (Dubios-Lang-Prade, 1991)

  rules: $(A \leftarrow B, \alpha)$

  interpretations: $N : 2^{HB} \rightarrow [0, 1]$ necessity function

  $N \models (A \leftarrow B, \alpha)$ iff $N(\neg B \vee A) \geq \alpha$

  Immediate Consequence operator based on weighted modus ponens: from $(A \leftarrow B, \alpha)$ and (B, β) derive $(A, \min(\alpha, \beta))$

Conditional / Implication -based approaches

- Fuzzy / many-valued logic programs
  rules: $(A \leftarrow B, \alpha)$

  $I : At \rightarrow [0, 1]$ extends to rules by $I(A \leftarrow B) = I(A) \Rightarrow I(B)$, where $\Rightarrow$ is the residuum of a conjunctive aggregation operator (t-norm) $*$

  $I \models (A \leftarrow B, \alpha)$ iff $I(A) \Rightarrow I(B) \geq \alpha$ iff $I(B) \geq I(A) * \alpha$

  Immediate Consequence operator based on fuzzy modus ponens:
  from $(A \leftarrow B, \alpha)$ and (B, β) derive $(A, \alpha * \beta)$

Implication-based logic programming languages

- Answer set semantics for possibilistic logic programs
  - (Nicolás et al., 2005, 2006)
  - (Bauters-Schockaert-De Cock-Vermeir, 2010)
  - (Nieves-Osorio, 2007)

- Residuated Logic programs (Damasio-Pereira, 2001)
  truth-values domain: abstract residuated latiice

- Normal logic programs over lattices and bilattices (Straccia, 2005)

- Answer set semantics for fuzzy L.P.s
  - (Madrid-Ojeda, 2009)
  - (Janssen, Schockaert, Vermeir, De Cock, 2009)

Discussion

- Annotated versus implication based approaches:
  - extendability?
  - expressiveness?
  - applicability? (Simari et al, SUM 2010)

- Fuzzy logic programming languages:

  - weak link to well-established systems of formal fuzzy logic (e.g. Łukasiewicz, Gödel, product logics)

  - answer set semantics: introducing non-monotonicity into fuzzy logics (fuzzy equilibrium logic - Schockaert et al.)

- Integration of uncertainty and fuzziness handling
  - disjunctive Fuzzy LP (Saad, SUM 2010)

- Scalability