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Timing is everything: Sparse sampling based on time-encoding machines

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Ack.

Interaction
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Joint work with
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The Problem

- Your primary school child has been assigned a nasty summer homework
- He needs to estimate rainfall over the summer break
- He is desperate because the summer vacation is at stake

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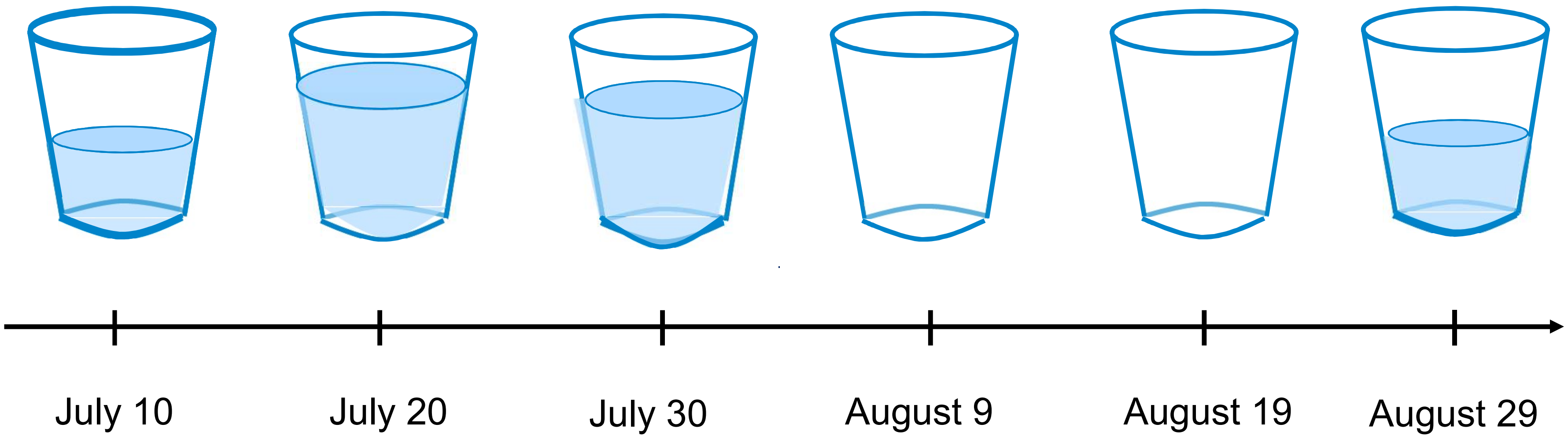
The Problem

Approach 1

- Put a bucket in the back garden and record rainfall at regular intervals (e.g. every 10 days record rainfall and empty bucket)



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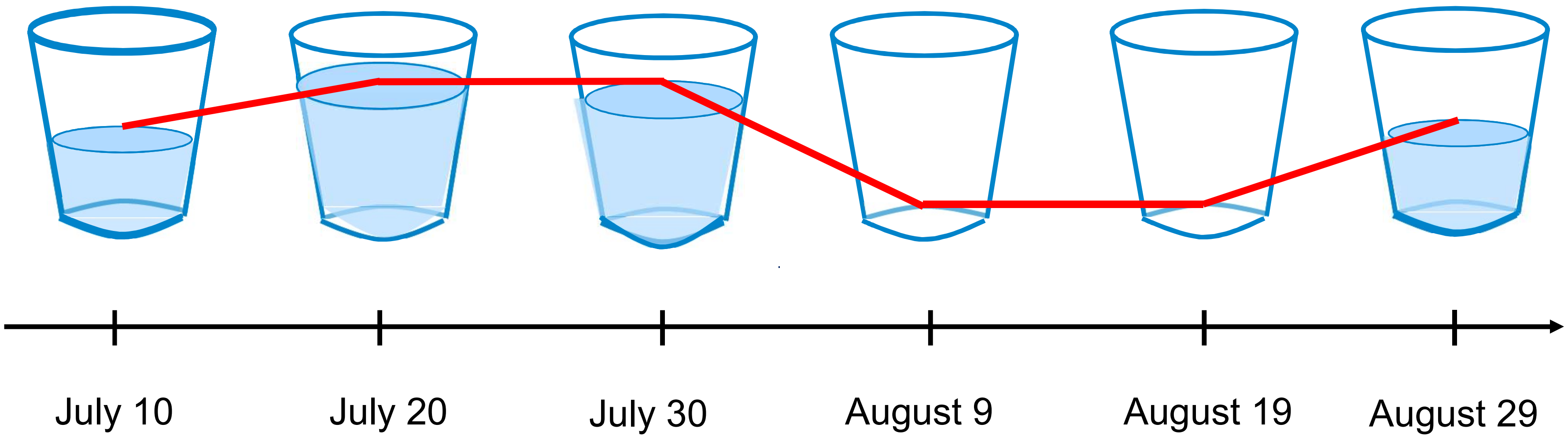
The Problem

Approach 1

- Advantage: Easy estimation
- Disadvantage: inefficient (need to check and empty the bucket regularly even during the dry season and so…no summer vacations!)



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The Problem

Approach 2

- Only record the day when the bucket is full and then empty it

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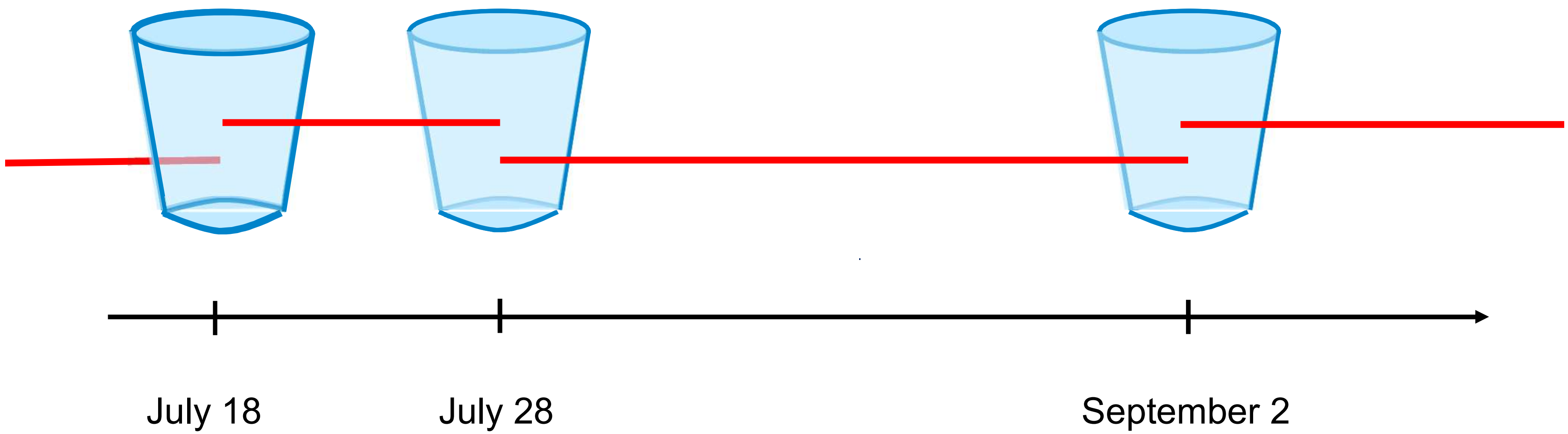
The Problem

Approach 2

- Advantage: Very efficient (we can go on holiday in August!)
- Disadvantage: estimation of rainfall over the period is more complicated



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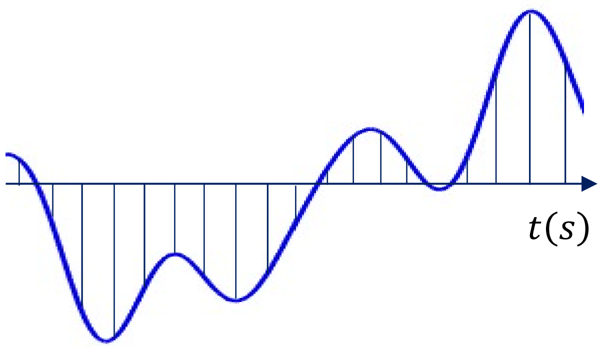
Sampling

- These two approaches represent two very different ways to sample a continuous phenomenon
- Approach 1 is what we engineers do and is equivalent to the traditional amplitude-based uniform sampling



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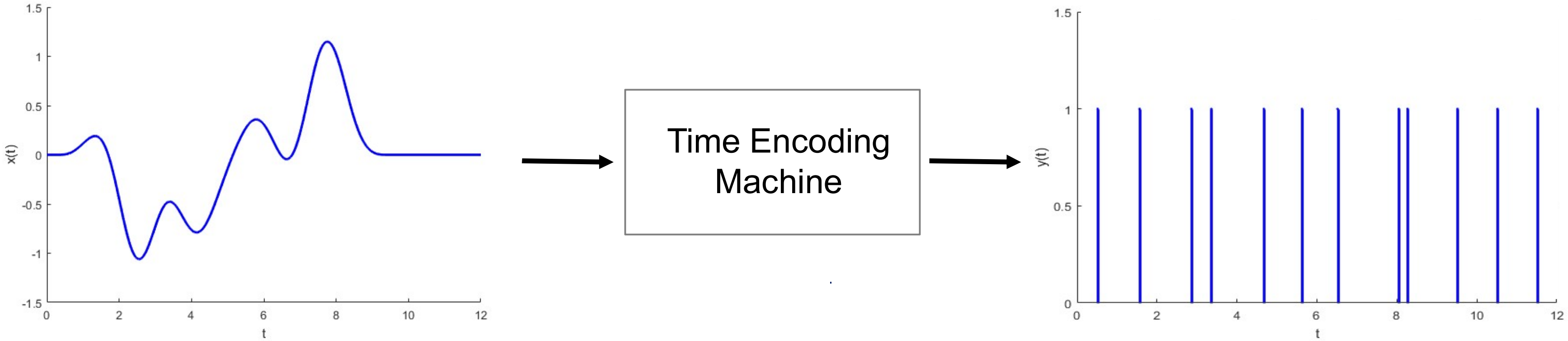
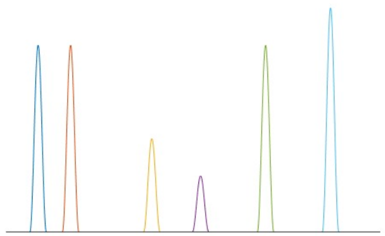
Sampling

- Approach 2 maps analogue information into a time sequence and is used by nature (e.g., integrate-and-fire neurons)

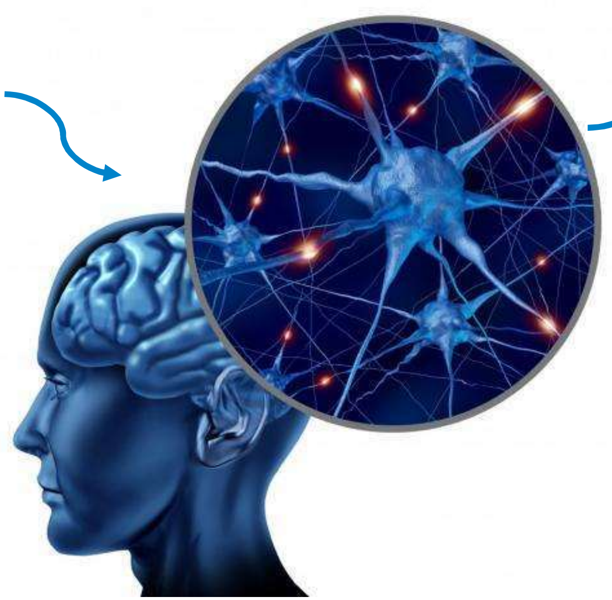


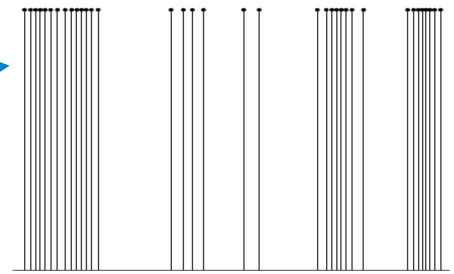
Motivation

Time encoding appears in nature, as a mechanism used by neurons to represent sensory information as a sequence of action potentials, allowing them to process information **very efficiently**.



Input signal





Action potentials

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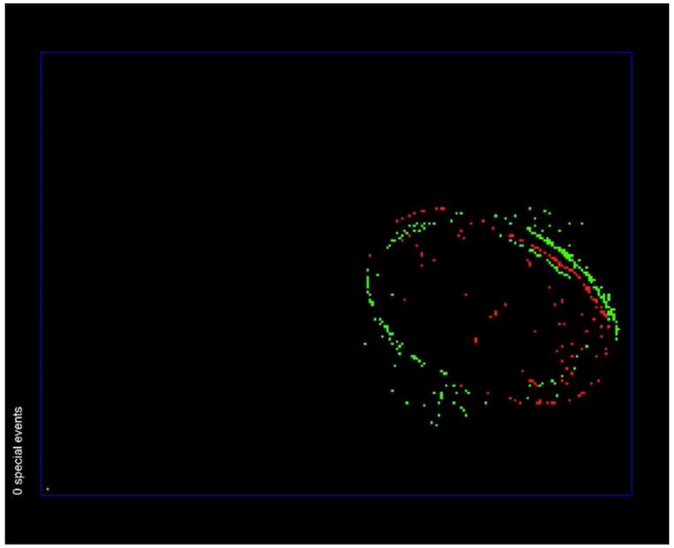
Time-Based Sampling

- Acquisition systems inspired by time-based sampling, such as event-based vision sensors, are emerging in a variety of new scenarios (e.g. see Toby Delbruck web page)



Videos taken from Inivation.com (see also Toby Delbruck web page)

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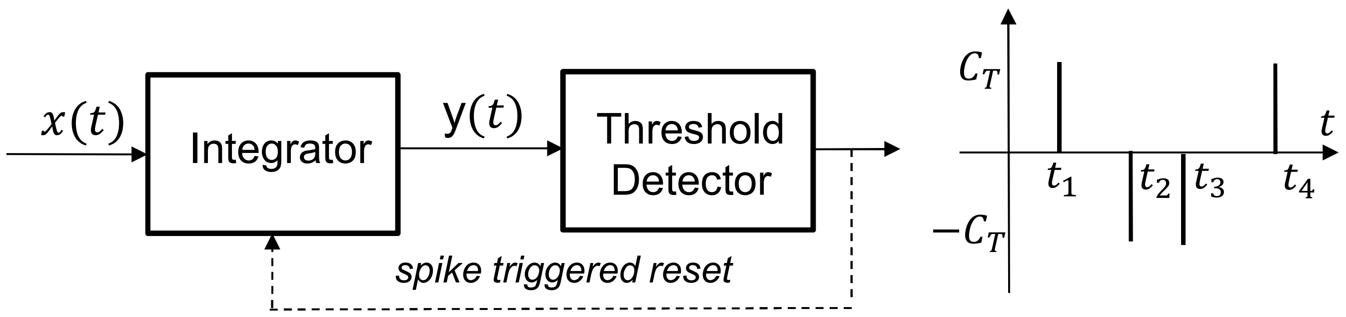
Time-encoding machines

Integrate-and-fire System



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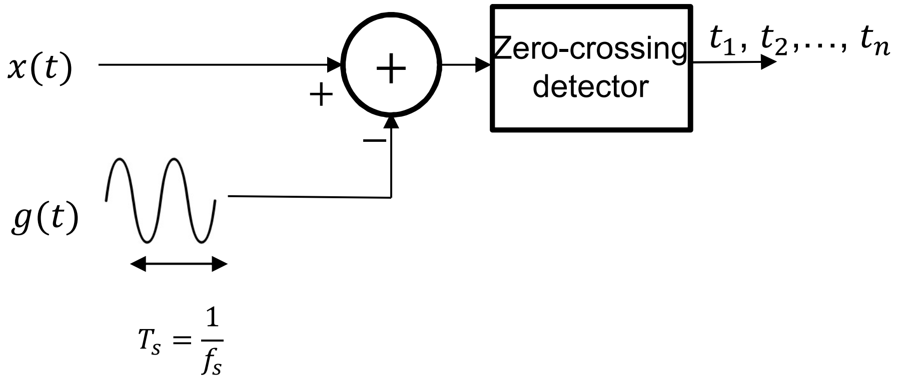
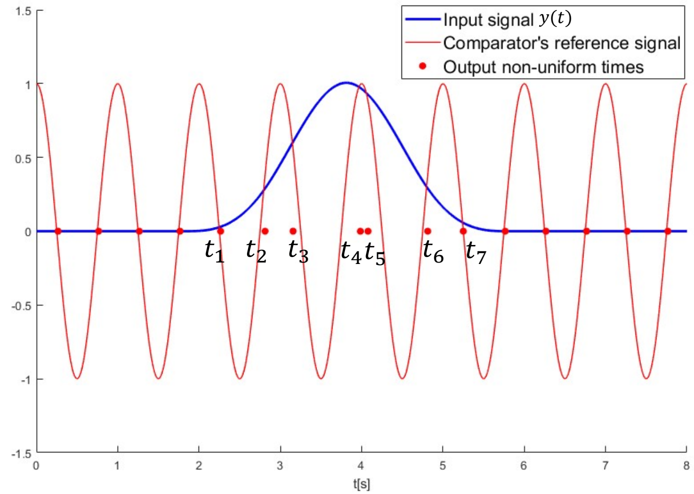
Time-encoding machines

Comparator System





- At the crossing times, $x(t_n) - g(t_n) = 0$ hence $x(t_n) = g(t_n)$.

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Reconstruction from time-encoded information

- Given the retrieved non-uniform samples $x(t_1), x(t_2), \ldots, x(t_n)$ can we reconstruct $x(t)$?

- This is a classical problem in non-uniform sampling

- Assume that $x(t)$ belongs to a shift-invariant space (e.g., $x(t)$ is bandlimited, $x(t) = \sum_k c_k \varphi(t-k)$) then, if the density of samples $\mathrm{D} \geq 1$, perfect reconstruction is possible[1]

[1]A. Aldroubi and K, Grochenig, “Non-Uniform Sampling and Reconstruction in shift-invariant spaces” SIAM Review 2001

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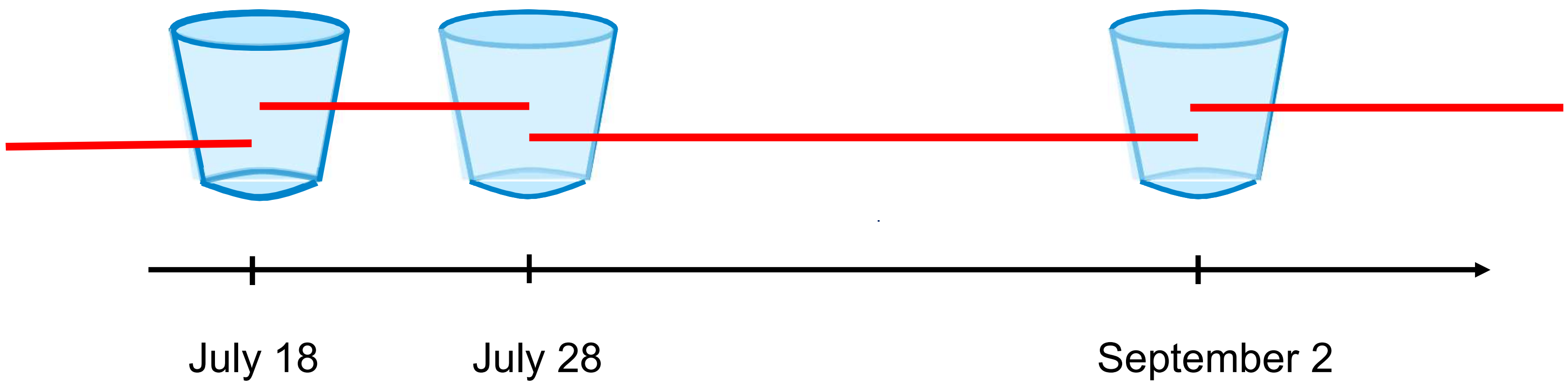
Reconstruction from time-encoded information

- **Key result**:[1] if the density of samples $D \geq 1$ then perfect reconstruction can be using an iterative approach proposed by Aldroubi and Grochenig[1]



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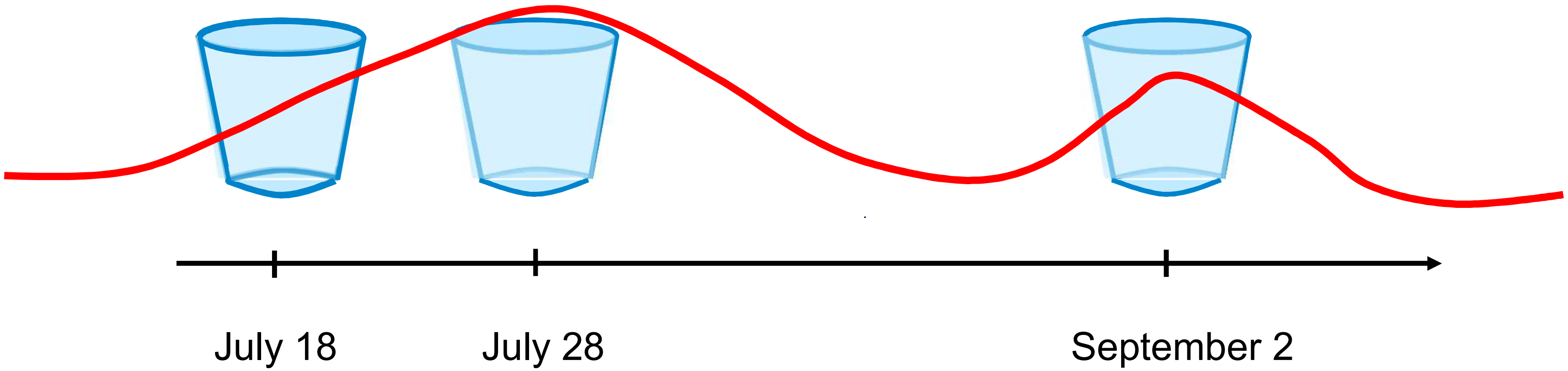

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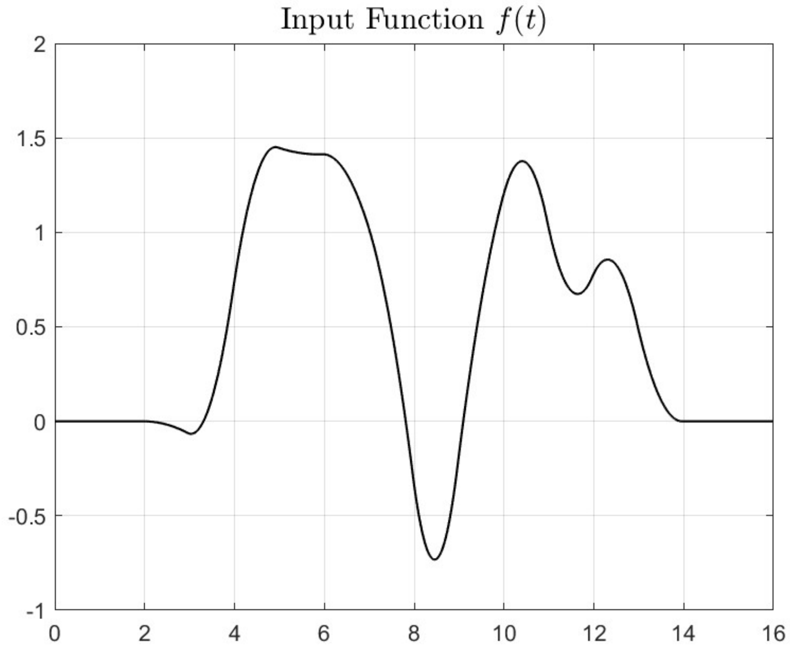
Reconstruction from time-encoded information

- The iterative approach proposed by Aldroubi and Grochenig[1]



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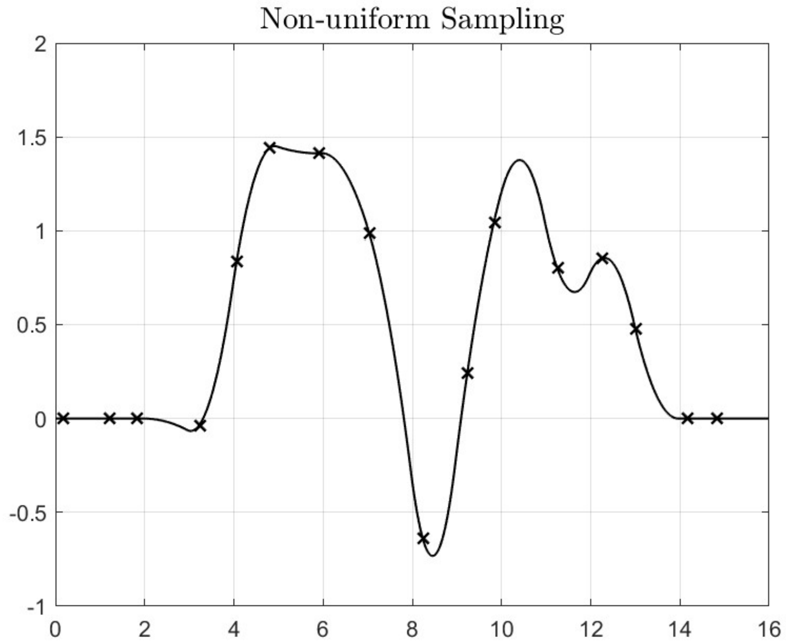

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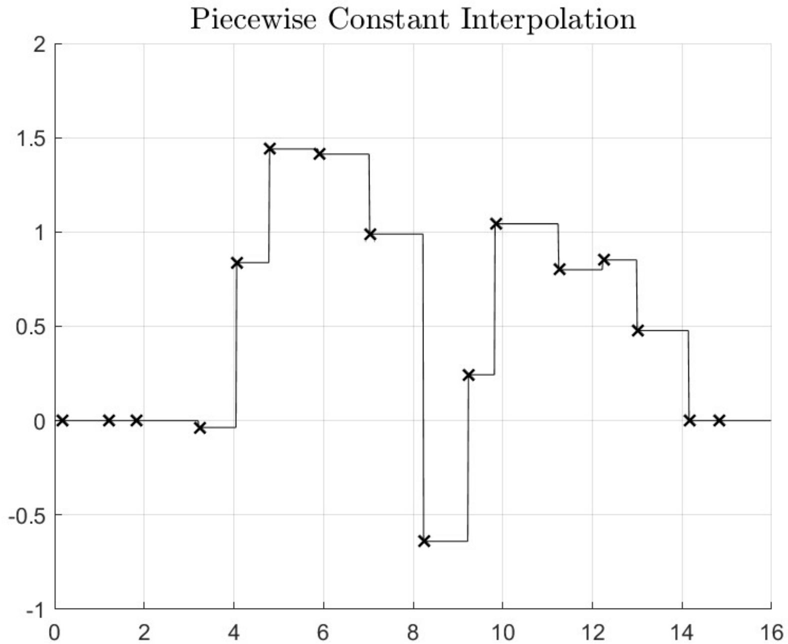

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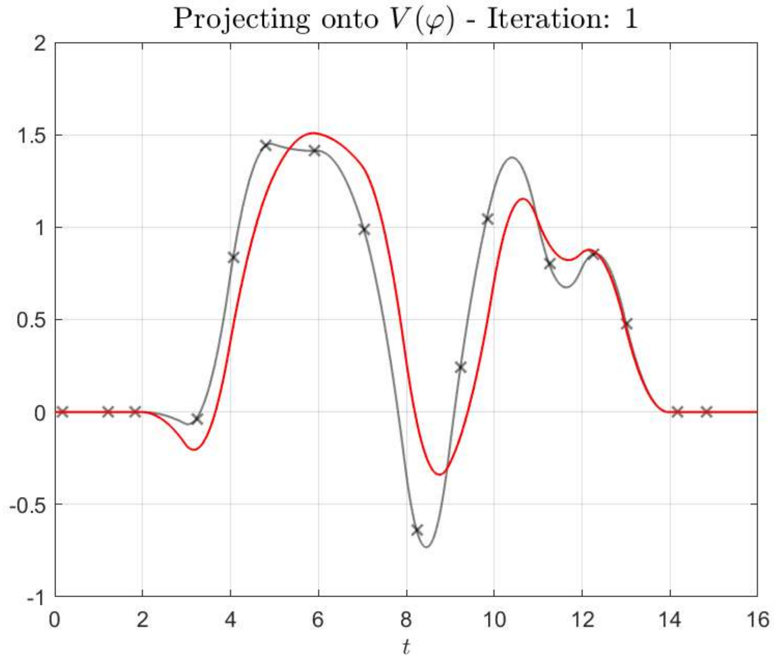

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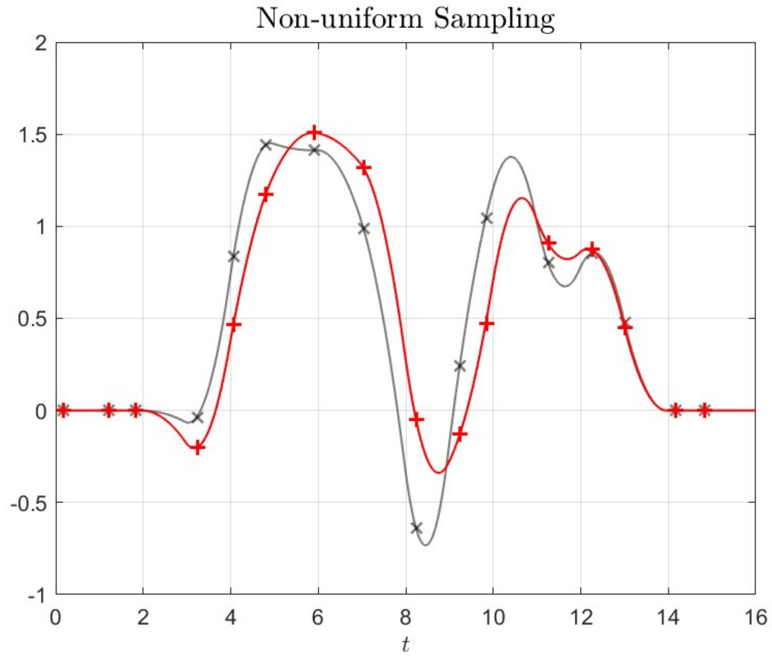

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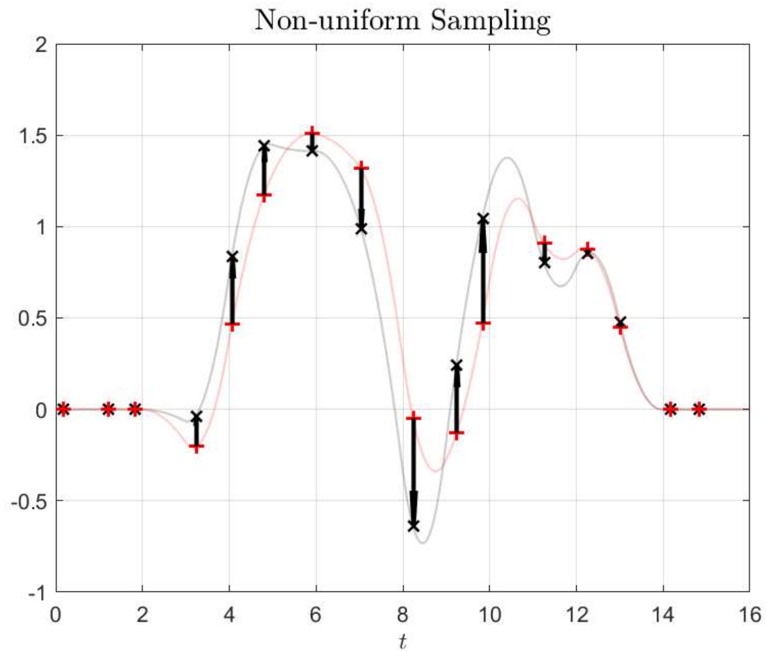

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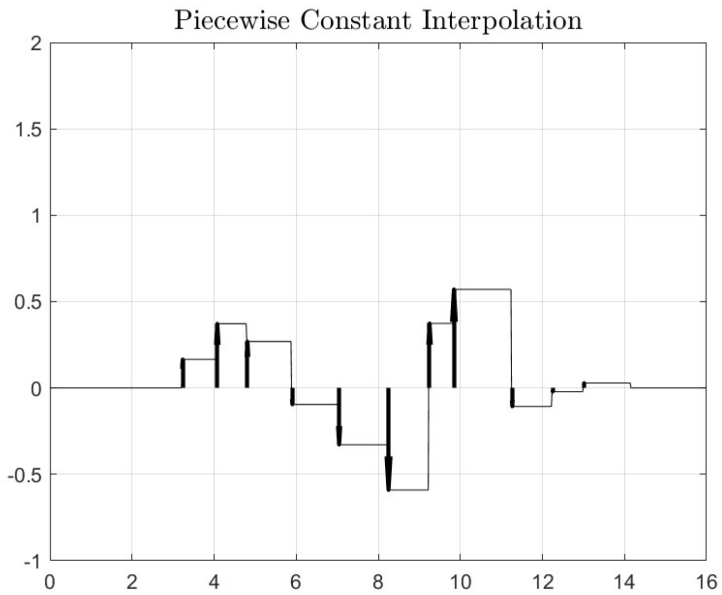

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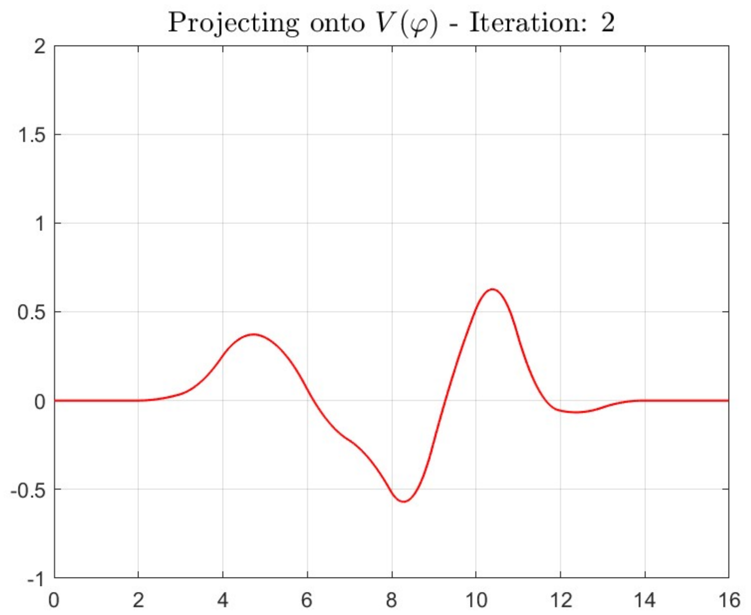

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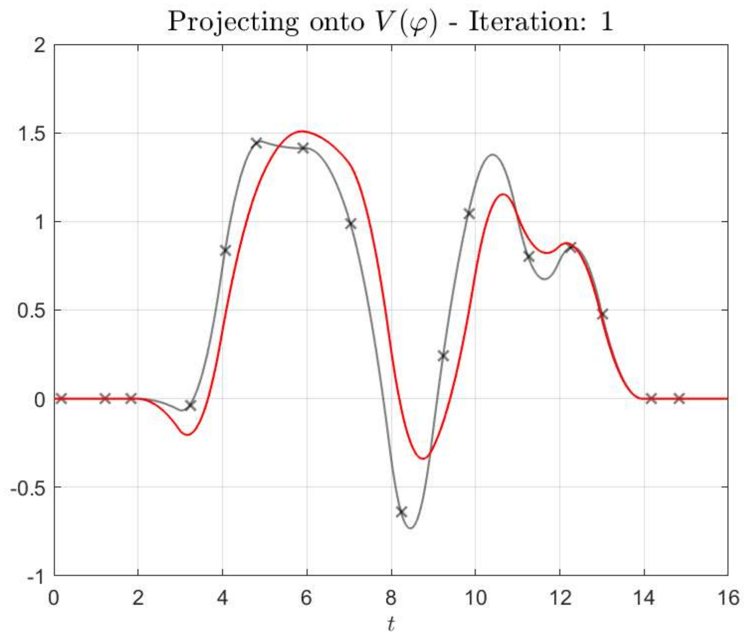

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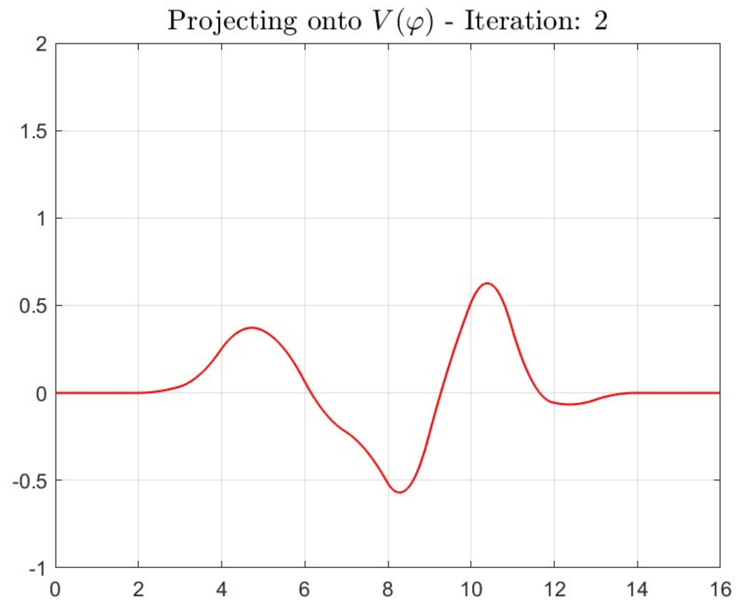

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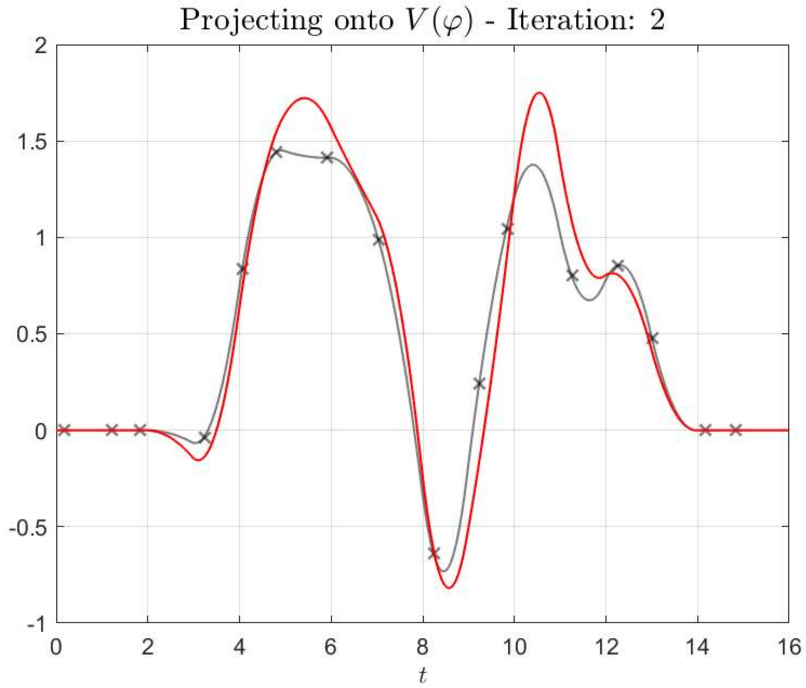

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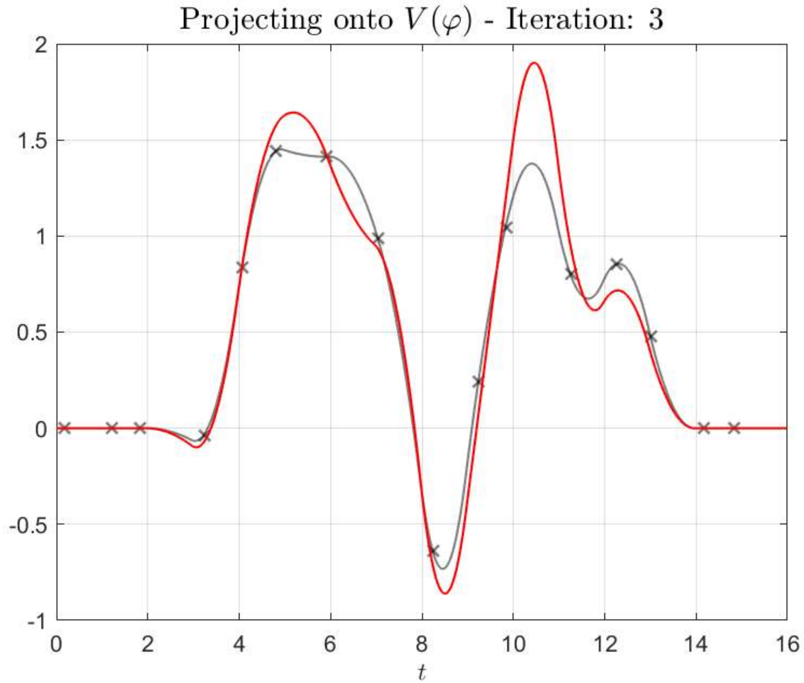

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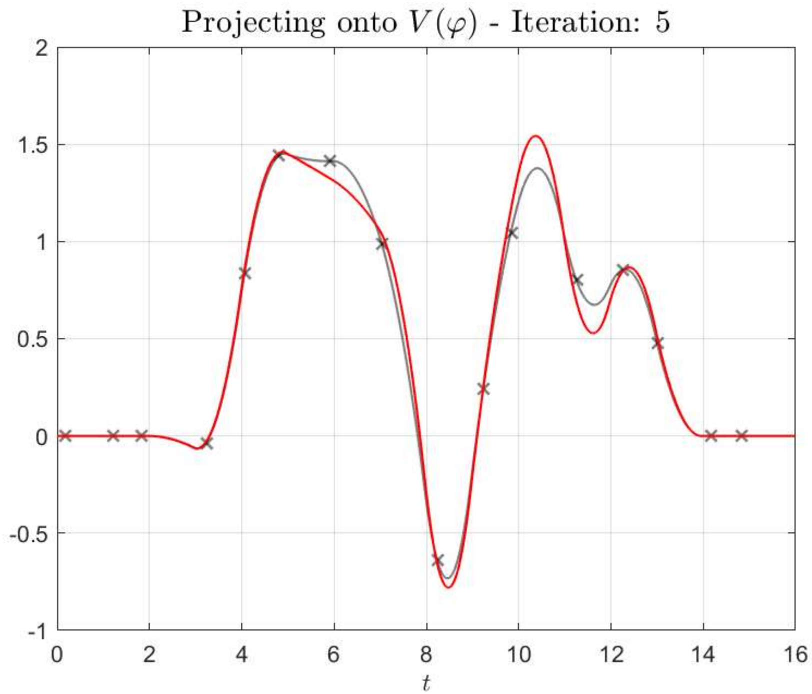

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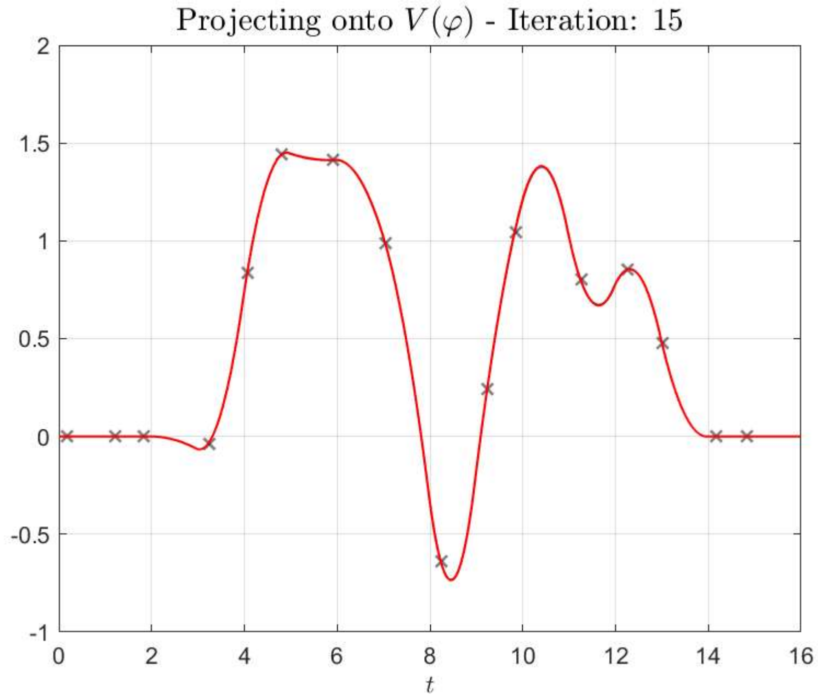

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Reconstruction from time-encoded information

- **Key result**: if the density of samples $\mathrm{D} \geq 1$ then $K_{t_j}(t)$ form a basis

- **Key Issue 1**: In the case of uniform sampling the density is $D = 1$. This means that current TEMs are **less** energy efficient than uniform sampling!

- **Key Issue 2:** Cannot sample sparse (non-bandlimited) signals with the current methods.

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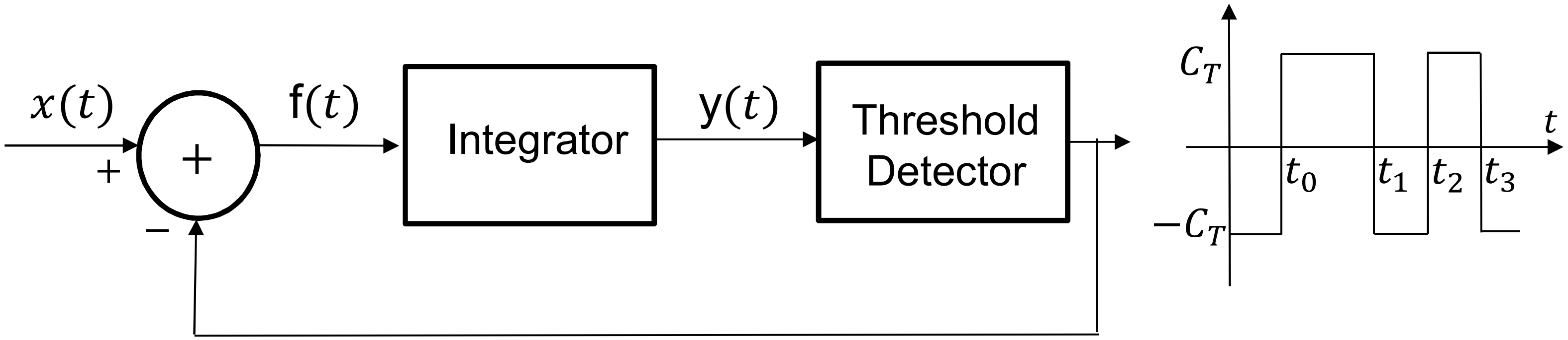
Reconstruction from time-encoded information

- For integrate-and-fire machines exact reconstruction proved here: A. A. Lazar and L. T. Toth, “Time encoding and perfect recovery of bandlimited signals”, ICASSP 2003



See also: Gauntier-Vetterli-2014, Adam et al 2019,

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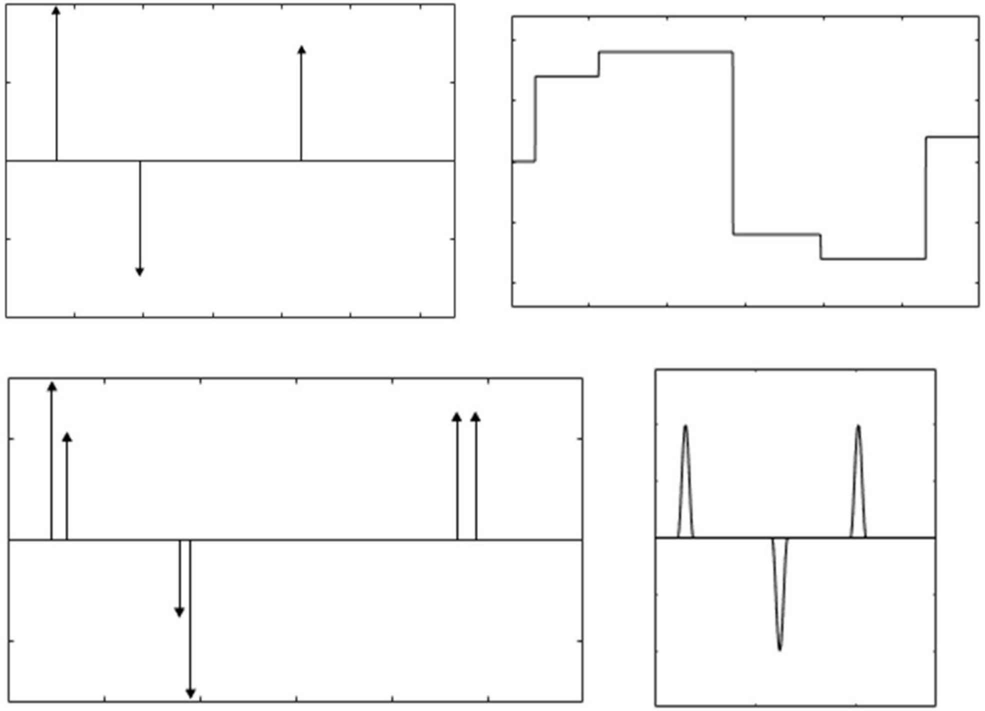
Sparse Sampling - Signals

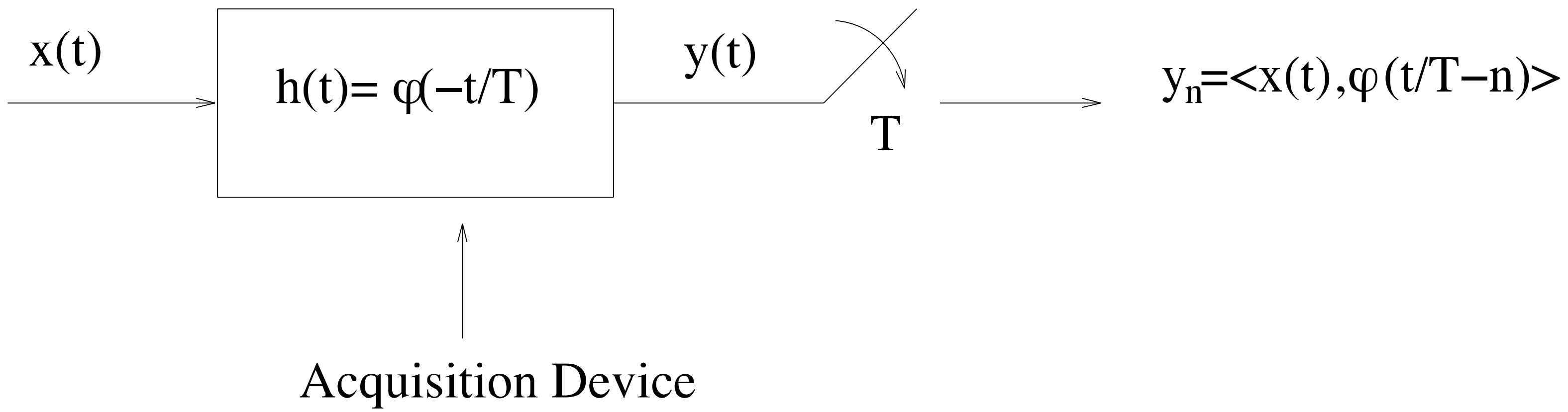
- We consider sparse parametric signals (i.e., signals with finite rate of innovation[2]).
- Key issue is how to retrieve the free parameters of these signals for time-based information



[2]Vetterli Marziliano Blu, Sampling Signals with Finite Rate of Innovation, IEEE Trans. on Signal Processing, June 2002

Sparse Sampling - Acquisition

- In sparse sampling, the acquisition device is used to 'spread the innovation'
- Reconstruction process is non-linear
- These two ingredients are necessary to time-encode sparse non-bandlimited signals



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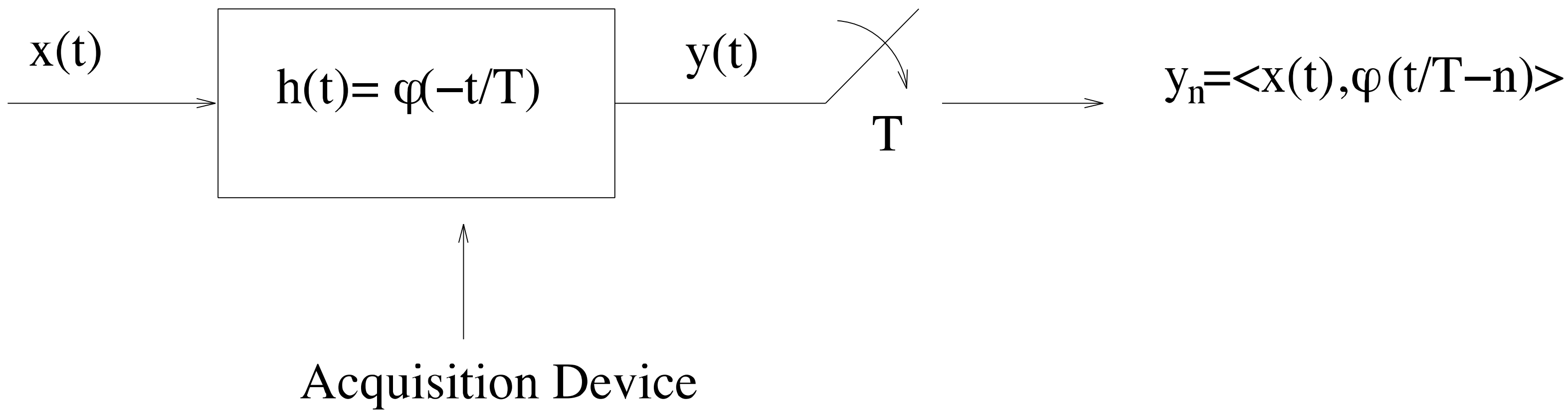
Sparse Sampling

- We leverage two main ideas from sampling sparse signals with finite rate of innovations:
  - The sampling kernels can reproduce polynomials or exponentials
  - Reconstruction is achieved using Prony's method



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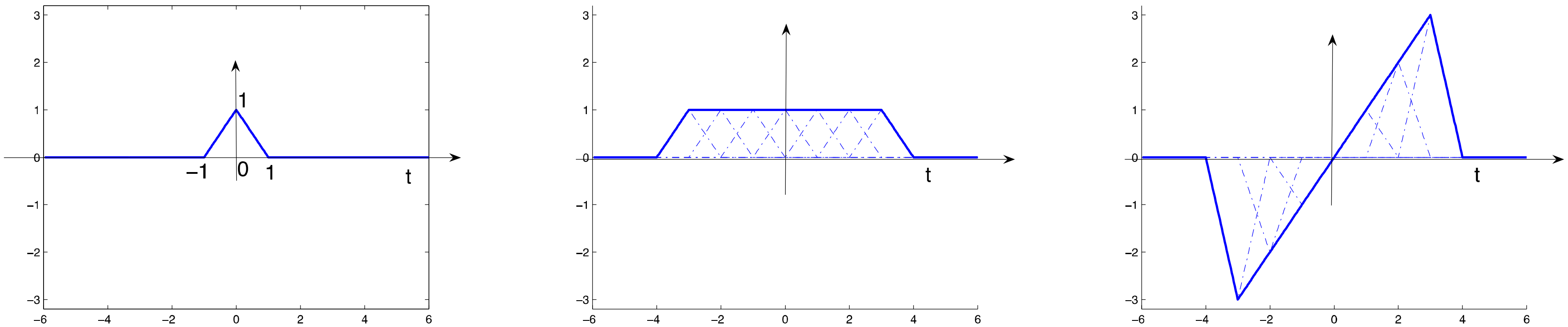
Reproduction of Polynomials

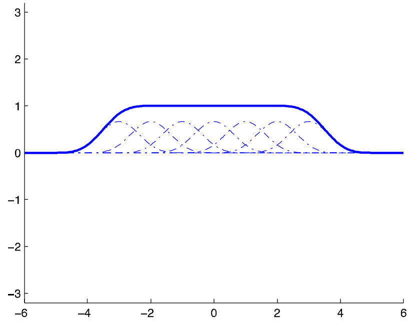


$\beta_1(t)$ $\quad c_{0,n} = (1, 1, 1, 1, 1, 1, 1)$ $\quad c_{1,n} = (-3, -2, -1, 0, 1, 2, 3)$

The linear spline reproduces polynomials up to degree L=1: $\sum_n c_{m,n}\beta_1(t-n) = t^m \quad m = 0, 1,$ for a proper choice of coefficients $c_{m,n}$ (in this example $n = -3, -2, \ldots, 1, 2, 3$).

Notice: $c_{m,n} = \langle \tilde{\varphi}(t-n), t^m \rangle$ where $\tilde{\varphi}(t)$ is biorthogonal to $\varphi(t)$: $\langle \tilde{\varphi}(t), \varphi(t-n) \rangle = \delta_n$.

Reproduction of Polynomials



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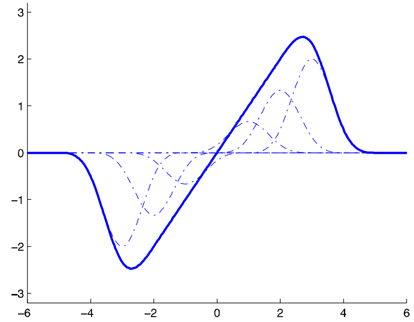
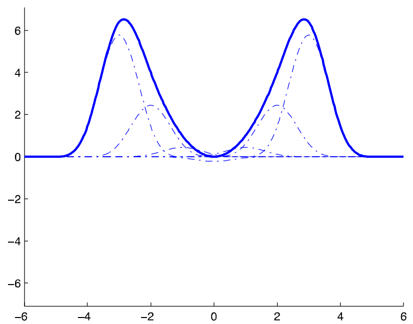
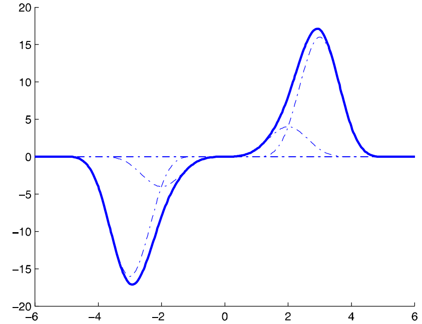


$c_{1,n} = (-3, -2, -1, 0, 1, 2, 3)$



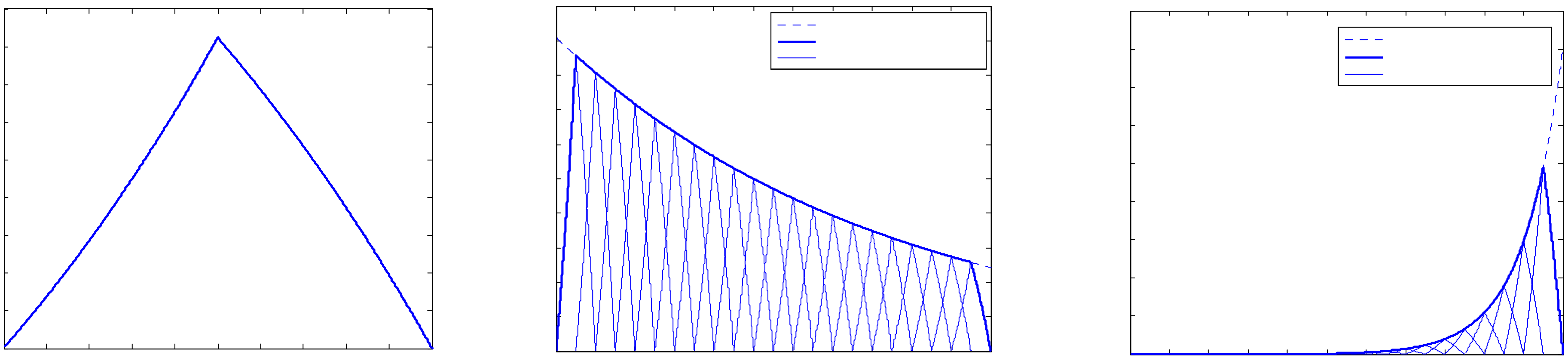
$c_{2,n} \sim (8.7, 3.7, 0.7, -0.333, 0.7, 3.7, 8.7)$



$c_{3,n} \sim (-24, -6, -0.001, 0, 0.001, 6, 24)$

The cubic spline reproduces polynomials up to degree L=3: $\sum_n c_{m,n}\beta_3(t-n) = t^m \quad m = 0, 1, 2, 3.$

Reproduction of Exponentials



Here the E-spline is of second order and reproduces the exponential $e^{\alpha_0 t}$, $e^{\alpha_1 t}$: with $\alpha_0 = -0.06$ and $\alpha_1 = 0.5$.

From Samples to Signals

- Compute a linear combination of the samples: $s_m = \sum_n c_{m,n} y_n$ for some choice of coefficients $c_{m,n}$ that reproduce polynomials or exponentials
- Because of linearity of inner product, we have that

$$
\begin{aligned}
s_m &= \sum_n c_{m,n} y_n \\
&= \sum_m c_{m,n} \langle x(t), \varphi(t/T - n) \rangle \qquad m = 0, 1, ..., L. \\
&= \langle x(t), \sum_n c_{m,n} \varphi(t/T - n) \rangle \qquad m = 0, 1, ..., L.
\end{aligned}
$$

- Given the proper choice of coefficients, we have that $\sum_n c_{m,n} \varphi(t/T - n) = e^{j\omega_m t/T}$

From Samples to Signals

Then

$$
\begin{aligned}
s_m &= \sum_n c_{m,n} y_n \\
&= \langle x(t), \sum_n c_{m,n} \varphi(t/T - n) \rangle \\
&= \int_{-\infty}^{\infty} x(t) e^{j\omega_m t} dt, \qquad m = 0, 1, ..., L.
\end{aligned}
$$

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Sampling a stream of Diracs

- Assume $x(t)$ is a stream of K Diracs on the interval of size N: $x(t) = \sum_{k=0}^{K-1} x_k \delta(t - t_k)$, $t_k \in [0, N)$.
- We restrict $j\omega_m = j\omega_0 + jm\lambda \qquad m = 1, ..., L \quad$ and $\quad L \geq 2K$.
- We have N samples: $y_n = \langle x(t), \varphi(t-n) \rangle$, $n = 0, 1, ...N-1$:
- We obtain

$$
\begin{aligned}
s_m &= \sum_{n=0}^{N-1} c_{m,n} y_n \\
&= \int_{-\infty}^{\infty} x(t) e^{j\omega_m t} dt, \\
&= \sum_{k=0}^{K-1} x_k e^{j\omega_m t_k} \\
&= \sum_{k=0}^{K-1} \hat{x}_k e^{j\lambda m t_k} = \sum_{k=0}^{K-1} \hat{x}_k u_k^m, \qquad m = 1, ..., L.
\end{aligned}
$$

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Prony's Method

- The quantity

$$s_m = \sum_{k=1}^{K} x_k e^{jm\omega_0\tau_m} = \sum_{k=1}^{K} x_k u_k^m \qquad m = 1, \dots, L$$

  is a sum of exponentials

- Retrieving the locations u_k and the amplitudes x_k from $\{s_m\}_{m=1}^{L}$ is a classical problem in spectral estimation and was first solved by Gaspard de Prony in 1795.[3]

- Given the pairs $\{x_k, u_k\}$ then $\tau_k = (\ln u_k)/\, j\omega_0$.

[3]P. Stoica and R. Moses. Spectral Analysis of Signals. 2005.

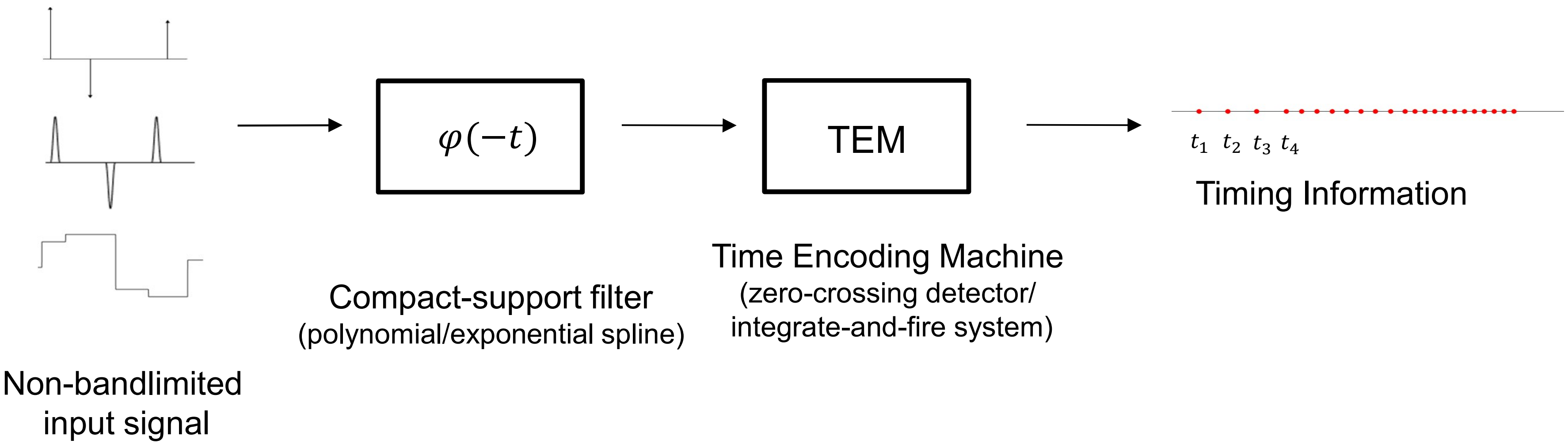
Our approach for time decoding of signals

Signals

- We consider sparse continuous-time signals like streams of diracs, stream of pulses or piecewise constant signals

Sensing Systems

- We filter before using a TEM



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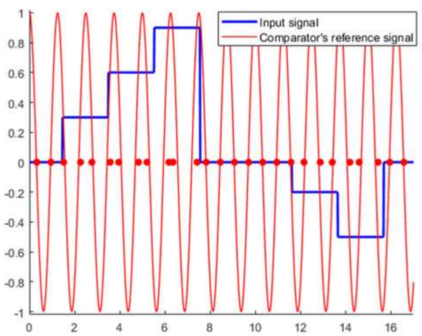
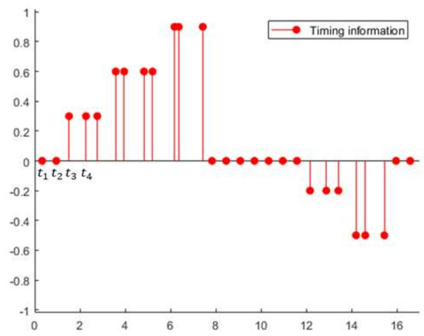
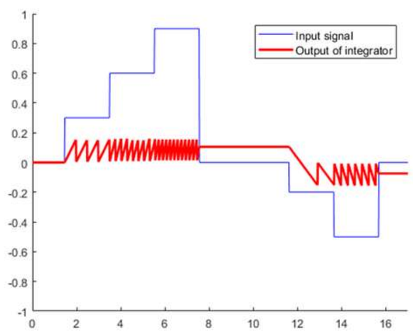
Our approach for time decoding of signals

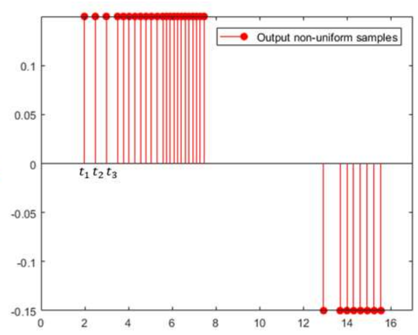
- Reconstruction of $x(t)$ depends on the
  - sampling kernel $\varphi(t)$
  - the density of time instants $\{t_n\}$

- We achieve a sufficient density of output samples by imposing conditions on:
  - The frequency of the comparator's sinusoidal signal (**crossing TEM**).
  - The trigger mark of the integrator (**integrate-and-re TEM**).









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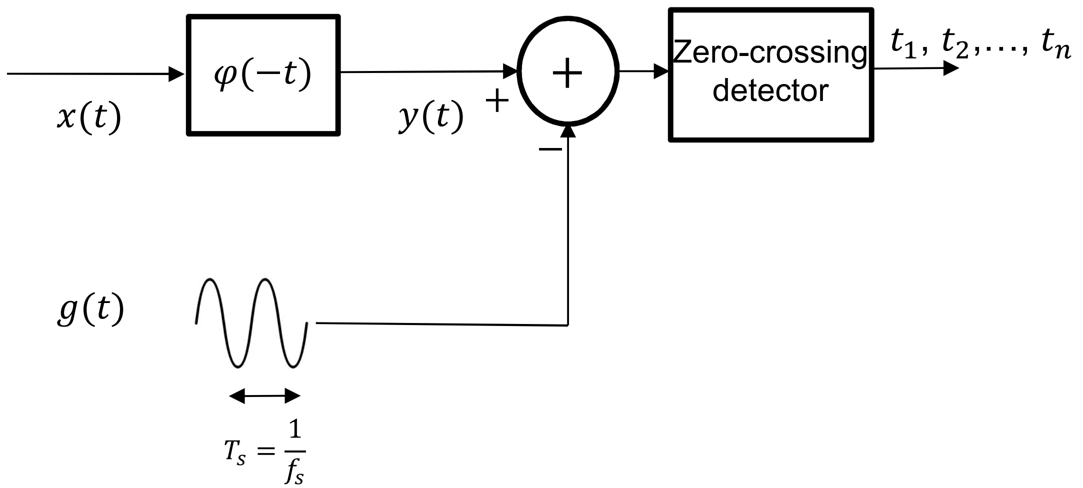
Our approach for time encoding of signals

Comparator System



- At the crossing times, $y(t_n) - g(t_n) = 0$ hence $y(t_n) = g(t_n)$.
- Moreover:

$$y(t_n) = \int x(\tau)\varphi(\tau - t_n)\, d\tau = \langle x(t), \varphi(t - t_n)\rangle$$

Sampling Kernels (B-splines)

- The anti-causal version of the zero-order B-spline is defined as:

$$\beta_0(t) = \begin{cases} 1, & -1 \leq t \leq 0, \\ 0, & \text{otherwise.} \end{cases}$$

- The P-order B-spline can be computed as:

$$\beta_P(t) = \underbrace{\beta_0(t) * \beta_0(t) .... * \beta_0(t)}_{P+1 \text{ times}},$$

- The P-order B-spline satisfies the Strang-Fix condition:

$$\sum_{n \in \mathbb{Z}} c_{m,n} \beta_P(t-n) = t^m,$$

where $m \in \{0, 1, ..., P\}$, and for a proper choice of coefficients $c_{m,n}$.

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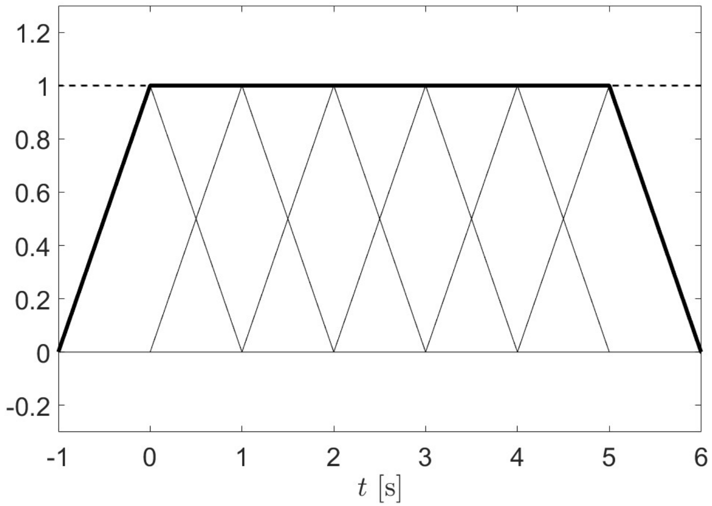
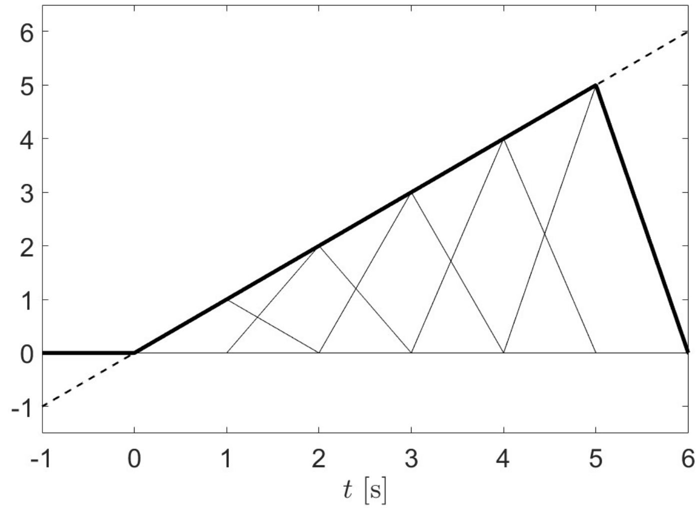
Sampling Kernels (B-splines)

Polynomial Splines

- Linear combinations of uniform shifts of B-splines reproduce polynomial because the ***'knots'*** overlap and ***'compensate'*** each other.





- **Key insight**: in the case of non-uniform shifts, reproduction of polynomials is still possible locally in ***'knot-free'*** regions

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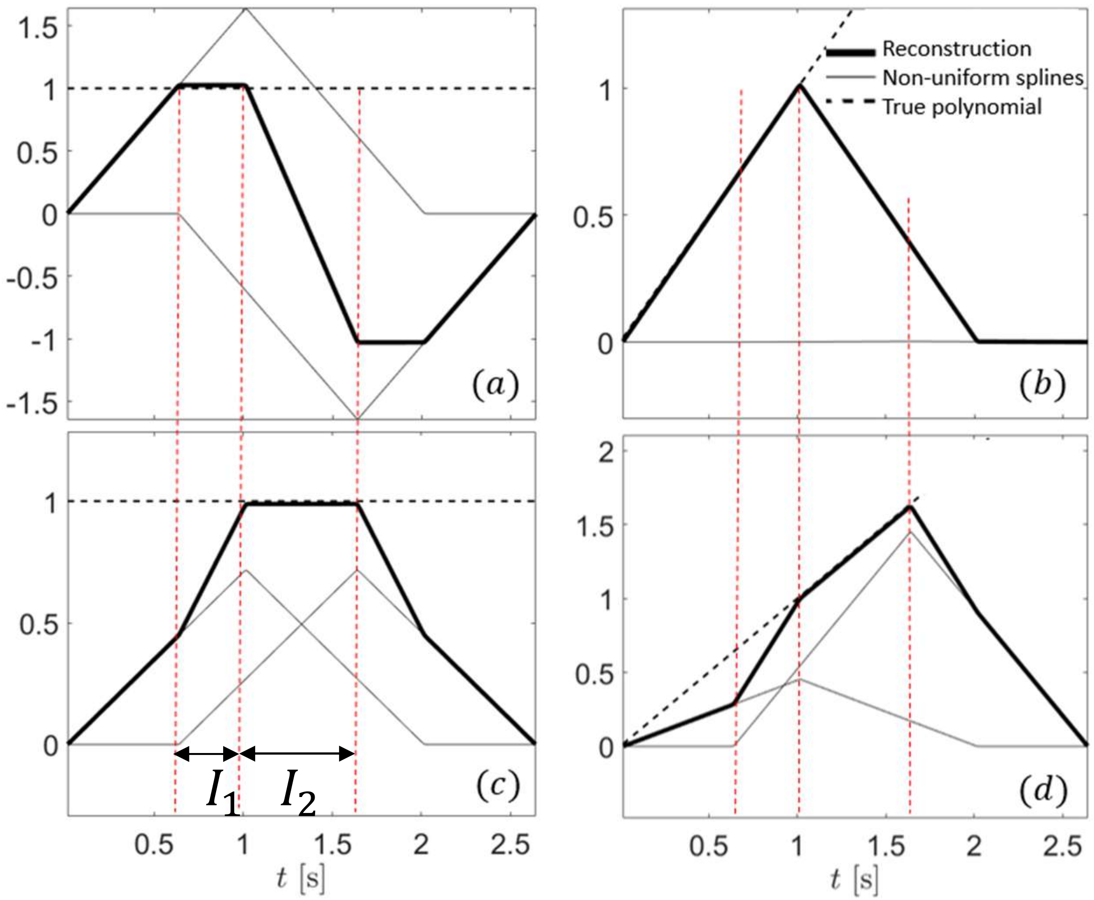
Sampling Kernels (B-splines)

Key insight: in the case of non-uniform shifts, reproduction of polynomials is still possible locally in 'knot-free' regions

Sketch of the argument:

- Each 'knot-free' piece of a spline of order d is a polynomial of degree d
- d overlapping splines can reproduce polynomial of maximum degree d in a 'knot-free' region



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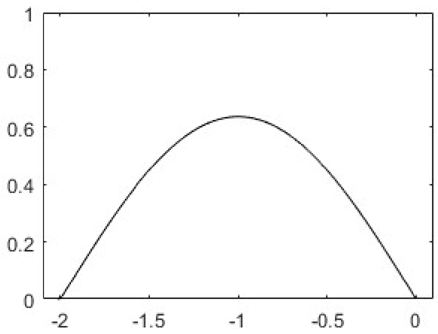
Sampling Kernels (E-splines)

- Exponential Splines (E-splines) can reproduce exponentials:

$$\sum_{n \epsilon \mathbb{Z}} c_{m,n} \varphi(t-n) = e^{-\alpha_m t}$$

- The first-order E-spline of support L is defined as:

$$\varphi(t) = \begin{cases} \dfrac{e^{\alpha_1-\alpha_0}}{\alpha_1-\alpha_0} \boldsymbol{e^{-\alpha_0 t}} + \dfrac{e^{-\alpha_1+\alpha_0}}{\alpha_0-\alpha_1} \boldsymbol{e^{-\alpha_1 t}}, & -L \leq t \leq \dfrac{-L}{2} \\ \dfrac{1}{\alpha_0-\alpha_1} \boldsymbol{e^{-\alpha_0 t}} + \dfrac{1}{\alpha_1-\alpha_0} \boldsymbol{e^{-\alpha_1 t}}, & \dfrac{-L}{2} \leq t \leq 0 \\ 0, & otherwise \end{cases}$$



- This function can reproduce exponentials $\boldsymbol{e^{-\alpha_0 t}}$ and $\boldsymbol{e^{-\alpha_1 t}}$.

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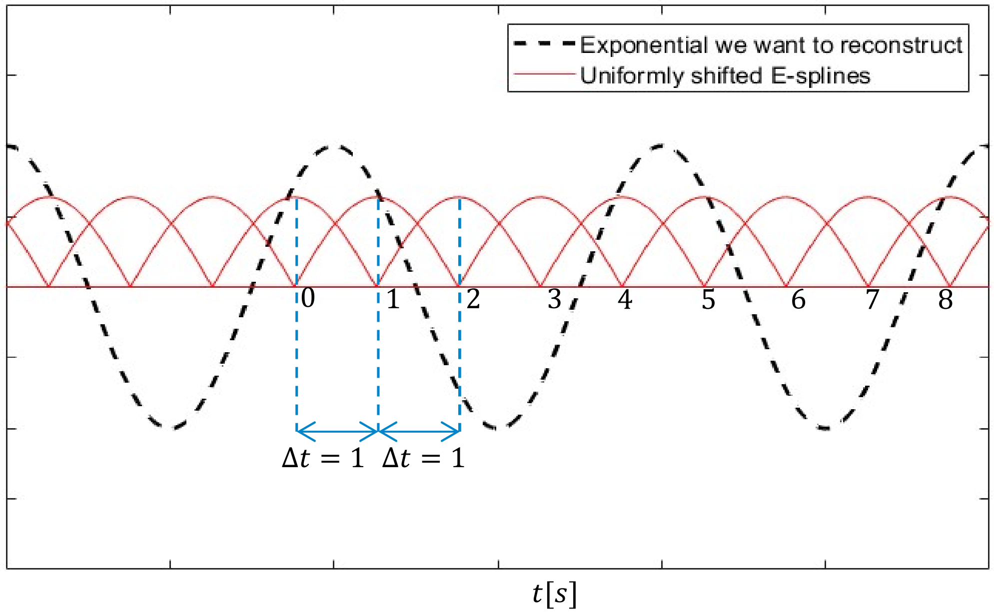
Sampling Kernels

- Reproduction of exponentials using uniform shifts of the first-order E-spline:

$$\sum_{n\in\mathbb{Z}} c_{m,n}\varphi(t-n) = e^{-\alpha_m t}$$



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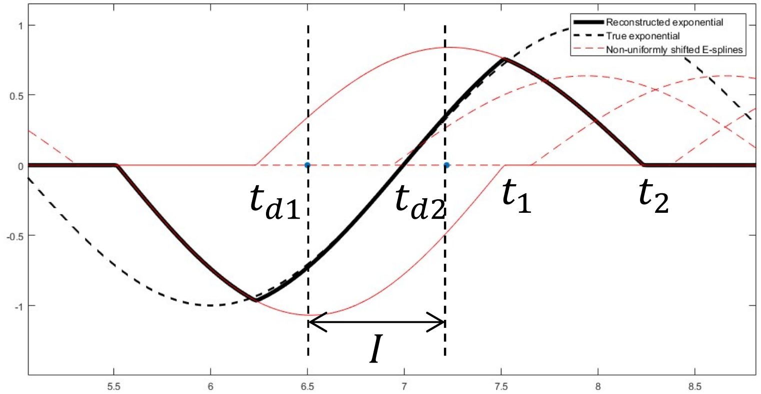
Sampling Kernels

- Reproduction of exponentials can be achieved locally in I, using at least two non-uniform shifts of the E-spline:

$$\sum_{n=1}^{N} c_{m,n}\varphi(t-t_n) = e^{-\alpha_m t}, N \geq 2$$

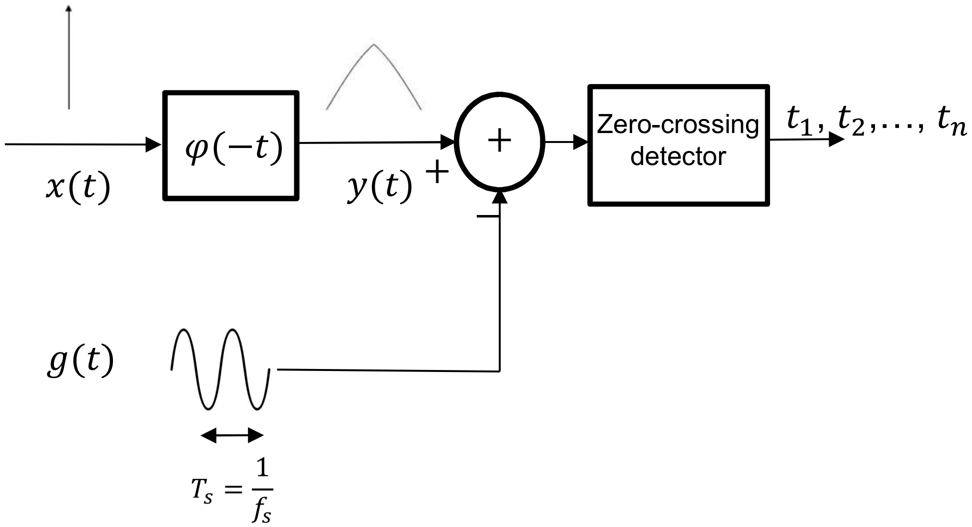
- The kernels should be continuous within that local interval I.

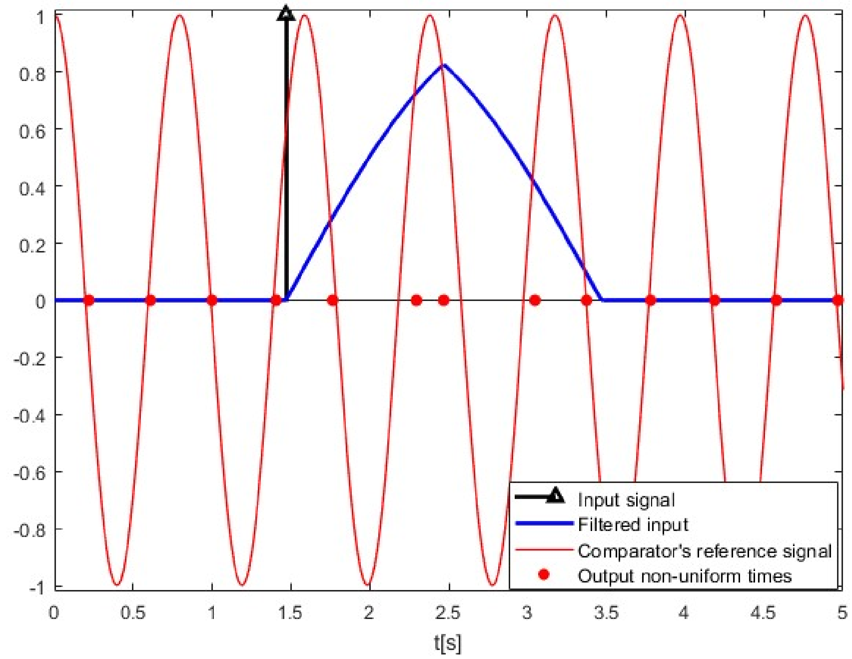


t_{d1} - discontinuity of $\varphi(t-t_1)$

t_{d2} - discontinuity of $\varphi(t-t_2)$

Comparator System – One Input Spike





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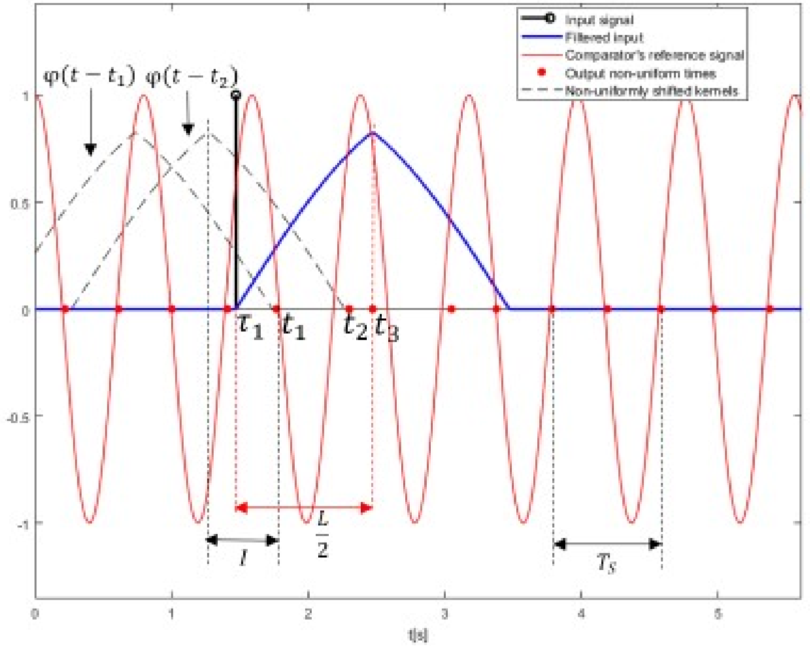
Comparator System – One Input Spike



$$y(t_1) = \langle x(t), \varphi(t - t_1) \rangle$$

$$y(t_2) = \langle x(t), \varphi(t - t_2) \rangle$$

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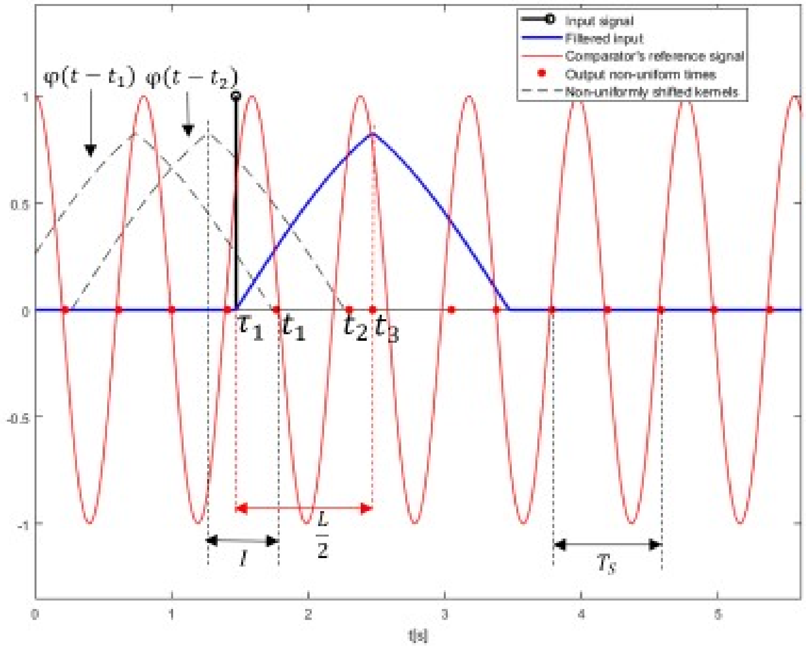
Comparator System – One Input Spike



We assume:

- Amplitude of the Dirac $|x_1| < 1$
- The sampling kernel $\varphi(t)$ and its non-uniform shifts reproduce $e^{j\omega_0 t}$and $e^{-j\omega_0 t}$ and $0 < \omega_0 \leq \frac{\pi}{L}$ where L is the support of $\varphi(t)$.
- The frequency of the sinusoidal signal satisfies $f_s > \frac{5}{2L}$.

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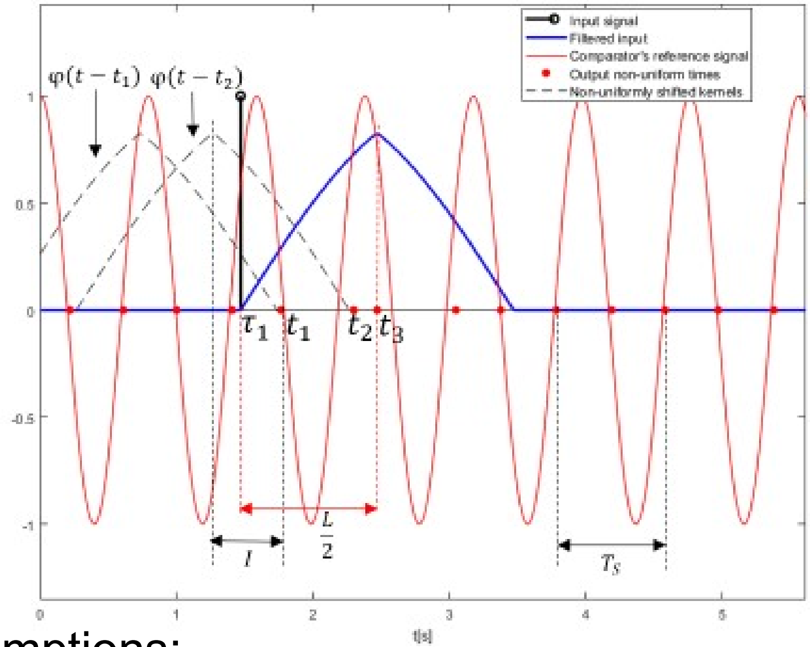
Comparator System – One Input Spike



Under these assumptions:

- The first two timing locations satisfy $t_1, t_2 \in [\tau_1, \tau_1 + \frac{L}{2}]$. This means that $\tau_1 \in \left[t_2 - \frac{L}{2}, t_1\right]$
- This is useful since in the interval $I = \left[t_2 - \frac{L}{2}, t_1\right]$, the shifted kernels $\varphi(t - t_1)$ and $\varphi(t - t_2)$ have no knots (so can reproduce exponentials or polynomials)

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Comparator System – One Input Spike

- We know we can find coefficients $c_{m,n}^{I}$ such that:

$\sum_{n=1}^{2} c_{m,n}^{I}\varphi(t-t_n) = e^{j\omega_m t}$, for $t \in I, m = 0,1.$

- We then have:

$$s_0 = \sum_{n=1}^{2} c_{0,n}^{I} y(t_n) = \sum_{n=1}^{2} c_{0,n}^{I}\langle x(t), \varphi(t-t_n)\rangle = \int_{-\infty}^{\infty} x(t) \sum_{n=1}^{2} c_{0,n}^{I}\varphi(t-t_n) = x_1 e^{j\omega_0\tau_1},$$

$$s_1 = \sum_{n=1}^{2} c_{1,n}^{I} y(t_n) = \sum_{n=1}^{2} c_{1,n}^{I}\langle x(t), \varphi(t-t_n)\rangle = \int_{-\infty}^{\infty} x(t) \sum_{n=1}^{2} c_{1,n}^{I}\varphi(t-t_n) = x_1 e^{j\omega_1\tau_1},$$

- Then $\tau_1 = \frac{1}{j(\omega_1-\omega_0)} \ln\frac{s_1}{s_2}$ and $x_1 = \mathrm{s}_0/\mathrm{e}^{j\omega_0\tau_1}$

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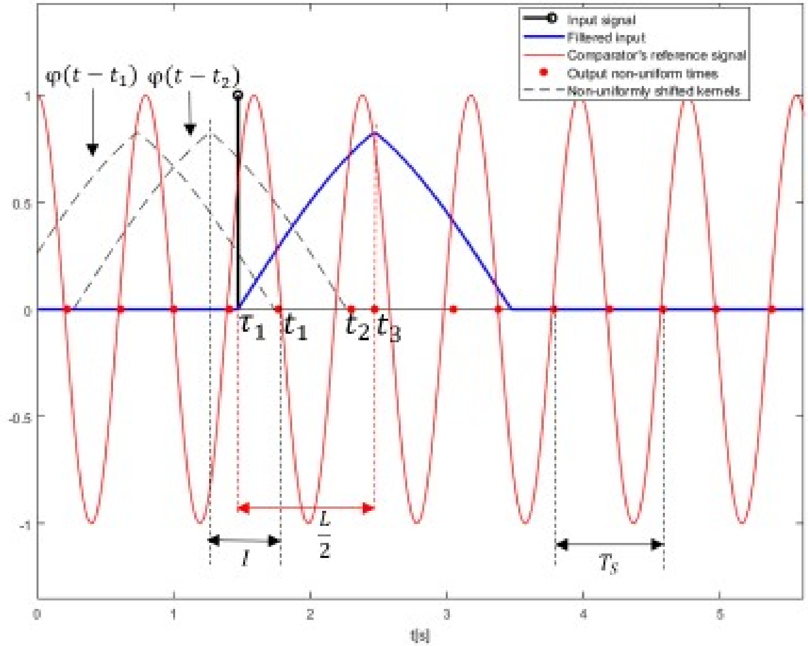
Comparator System – One Input Spike



- We use Bolzano's intermediate value theorem to show that $t_1, t_2 \in [\tau_1, \tau_1 + \frac{L}{2}]$
- Denote with $h(t) = g(t) - y(t)$, assume $g(\tau_1) > 0$ and $x_1 > 0$ then $h(\tau_1) > 0$ and $h\left(\tau_1 + \frac{T_s}{2}\right) = g\left(\tau_1 + \frac{T_s}{2}\right) - y\left(\tau_1 + \frac{T_s}{2}\right) < 0$, this implies $h(t_1) = 0$ for some $t_1 \in [\tau_1, \tau_1 + \frac{T}{2}]$
- Similarly $t_2 \in [\tau_1 + \frac{T_s}{2}, \tau_1 + \frac{5T_s}{4}]$
- Since $T_s = \frac{1}{f_s} < \frac{2L}{5}$ then $t_1, t_2 \in [\tau_1, \tau_1 + \frac{L}{2}]$

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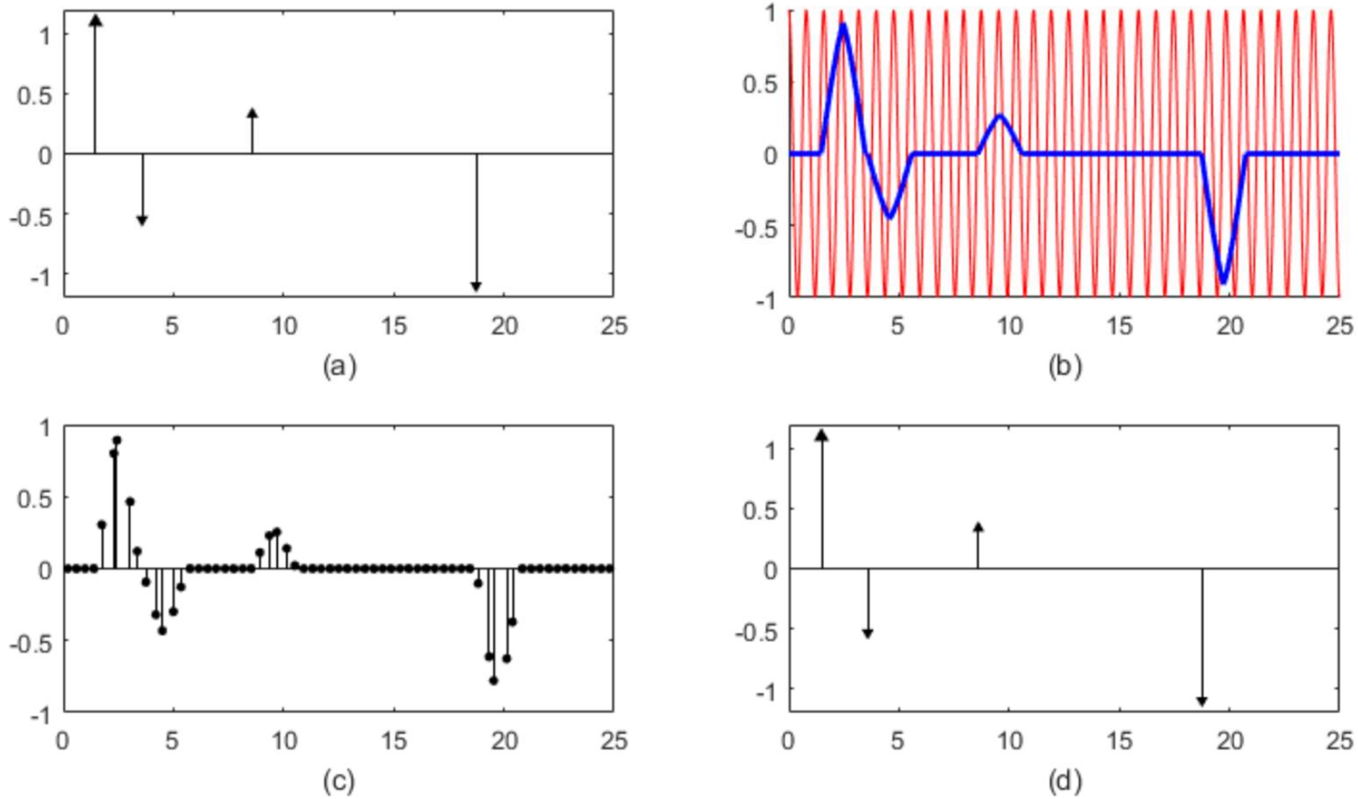
Comparator System – Example



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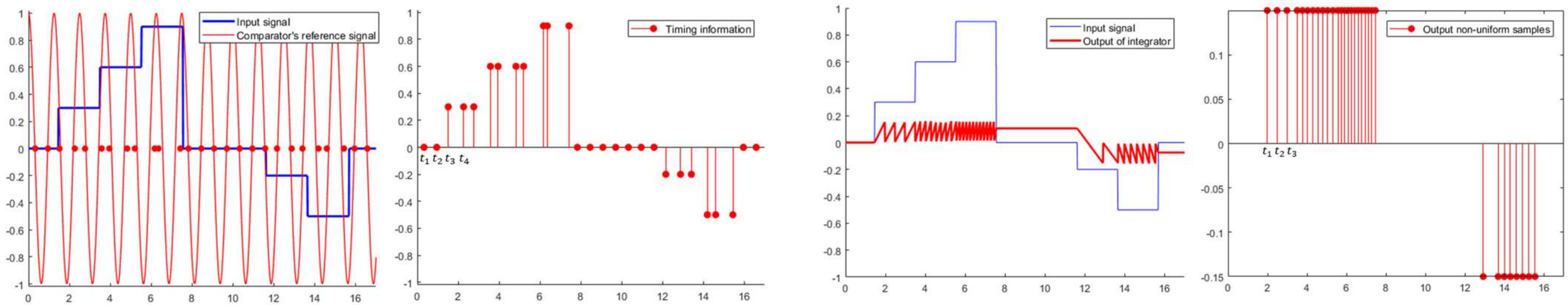
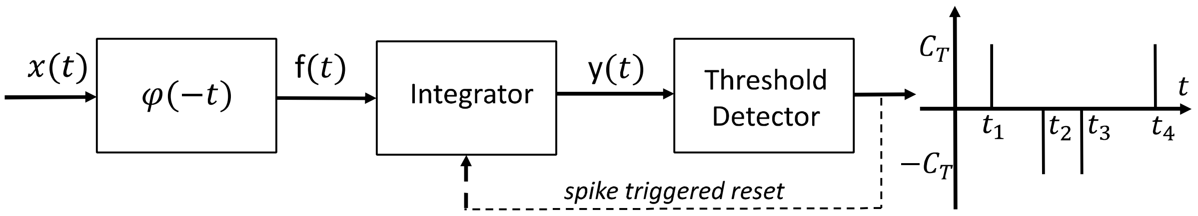
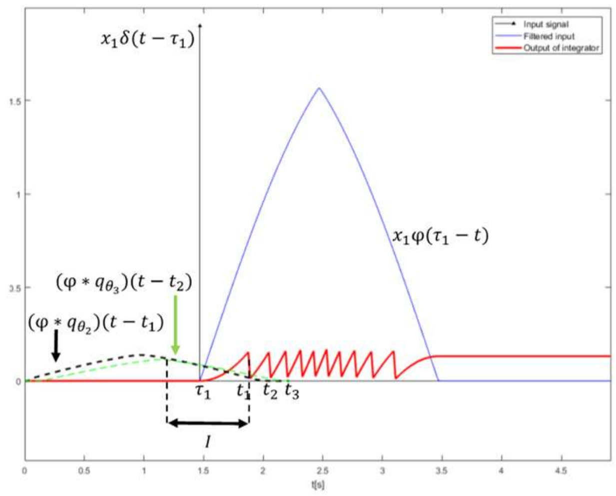
Summary on Sparse Sampling with Comparator

- We can sample and perfectly reconstruct non-bandlimited signals
- Number of time samples still large (time information provided also when signal is zero)
- Use the new framework but with the Integrate-and-Fire TEM



Integrate and Fire TEM





- The sampling kernel $\varphi(t)$ and its non-uniform shifts reproduce $e^{j\omega_0 t}$and $e^{-j\omega_0 t}$ and $0 < \omega_0 \le \frac{\pi}{L}$ where L is the support of $\varphi(t)$.
- What is the minimum value of the trigger mark C_T that would allow the perfect reconstruction of stream of pulses or piecewise constant signals?

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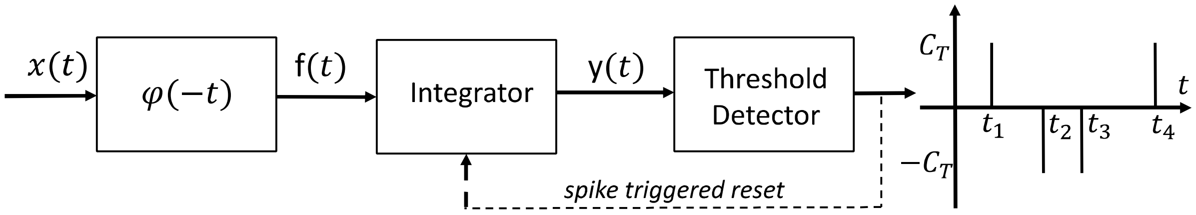
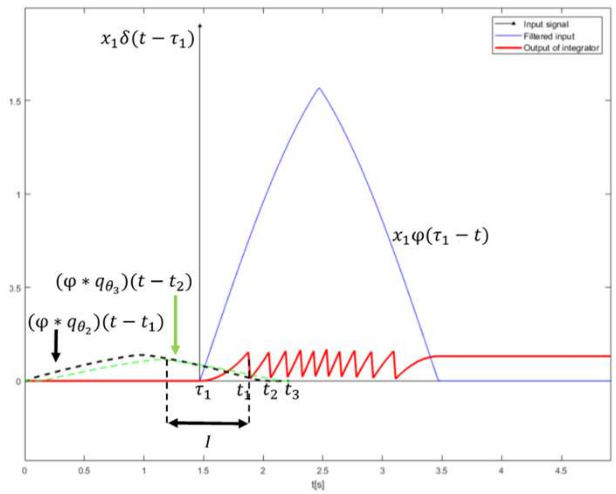
Integrate and Fire TEM





- Given the times $t_1, t_2, \ldots, t_n$, the amplitude values are

$$y_n = y(t_n) = \pm C_T = \int_{t_{n-1}}^{t_n} f(\tau)d\tau = \int_{t_{n-1}}^{t_n} \int x(\alpha)\phi(\alpha - t)d\alpha d\tau.$$

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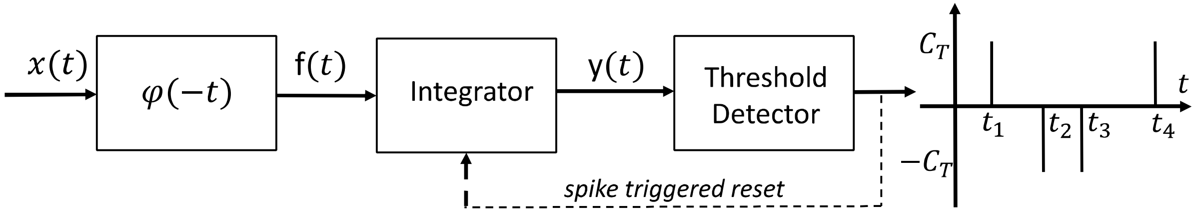
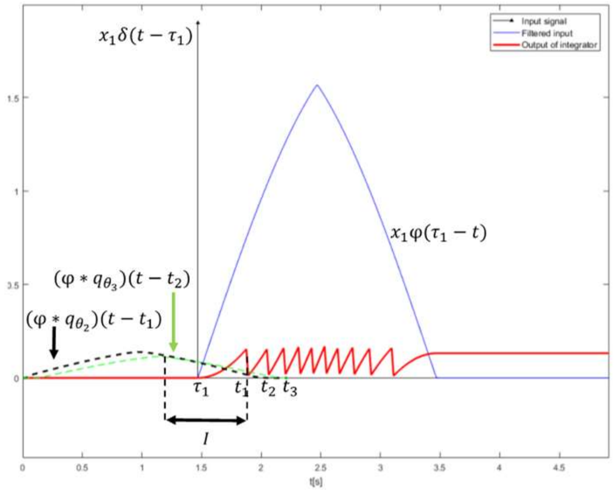
Integrate and Fire TEM





- Equivalently the output samples can be expressed as:

$$y(t_n) = \langle x(t), (\phi * q_{\theta_n})(t - t_{n-1})\rangle,$$

where $\theta_n = t_n - t_{n-1}$ and $q_{\theta_n}(t)$ is defined as:

$$q_{\theta_n}(t) = \begin{cases} 1, & 0 \leq t \leq \theta_n, \\ 0, & otherwise. \end{cases}$$

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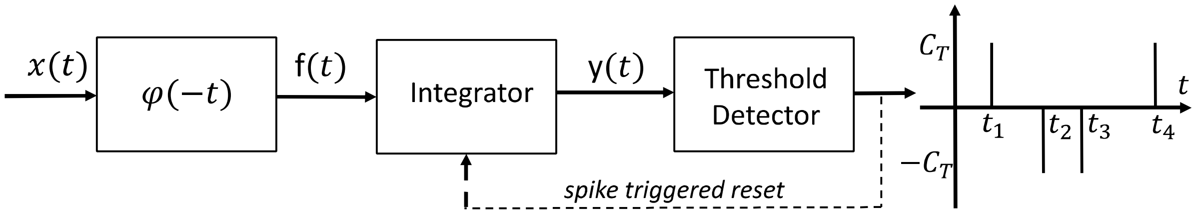
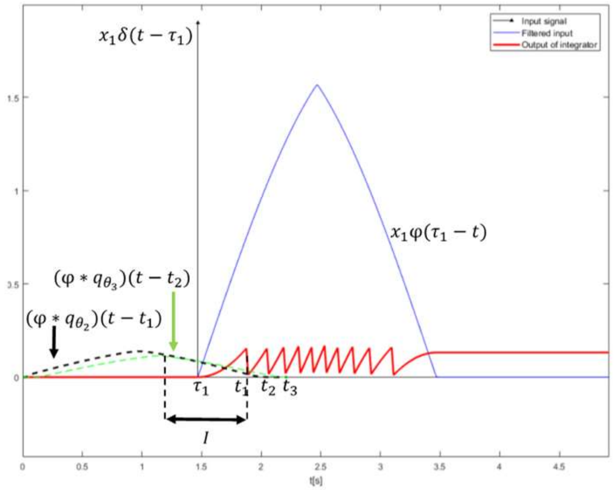
Integrate and Fire TEM

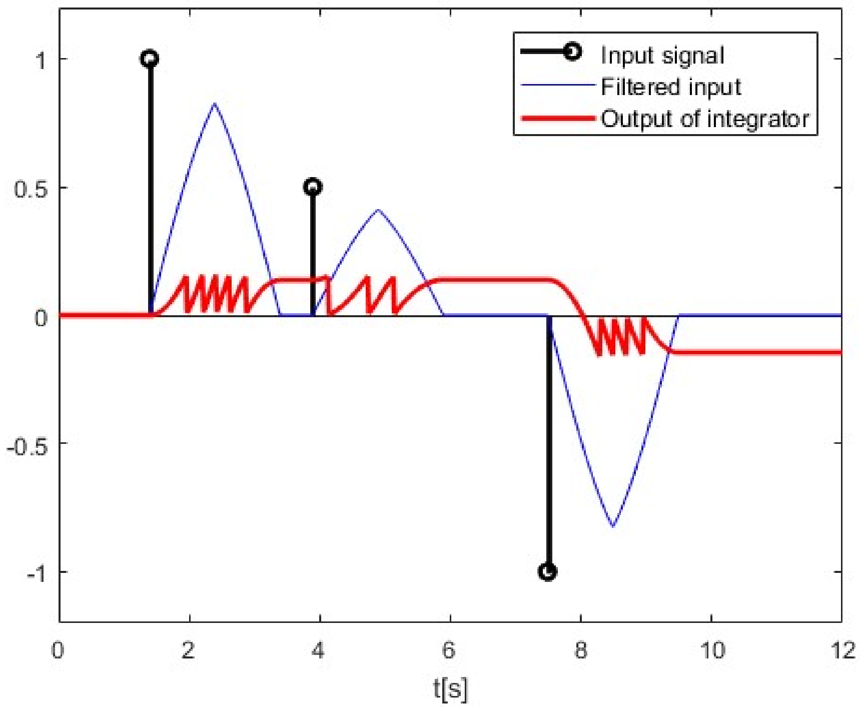


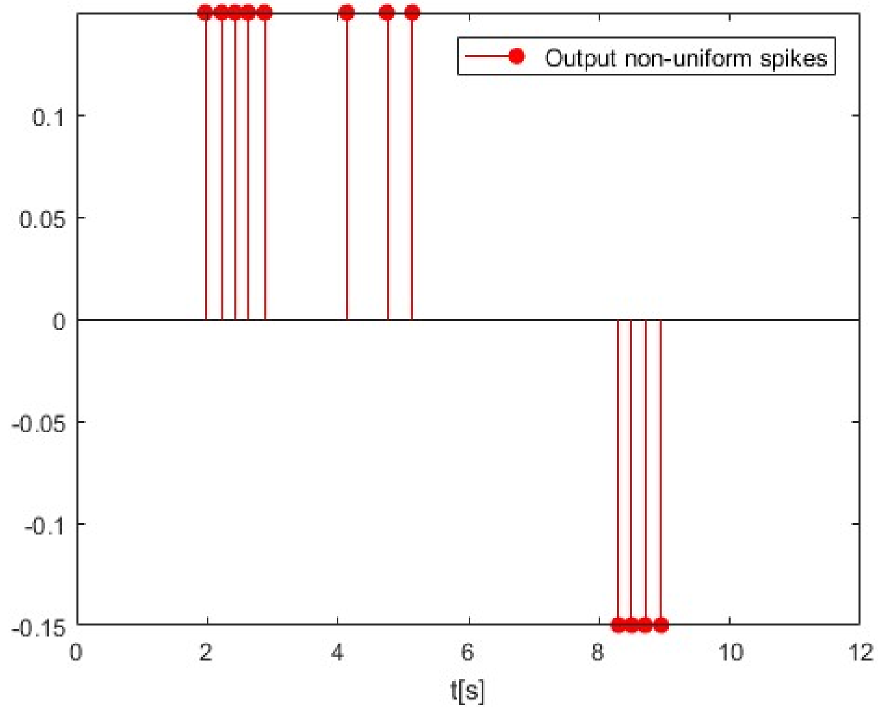


- The equivalent kernel $(\varphi * q_{\theta_n})(t - t_{n-1})$ is still able to reproduce exponentials
- So trigger mark must guarantee enough samples in a short interval

- *Proposition:* when $C_T < \frac{A_{min}}{4\omega_0^2}\left(1 - \cos\left(\frac{\omega_0 L}{2}\right)\right)$ then $t_1, t_2, t_3 \in \left[\tau_1, \tau_1 + \frac{L}{2}\right]$ and perfect reconstruction is possible

Integrate and Fire – Reconstruction of Pulses





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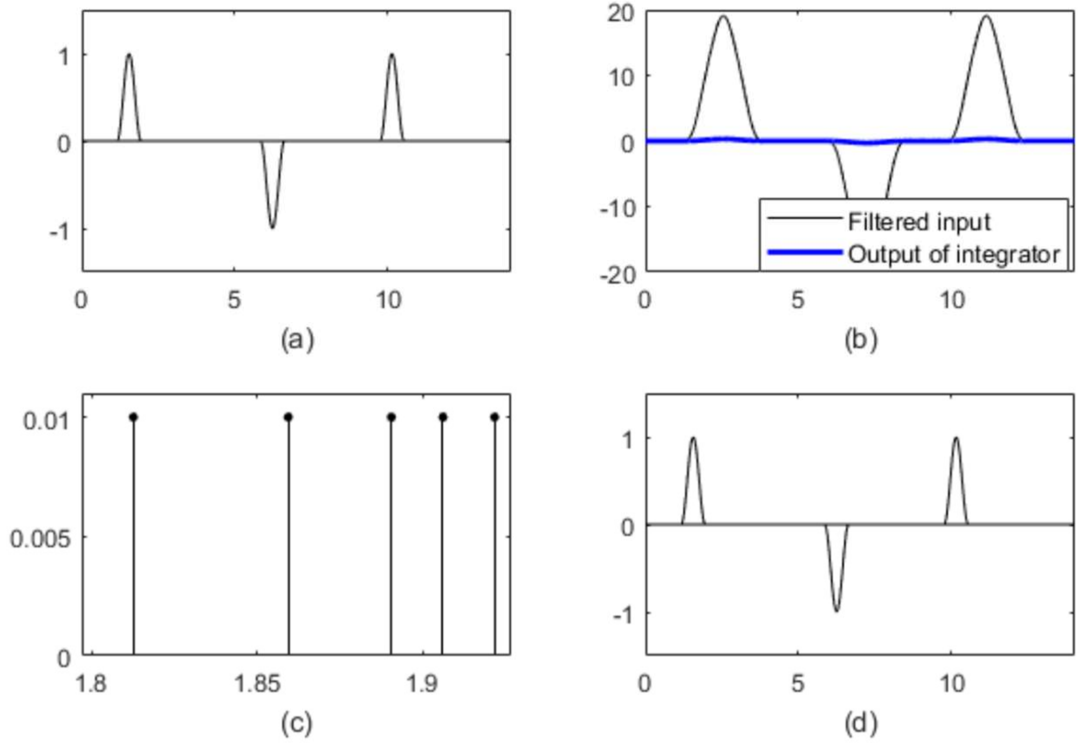
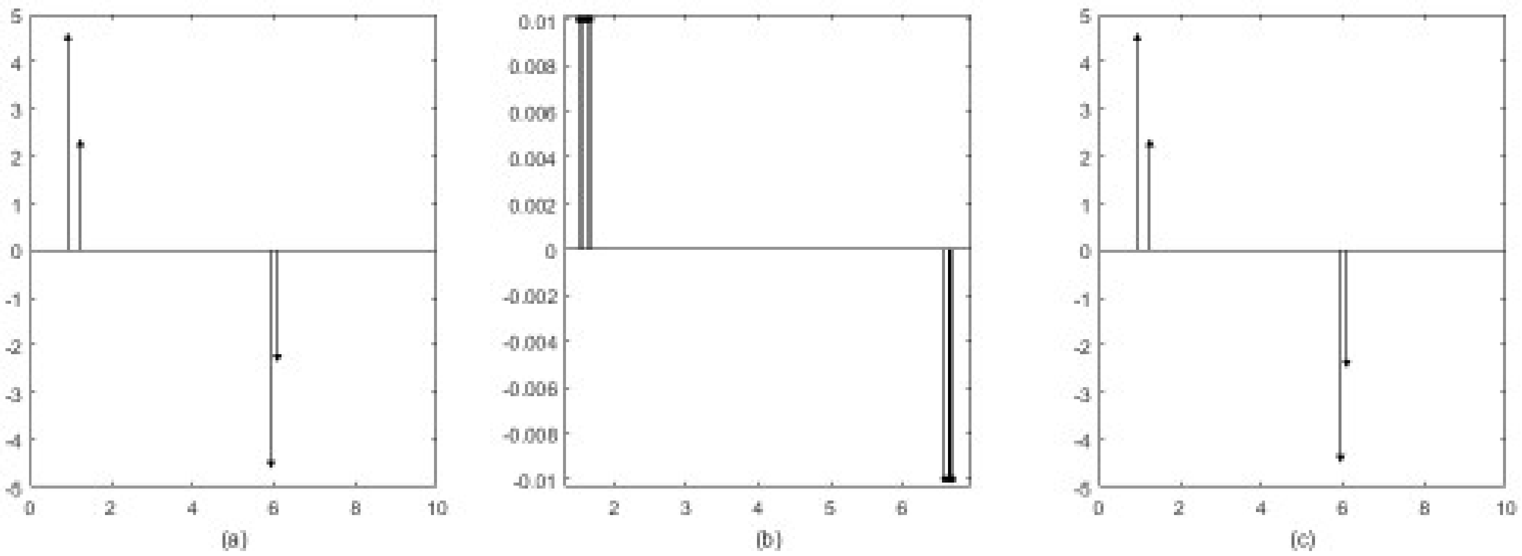
Integrate and Fire – Reconstruction of Pulses



Reconstruction with Arbitrary Kernels

- Sufficient conditions for perfect reconstruction may appear restrictive, but they can be relaxed with minimum loss in reconstruction quality
- The proposed reconstruction framework can be used with *any* acquisition device
- If reproduction of exponentials is not satisfied use LS methods to find the coefficients $c_{m,n}^{I}$ that achieve best fit:

$$\langle f(t), \tilde{\varphi}(t-t_n)\rangle = \sum_{k=1}^{N} c_{m,k}^{I} \langle \tilde{\varphi}(t-t_k), \tilde{\varphi}(t-t_n)\rangle, \quad \text{so that} \quad \sum_{n=1}^{N} c_{m,n}^{I} \tilde{\varphi}(t-t_n) \approx e^{j\omega_m t},$$



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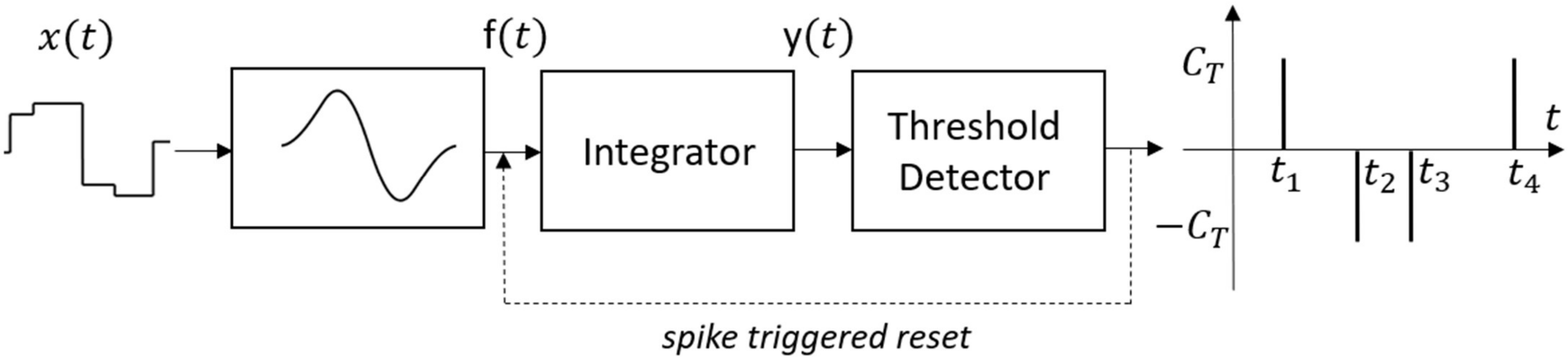
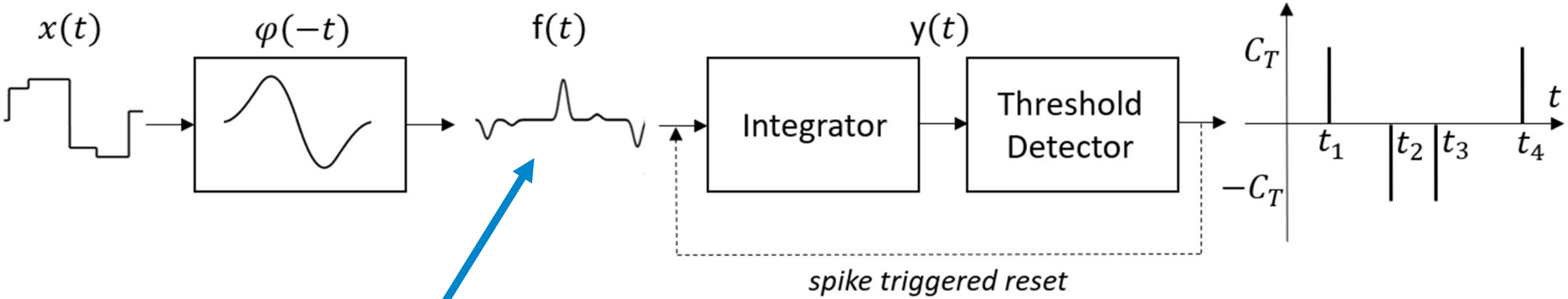
Integrate and Fire – Piecewise Constant Signals



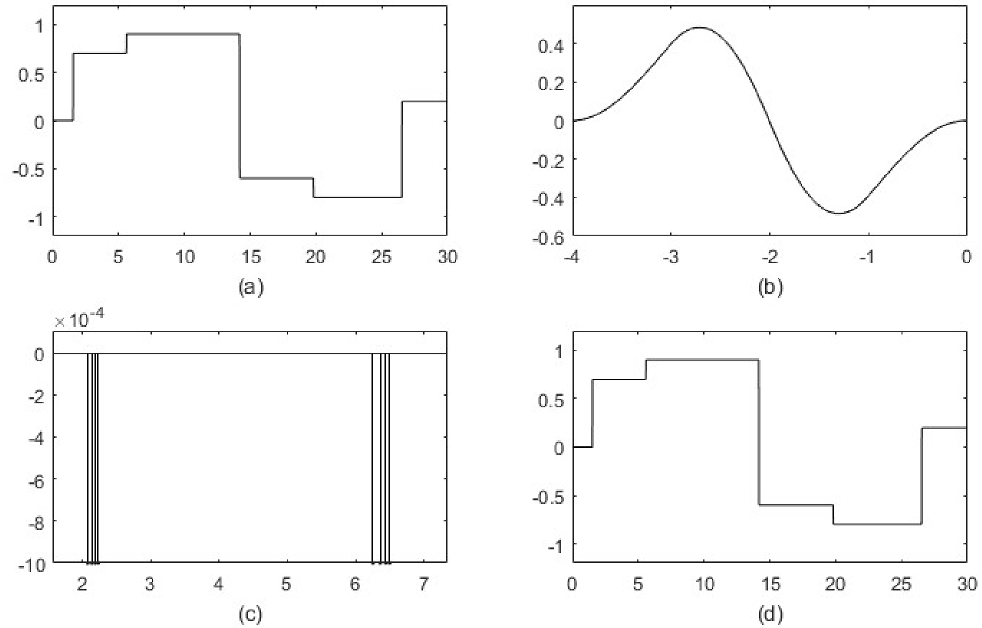
This is equivalent to the way a pixel operates
in neuromorphic video cameras

Integrate and Fire – Piecewise Constant Signals



Filtered Stream of Diracs

Integrate and Fire – Piecewise Constant Signals



If the distance S between discontinuities is on average $S > (L-1)T$ with T being the sampling period in uniform sparse sampling[4] then our time encoding framework is **more efficient** than uniform sampling (lower sampling density)

[4]P.L. Dragotti, M. Vetterli and T. Blu, Sampling Moments and Reconstructing Signals of Finite Rate of Innovation: Shannon meets Strang-Fix, IEEE Trans. on Signal Processing, vol.55 (5), pp. 1741-1757, May 2007.

Spike-Based Processing

- Sensing efficiently is only half of the story

- Once a signal has been converted into spikes, how do we process it efficiently?
  - Creating an AI can be five times worse for the planet than a car (resource NewScientist)
  - How do we compute fundamental transforms (e.g., Fourier or Wavelet Transforms)
  - Can we find the sparse representation of a signal using spiking neuron models? (Some results based on spike rates[4,5])
  - Deep learning with spiking signals?[6]

[4]P.T.P. Tang, T.-H. Lin, and M. Davies, “Sparse coding by spiking neural networks: Convergence theory and computational results” arXiv:1705.05475 , 2017.
[5]C. Pehlevan, “A Spiking Neural Network with Local Learning Rules Derived From Nonnegative Similarity Matching”, ICASSP 2019.
[6]E. Neftci, “Surrogate Gradient Learning in Spiking Neural Networks,”, arXiv:1901.09948, 2019.

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Conclusions

- Event-based sensing and processing is an emerging and exciting research area!
- Topic at the intersection of signal processing, computational neuroscience and machine learning

- Proved sufficient conditions for the exact reconstruction of classes of sparse signals from time-based information

- Many open questions on both the sensing and the processing front
  - Multi-dimensional case
  - Adaptive acquisition
  - L_1 optimization strategies
  - Learning sparsifying representations for spiking signals

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References

[1] Roxana Alexandru and Pier Luigi Dragotti, "Reconstructing classes of non-bandlimited signals from time encoded information", Available online at arXiv:1905.03183.

[2] Roxana Alexandru and Pier Luigi Dragotti, "Time encoding and perfect recovery of non-bandlimited signals with an integrate-and-fire system.",, International Conference on Sampling Theory and Applications, Bordeaux, France.

[3] Roxana Alexandru and Pier Luigi Dragotti "Time-based sampling and reconstruction of non-bandlimited signals", IEEE International Conference on Acoustics, Speech and Signal Processing, Brighton, United Kingdom.

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Thank you.