On real-time physical systems

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Outline

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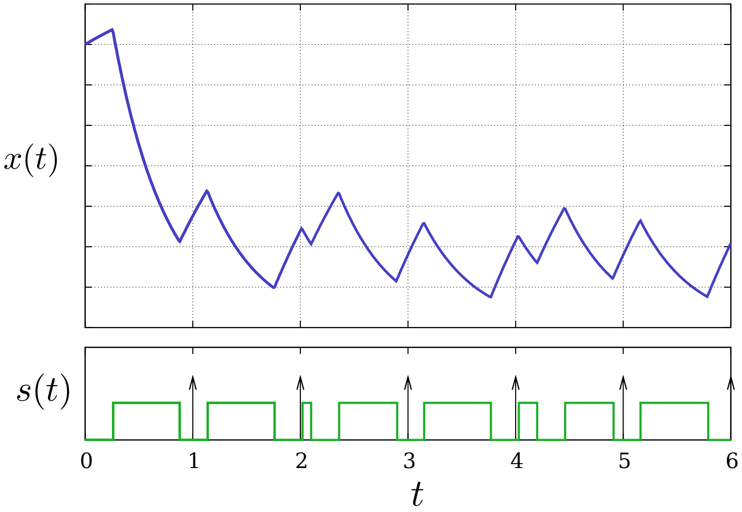
Basic definition

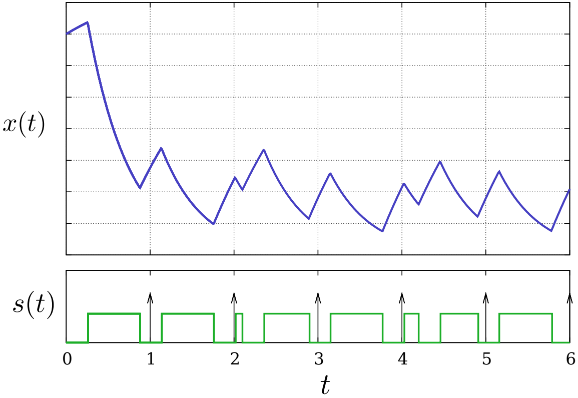
A Real-Time Physical System (RTPS) is defined by

- a set of real-time resources to be timely allocated for using a limited resource
- real-time resources are defined in terms of timing parameters and constraints
- a physical quantity is associated with each resource
- the physical quantity variation is bounded to the schedule of resources (activation/deactivation)

Example of real-time resource schedule

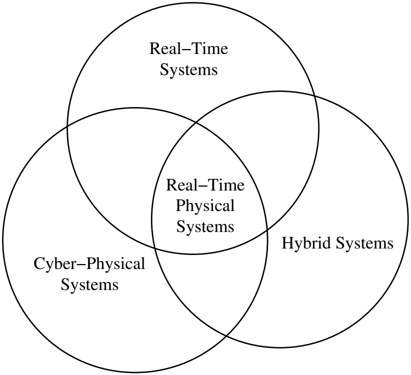


Example of real-time resource schedule



a real-time resource can be anything: a processing task, an electric device, a battery (charge/discharge), other physical processes

Collocation among research fields



a RTPS is:

1. built on top of Real-Time parameters, constraints, analysis techniques, and scheduling algorithms
2. a special class of (Switched) Hybrid system
3. a powerful modeling technique for Cyber-Physical (Energy) Systems

RTPSs and real-time systems

- in power-aware real-time systems the physical quantity of interest is consumed power/energy
- in temperature-aware real-time systems the physical quantity of interest is the temperature

> with respect to real-time systems, RTPS can be seen as generalization of power- and temperature-aware systems

Outline

A set of resources

- a set $\Lambda = \{\lambda_1, \cdots, \lambda_n\}$ of n *resources*
- turned on and off (or activated/deactivated)
- resource activity is controlled by a *resource scheduler* that decides when each resource is activated/deactivated

Definition of schedule

the scheduler assigns to each resource λ_i a schedule that is modeled by the function $s_i(t)$:

$$s_i(t) = \begin{cases} 1 & \lambda_i \text{ is active at } t \\ 0 & \text{otherwise} \end{cases}$$

the schedule of all resources is then given by $s(t) = \{s_1(t), \ldots, s_n(t)\}$

Dynamic system

- one physical system Γ_i is associated to each resource λ_i
- a state variable x_i (i.e., the physical quantity of interest) evolves as a function of the activity of resource λ_i
- a dynamic system Φ_i determines the behavior of the state variable, which is defined by the following equation:

$$\Phi_i : \frac{dx_i(t)}{dt} = k_i^{\text{off}} \left(h_i^{\text{off}} - x_i(t) \right) + k_i^{\text{on}} \left(h_i^{\text{on}} - x_i(t) \right) s_i(t)$$

Dynamic system: differential equation and time domain

$$\Phi_i : \frac{dx_i(t)}{dt} = k_i^{\text{off}} \left(h_i^{\text{off}} - x_i(t) \right) + k_i^{\text{on}} \left(h_i^{\text{on}} - x_i(t) \right) s_i(t)$$

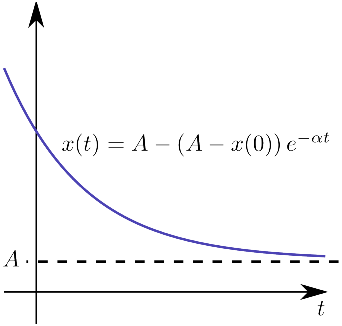
$$\begin{aligned} A =& (k^{\text{on}} h^{\text{on}} + k^{\text{off}} h^{\text{off}}) / (k^{\text{on}} + k^{\text{off}}) \\ \alpha =& k^{\text{on}} + k^{\text{off}} \\ B =& h^{\text{off}} \\ \beta =& k^{\text{off}} \end{aligned}$$

$$x(t) = \begin{cases} A - (A - x(0))\, e^{-\alpha t} & \text{if } s(t) \equiv 1 \\ B - (B - x(0))\, e^{-\beta t} & \text{if } s(t) \equiv 0 \end{cases}$$

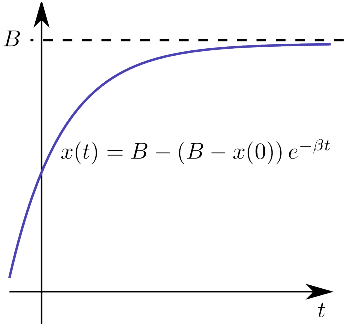
Dynamic system behavior

two exponential decay behaviours

$$x(t) = \begin{cases} A - (A - x(0))\, e^{-\alpha t} & \text{if } s(t) \equiv 1 \\ B - (B - x(0))\, e^{-\beta t} & \text{if } s(t) \equiv 0 \end{cases}$$



$s(t) \equiv 1$



$s(t) \equiv 0$

Constraints on state variables

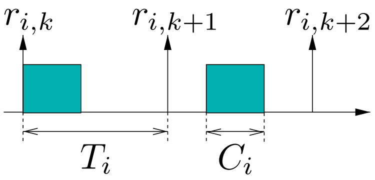
- each physical system is characterized by a set of *constraints* Ψ_i on the state variable
- constraints considered in this work:

$$\Psi_i : \begin{cases} x_i(t) \leq x_i^{\max} & \forall t > t_i^\star \\ x_i(t) \geq x_i^{\min} & \forall t > t_i^\star \end{cases}$$

the state variable x_i is required to be bounded in the range $[x_i^{\min}, x_i^{\max}]$ after a certain time instant $t_i^\star$

Real-time parameters

$$\lambda_i : \{T_i, C_i\}$$



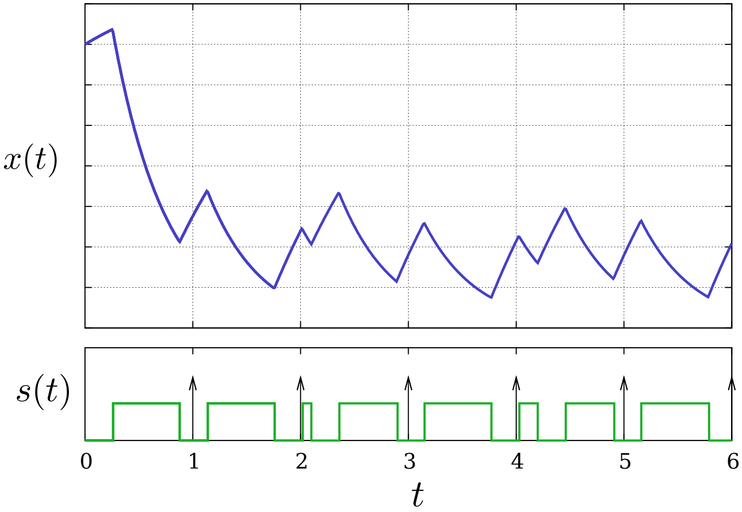
- T_i: time frame between two consecutive request times
- $C_i (\leq T_i)$: activation time
- deadline = period

utilization of λ_i $\quad U_i = \frac{C_i}{T_i}$

request time $\quad r_{i,k} = kT_i,\ k \in \mathbb{N}$

valid schedule $\mathcal{S}$ $\quad \forall \lambda_i, \forall k \quad \int_{r_{i,k}}^{r_{i,k+1}} s_i(t)\, dt = C_i$

Example of behavior of x_i



Feasible RTPS

we seek the relationship between

- physical system parameters $(A_i, \alpha_i, B_i, \beta_i)$
- real-time parameters (T_i and C_i)

in order to obtain a feasible RTPS:

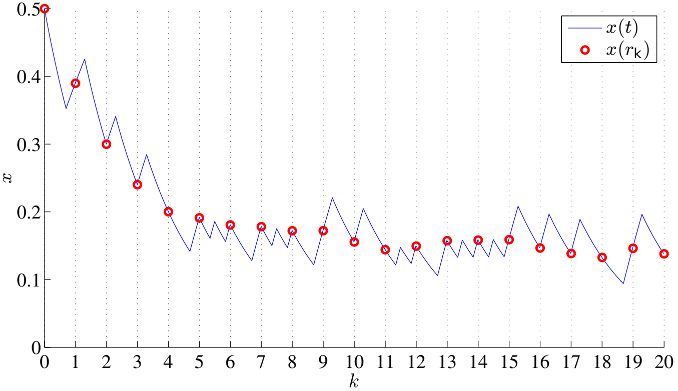
$$\begin{cases} \mathcal{S} \text{ is a valid schedule} \\ \Psi_i \text{ are satisfied} \end{cases}$$

Outline

Succession S_k

Definition

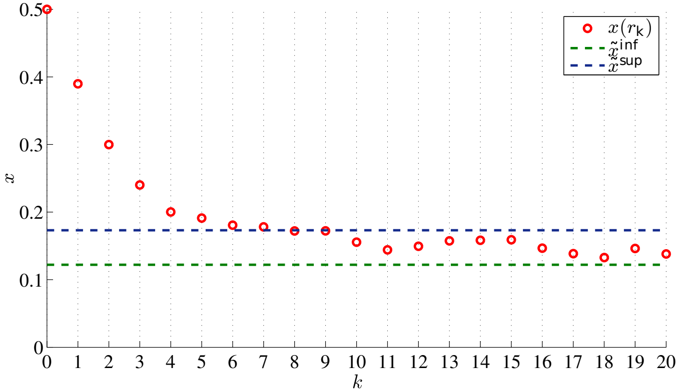
the succession of values of the state variable in correspondence of request times is $S_k = \{x(t)\}$ $(t = r_k,\ k = 0, 1, 2, \ldots)$.



Bounds for the succession S_k

Theorem

For any valid activation function $s(t)$, the succession S_k is bounded between $\tilde{x}^{\text{inf}}$ and $\tilde{x}^{\text{sup}}$ for $k \geq k^*$.



Bounds for the succession S_k

Equations

$$\tilde{x}^{\inf} = \frac{A + (B - A)\, e^{-\alpha U T} - B e^{-(\alpha U + \beta(1-U))T}}{1 - e^{-(\alpha U + \beta(1-U))T}}$$

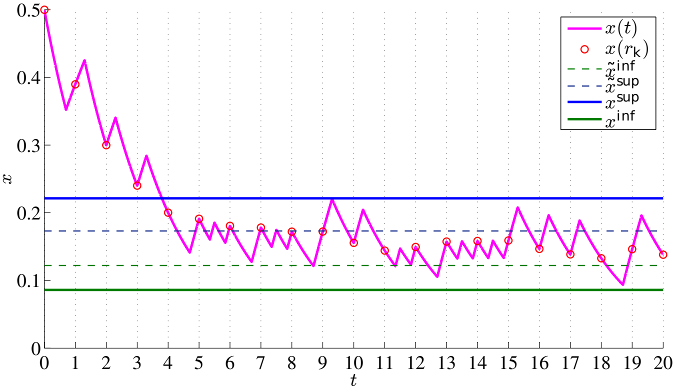
$$\tilde{x}^{\sup} = \frac{B + (A - B)\, e^{-\beta(1-U) T} - A e^{-(\alpha U + \beta(1-U))T}}{1 - e^{-(\alpha U + \beta(1-U))T}}$$

$\tilde{x}^{\inf}$ and $\tilde{x}^{\sup}$ are expressed as functions of physical parameters A, α, B, β and real-time parameters U, T

Bounds for the state variable dynamics

Theorem

For any valid activation function $s(t)$, the function $x(t)$ is bounded between x^{inf} and x^{sup} for $t \geq t^*$.



Bounds for the state variable dynamics

Equations

$$x^{\inf} = A - \left(A - \tilde{x}^{\inf}\right) e^{-\alpha U T}$$
$$x^{\sup} = B - \left(B - \tilde{x}^{\sup}\right) e^{-\beta (1-U) T}$$

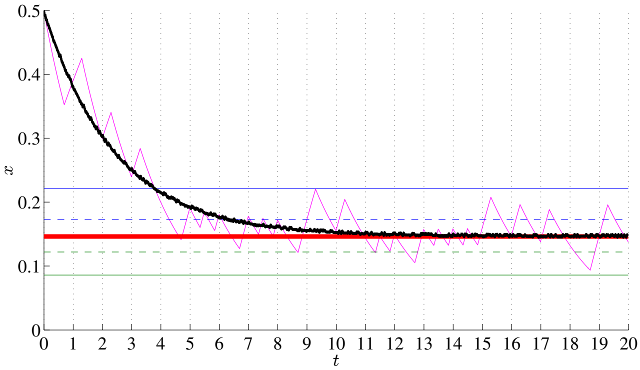
$x^{\inf}$ and $x^{\sup}$ are expressed as functions of physical parameters A, α, B, β and real-time parameters U, T

Asymptotic behavior for $T \to 0$

Fact

for any valid activation function $s(t)$, it holds

$$T \to 0 \Rightarrow x(t) \to \bar{x}$$



Asymptotic behavior for $T \to 0$

Equation

$$\bar{x} = \lim_{t \to \infty, T \to 0} x(t) = \frac{A\alpha U + B\beta(1-U)}{\alpha U + \beta(1-U)}$$

Equation of $\bar{x}$ is function of physical parameters A, α, B, β and real-time parameters U, T.

Feasibility region

Definition

the *feasibility region* Ω is a region in the $U - T$ plane composed by all and only pairs (U, T) such that:

- $s(t)$ is a valid schedule
- constraints Ψ are satisfied

in other words, in the feasibility region holds:

$$x^{\text{inf}} \geq x^{\text{min}} \quad \text{and} \quad x^{\text{sup}} \leq x^{\text{max}}$$

Feasibility region

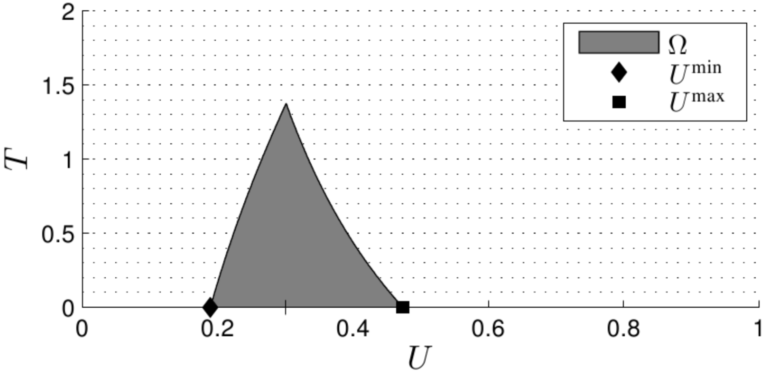
it is not possible to find values of U and T in closed form

since:

- $x^{\min} \leq x^{\inf}$
- $x^{\inf} = A - \left(A - \tilde{x}^{\inf}\right) e^{-\alpha U T}$
- $\tilde{x}^{\inf} = \frac{A+(B-A)e^{-\alpha U T} - Be^{-(\alpha U + \beta(1-U))T}}{1-e^{-(\alpha U+\beta(1-U))T}}$

pairs $(U, T) \in \Omega$ need to be found using numerical techniques

Example of feasible region Ω



Buonds on the utilization

range bounds $U^{\min}$ and $U^{\max}$ can be determined in closed form

by imposing $x^{\max} = \bar{x}$ and $x^{\min} = \bar{x}$:

$$U^{\max} = \frac{\beta(B - x^{\max})}{(\alpha - \beta)x^{\max} - (A\alpha - B\beta)}$$

$$U^{\min} = \frac{\beta(B - x^{\min})}{(\alpha - \beta)x^{\min} - (A\alpha - B\beta)}$$

Outline

Example of application

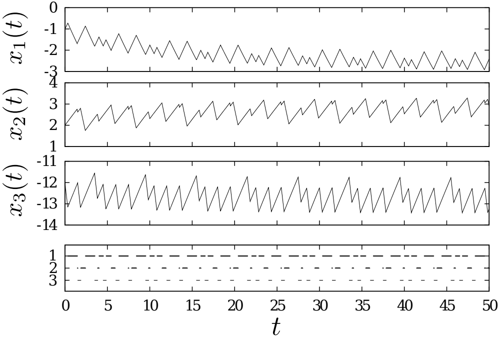
- the physical system associated with each resource is a fridge where the state variable x_i is the internal temperature
- the scheduler must keep temperature within bounds
- thermal phenomena have exponential decays:
  - A_i is the refrigerant temperature
  - B_i is the environmental temperature
  - α_i and β_i are related with physical properties such as thermal capacities and heat transfer coefficients

Example of application

i	A	α	B	β	$x^{\min}$	$x^{\max}$	$x(0)$	$U^{\min}$	$U^{\max}$	U	T
1	-10	0.10	20	0.04	-4	-1	-1	0.48	0.62	0.55	2.0
2	-10	0.15	20	0.03	1	5	2	0.17	0.26	0.21	3.0
3	-30	0.20	20	0.03	-15	-10	-12	0.18	0.26	0.22	1.5

$U^{\text{tot}} = 0.98 < 1$
EDF activates at most one resource for any t

in general, it is possible to strongly limit the simultaneous activation of multiple resources, even if $U^{\text{tot}} > 1$



Outline

Conclusions

- a new class of real-time systems has been introduced
- this class includes existing well-known models (power-aware, temperature-aware)
- a particular case of RTPS has been modeled and analyzed
- the focus has been put on the relationship between physical and real-time parameters

Future works

- accounting for event-driven (aperiodic) resource activations
- integration of feed-back techniques to cope with imprecision and inaccuracies in the model
- experimental validation required with practical implementation (promising on-going works)

Merci

pour votre attention

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