# Implicit Extrusion Fields


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**Abstract**

*This paper presents an implicit function fields based approach for the extrusion of 2-dimensional profiles. According to the fields used, the extrusion follows an implicit object shape or performs the transition between combined primitives. The 1-dimensional polynomial spline properties allow the point-by-point control of the profile. Used on the transition of combined primitives, free form functions allow us to introduce the notion of free form blending. The result is a powerful and precise tool to model and blend implicit surfaces.*



**Keywords:** Implicit modeling, free form blending, CSG trees.


## 1. Introduction

The wide variety of implicit models developed in different fundamental research topics (like 3D reconstruction, animation of highly deformable materials, morphing, smooth and round shape modeling) make implicit surfaces a useful and powerful tool. These applications are based on a property of implicit surfaces: automatic continuous blending. This blending notion is usually seen and computed as a smooth and regular curved or inflated transition.

After blobs[1], soft object[2] and metaballs[3], numerous models have proposed new implicit primitives. But improvement of blend control through the use of surface combination models[4,5,6] has been an important area of investigation. 3D approximation data[7,8] requires manipulation of many basic primitives joined by regular and automatic blending. Interactive modeling requires manipulation of a small number but a large variety of primitives and precise control at the transition level. Skeletons[9], convolution surfaces[10,11,12], algebraic surfaces[13,14,15] and sweep objects[16,17] are different implicit models which produce a large panel of shapes. Implicit models are usually presented in their basic form with the blend operator. This operator is equivalent to the Boolean union operator with soft transitions. To increase the panel of forms, models like R-functions[18] or 'The blob tree'[19] integrate CSG trees[20]. The blend operation is then available with intersection and difference.

An implicit function $F_i$ transforms the Euclidean 3D space into a potential field. A fixed value of potential C defines the surface $S_i$:

$$F_i : R^3 \to R$$
$$S_i = \{(x, y, z) \in R^3 / F_i(x, y, z) = C\}.$$

To handle this field intuitively, a few graphic parameters control the definition of the implicit primitives. The composition of functions $F_1,\ldots,F_n$ in a specific equation g generates the blended function F:

$$P(x,y,z) \in R^3$$
$$F(P) = g \circ (F_1(P),...,F_n(P)).$$

Parameters allowing the control of blend smoothness are not directly linked to graphic data and transition creation is an iterative succession of 'adapting the value of the parameters and visualization'. The final object is obtained after an expensive task due to visualization process time and the difficulty to adjust the blend.

To get round this visualization problem, discrete representation of implicit fields is used. Combined with local operators[21], it allows us to add or remove matter in a small region of space and then to interactively model the implicit object.

Our goal is to increase the interactivity, to allow precise shape control and to extend processing of regular smooth shapes to that of free form shapes at the transition level of combined implicit primitives in CSG trees.

This paper presents a generic vision of field manipulation. Two implicit primitives generate a 2D extrusion field. Objects and transitions are the extrusion result of a 2D profile in this field. The properties of the extrusion field and of the point-by-point defined 2D curve allow precise control of the shape outline in a 2D visualization interface. 3D visualization with an octree is used to validate the resulting object shape if necessary. General field extrusion definition and properties are developed in Section 2. Precise and easy profile creation in a 2D interface gives flexibility and interactivity to our model. Such profiles are presented in Section 3. Depending on the implicit functions instantiating the model, different extrusion fields with different properties are produced. Two examples are presented in Section 4 to illustrate the possibilities offered by this approach to control transitions in CSG trees and model primitives. The first is an adaptation of the model developed by Dekkers[22]. The second extrudes the profile around an implicit function.

## 2. Implicit extrusion fields

Implicit functions F split the space into two half spaces. One where F(P)>0 and one where F(P)<0. If F defines a closed object, the convention of inside/outside is chosen as follows:

- If F(x,y,z) > 0, the point P (x,y,z) is outside the volume defined by the surface.
- If F(x,y,z) < 0, the point P (x,y,z) is inside the volume defined by the surface.

Surfaces are 0 iso-surfaces:

$$S = \{(x,y,z) \in R^3 / F(x,y,z) = 0\}.$$

In common models of implicit primitive combination with soft blending, primitive equations $f_i$ are composed with blending functions $g_i$. The final object is the result of the $g_i(f_i)$ summation. These approaches are efficient but it remains difficult to control the blending shape accurately. We propose to develop the model presented in Equation 1 (based on a very simple formulation) which allows the points of 3D Euclidean space to be linked to the implicit surface shape.

$$F(x,y,z) = f_2(x,y,z) - g(f_1(x,y,z))$$

**Equation 1:** *$f_1$ and $f_2$ are two 0 iso-surface implicit functions and g is the matter adding function.*

An interesting way to understand how this link is performed is to compare this equation to that of a function in a 2D space. It can easily be written under its implicit form (Equation 2). The comparison allows us to see function $f_1$ as the X axis and function $f_2$ as the Y axis. Indeed, $f_1$ and $f_2$ are functions of $R^3 \rightarrow R$, g remains a function of $R \rightarrow R$. The new space defined by $f_1$ and $f_2$ is called the implicit extrusion field.

$$Y = g(X) \Leftrightarrow Y - g(X) = 0$$
$$F = Y - g(X)$$

**Equation 2:** *F is written as a function of $R \rightarrow R$. The 3D surface is the result of the extrusion of g in the implicit field generated by X and Y (respectively $f_1$ and $f_2$).*

A fixed value of the abscissa $X=X_0$ is then the set of points $p'(x,y,z)$ of the Euclidean space such that $f_1(p') = X_0$. This set of points is the $X_0$ iso-surface of the implicit field defined by $f_1$. In the same way, the ordinate is the $Y_0$ iso-surface of the implicit field defined by $f_2$. A point $(X_0,Y_0)$ of the implicit space is then the set of points $p'(x,y,z)$ such that $f_1(p') = X_0$ and $f_2(p') = Y_0$. This set of points is the resulting curve $P_0$ of the intersection of the $X_0$ iso-surface and the $Y_0$ iso-surface.

In choosing a point $p(x,y,z)$ in Euclidean space, a $P_0$ curve of the implicit extrusion field is defined with the coordinates: $P_0(X_0=f_1(p), Y_0=f_2(p))$. The curve is then considered to be the extrusion of point p in the implicit extrusion field. The curve follows the form of the intersection between the two implicit surfaces.

Function g is a continuous set of points $P'(X, Y=g(X))$. A surface is then generated in Euclidean space which is the result of the extrusion of curve g in implicit space.

Equation 1 allows the shape defined by $f_2$ to be partially or completely conserved in the final object. Indeed, the surface is defined when implicit function F equals 0 and it generates the following properties:

- When $g(f_1(x,y,z))=0$: $F(x,y,z)=f_2(x,y,z)$. The field defined by F is that defined by $f_2$. If $f_2=0$ then $F=0$: the points of the resulting surface are the ones of the primitive defined by $f_2$.

- At point $P_2(X_2,0)$, $X_2$ is the smallest abscissa where $g(X)=0$ (see Figure 1 and 2), a null value of the tangency ($g'(X)=0$) ensures $C^1$ continuity between the primitive defined by $f_2$ and the blend defined by g.

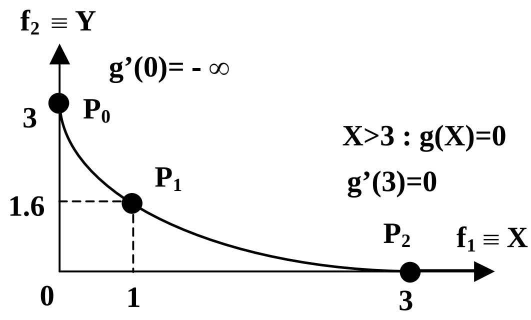


**Figure 2:** *Example of a g profile function.*

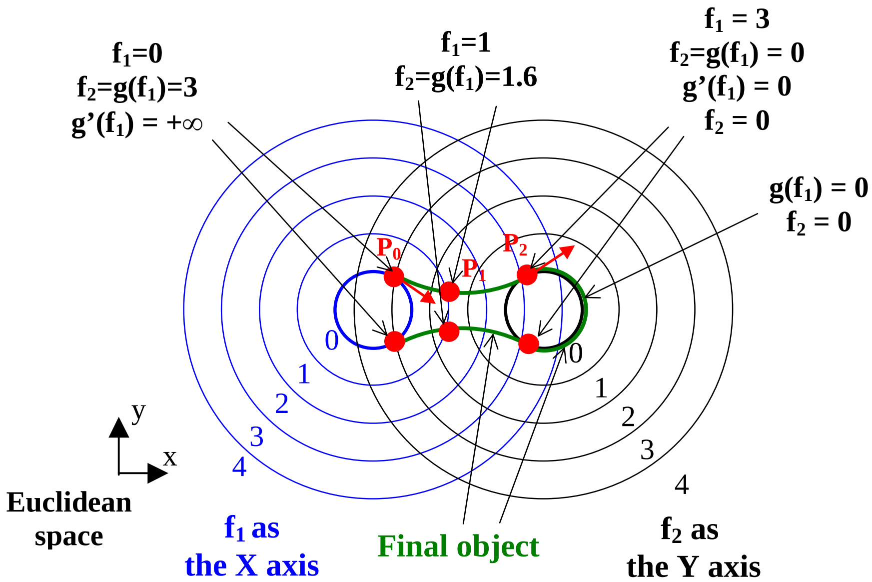


**Figure 1:** *Illustration of the g profile shown in Figure 2 extruded in a field instantiated with two spheres.*

One solution to interactively create this surface with a graphic interface, is to visualize this 2D section of the implicit field and to select control points $p_i$ using the mouse. Then the point coordinates are computed from the 2D selection space to the 3D Euclidean working space ($p_i(x_i,y_i,z_i)$). Their coordinates are computed in the implicit extrusion field ($P_i(X_i,Y_i)$ with $Xi = f_1(x_i,y_i,z_i)$, $Y_i = f_2(x_i,y_i,z_i)$). The g function is defined by interpolation of the control points $P_i$. A 2D interactive visualization of the final implicit object outline is proposed (using interval arithmetic). The user can act on the shape outline by moving, adding or removing control points or by acting on other control parameters (tangent, curvature, etc), depending on the interpolation function used. The implicit field visualization and g interpolation functions are presented in the following section.

Used as shown in Figure 1 (sphere instantiation), the model is not efficient enough. Indeed, the only primitive which can be a part of the final object is the one defined by $f_2$. Improved instantiations of abscissa and ordinate implicit functions are presented in Section 4.

# 3. 2D surface outline manipulation

## 3.1. 2D implicit field visualization

The 2D space visualized is a plane section of the 3D working space. It is important to choose a plane which intersects the implicit fields correctly. The plane is set where the outline of the final shape is to be controlled. A bad choice of the plane will considerably decrease the intuitive link between the 2D outline and the 3D shape. This condition obliges the user to have a working knowledge of implicit functions and the fields generated by implicit primitives.

The g functions are called “master profiles”. These master profiles are functions of R→R. To respect the function properties, the X axis must be visualized. The $f_1$ implicit field is visualized as a background picture using gray graduations (black when $f_1 = 0$ and white when $|f_1|$ is max). To complete the field reference, outlines of $f_1$ and $f_2$ 0 iso-surfaces can be visualized. We will see that the $f_1$ field background picture can be sufficient (Figure 3).

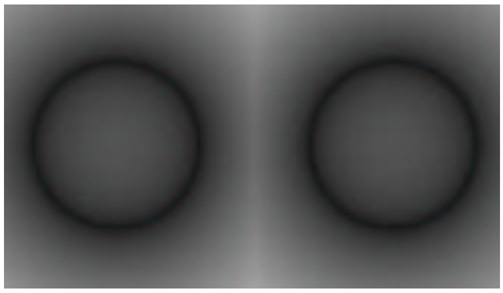

**Figure 3:** *The X axis is instantiated with the minimum of two spheres.*

## 3.2. 1D cubic polynomial splines

Control points have been presented as points of the master profile. They can also be control points like the ones used in Bézier curves for example.

The interpolation function proposed has been chosen for its good smoothness and oscillation properties. If we consider a single interval $X_i \leq X \leq X_{i+1}$ with both Y and g’(x) specified at each end, we can construct a cubic interpolating polynomial over that interval. If this is done over every interval, the resulting piecewise cubic function has continuity of slope at all data points and interpolates data smoothly. Low degree polynomials reduce problems of oscillation. These functions are called 1D polynomial splines[23] and were used to define our master profiles.

The first derivative can be computed interactively as well. This is done by choosing the beginning point (one of the control points of the 2D space) and selecting another point (with the mouse) giving the tangent direction (Figure 4).

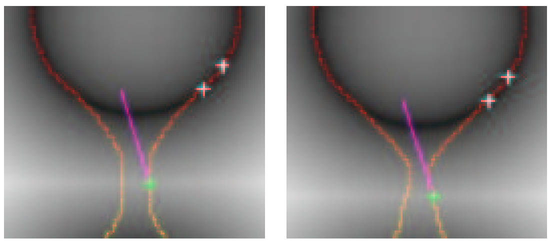

**Figure 4:** *The line moves with the mouse pointer until the user clicks to select the desired tangent.*

It is then computed in the implicit space. The first derivative value can also be directly given (to allow the user to input an exact value).

# 4. Examples of extrusion field instantiations

## 4.1. Union Boolean operator

D. Dekkers, K. van Overveld, and R. Golsteijn[22] propose a method to combine CSG modeling with soft blending using implicit surfaces. Their model is optimised for Lipschitz-based implicit surfaces and allows soft transitions for primitives having homogeneous fields (a plane combined with a sphere generates $C^1$ discontinuities in the transition). The union Boolean operator is computed using the following equation:

$$O_1 \cup O_2 : F = \min(f_1, f_2) - f_b(|f_1 - f_2|, n),$$

where $O_1$ and $O_2$ are the primitive objects respectively defined by the $f_1$ and $f_2$ implicit functions. Function $f_b$ is the blending function and n its softness control parameter. The transposition of this equation to our model gives:

- The extrusion field is instantiated with $|f_1\text{-}f_2|$ as the abscissa and $\min(f_1,f_2)$ as the ordinate.
- The blending function $f_b$ is replaced by a master profile g.

We then obtain:

$$O_1 \cup O_2 : F = \min(f_1, f_2) - g(|f_1 - f_2|).$$

When $g(|f_1\text{-}f_2|) = 0$, $F = \min(f_1,f_2)$. Outside the master profile boundaries, F defines the classical union operator (without blending) and inside the boundaries, the object is defined by the extrusion of the master profile in the fields. The profile is joined to the surface in $f_i = 0$. A null first derivative at this point ensures $C^1$ continuity between the surface and the profile (performing the blend).

When $|f_1\text{-}f_2|=0$, the min function creates a discontinuity in the ordinate field ($f_1$ is selected on one side of the frontier and $f_2$ on the other). For each side, a master profile has to be defined. The junction points are in $|f_1\text{-}f_2| = 0$ ($f_1 = f_2$). These points are located between the two blended objects. Functions $f_1$ and $f_2$ generally produce different fields. This is why we need to control the first derivatives on each side of the frontier to avoid $C^1$ discontinuities. Our computation method provides the same tangent on each side if desired. The first derivative value is then computed independently for each profile in its own field. Figure 5 illustrates the different sections composing the final object.

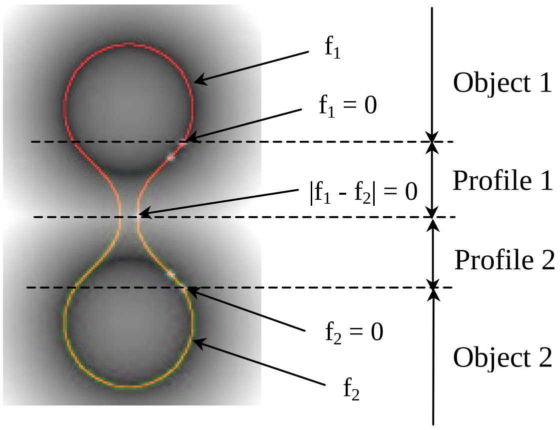


**Figure 5:** *Union of two spheres.*

All the classic continuous blending shapes are easily built by applying these constraints to the master profiles. But, the interactive manipulation of the control points and the first end derivatives allow free form blending (Figure 6).

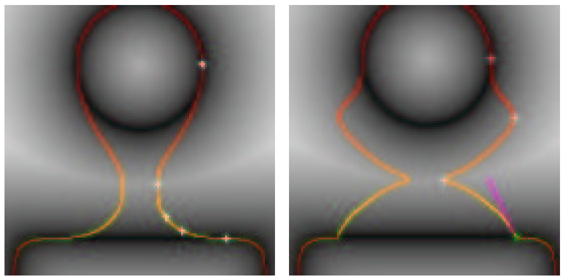

**Figure 6:** *Union of a parallelepiped and a sphere. On the left, a common blend and on the right, a free form blend.*

## 4.2. Intersection and difference operators

In the same way, master profiles allow the introduction of soft transitions into the intersection and difference Boolean operators.

From the Dekkers form, we obtain the following equations:

$$O_1 \cap O_2 : F = \max(f_1, f_2) - g(|f_1 - f_2|)$$
$$O_1 / O_2 : F = \max(f_1, -f_2) - g(|f_1 + f_2|)$$

All the binary CSG operators are now available with all the advantages and extensions seen in the union operator. Moreover, owing to the precision of the transition control, many complex implicit objects created with their own implicit model, can be used as a basic primitive. This extends the versatility of our model.

### 4.3. Extrusion of the master profile around an implicit object

Simple instantiation for the $f_1$ implicit function is a plane surface. The iso-value field is a set of parallel plane surfaces. The extrusion profile is then given by the intersection between an iso-value plane surface and an iso-value surface defined in $f_2$ field. According to these considerations, the final object is the master profile extrusion 'around' the implicit object defined by function $f_2$ (Figure 7).

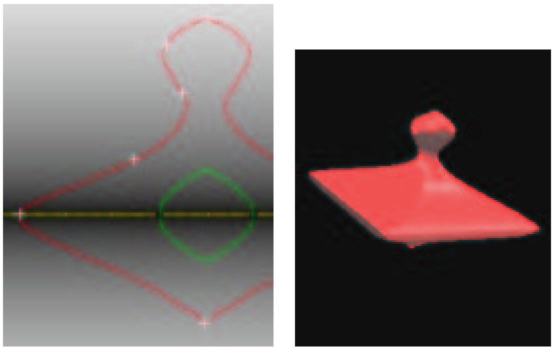

**Figure 7:** *Extrusion of the profile in an octagonal field.*

The chimney in Figure 8 is the result of an extrusion around a parallelepiped. The main part of the roof is the result of an extrusion around a cylinder.

The iso-surface abscissa can be modified. Simple functions like the intersection of inclined planes (with or without soft transition) produce interesting effects.

**Figure 8:** *Octree (256 $\times$ 256 $\times$ 256) of Smurf's house visualized with OpenGL.*

## 5. Conclusion

Extruding 1D cubic polynomial splines in implicit fields allow us to introduce precise control and free form blending. Through the instantiation fields, we propose a new class of implicit functions, extruding profiles 'around' an implicit object. The use of a 2D selection profile interface decreases the number of 3D visualizations. This adds interactivity to our model. At high levels of the CSG tree, the 2D visualization process time of object outlines increases (a few seconds) due to surface equation complexity.

The visualization of implicit fields has to be improved to precisely and interactively select the 2D selection profile plane. Splines create bounded modifications on the primitives. Algorithms computing only these modifications in the 2D/3D visualization structure can increase interactivity.

Extruding 2D profiles in implicit fields is an interesting topic of investigation. Interactive and accurate tools for implicit modeling can be developed and integrated in other models.

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## References


[1] J.F. Blinn. A Generalization of algebraic surface drawing. ACM Transaction on Graphics, **1**(3):235-256, July 1982.

[2] H. Nishimura, M. Hirai, T. Kawai, T. Kawata, I. Shirakara, and K. Omura. Object modeling by distribution functions. Electronics Communications, **68D**(4):718-725, 1985 in Japanese.

[3] G. Wyvill, C. McPheeters and B. Wyvill. Data structure for soft objects. The Visual Computer, **2**(4):227-234, February 1986.

[4] J. Warren. Blending algebraic surfaces. ACM Transaction on Graphics, **8**(4):263-278, 1989.

[5] Z. Kacic-Alesic and B. Wyvill. Controlled blending of procedural implicit surfaces. Proc. Graphics Interface 1991, pp. 236-245, 1991.

[6] M.P. Gascuel. An implicit formulation for precise contact modeling between flexible solids. Computer Graphics (proc. of SIGGRAPH '93), pp. 313-320, August 1993.

[7] S. Muraki. Volumetric shape description of range data using "Blobby Model". Computer Graphics (proc. of SIGGRAPH '91), **25**(4):227-235,1991.

[8] R. Whitaker and D. Breen. Level-Set models for the deformation of solid objects. Proc. of Implicit Surfaces '98, pp 18-34, 1998.

[9] J. Bloomenthal and B. Wyvill. Interactive techniques for implicit modeling. Computer Graphics (proc. of SIGGRAPH '90), **24**(2):109-116, 1990.

[10] J. Bloomenthal and K. Shoemake. Convolution surfaces. Computer Graphics (proc. of SIGGRAPH '91), **25**(4):251-256, 1991.

[11] G. Sealy and G. Wyvill. Smoothing of three dimensional models by convolution. In Computer Graphics International'96, pp 184-190, June 1996.

[12] A. Sherstyuck. Interactive shape design with convolution surfaces. Shape Modeling International '99. March 1999.

[13] P. Hanrahan. Ray tracing algebraic surfaces. Computer Graphics, **17**(3), July 1983.

[14] T. Sedeberg. Techniques for cubic algebraic surfaces. Computer Graphics & Applications, 1989.

[15] C. Blanc and C. Schlick. Ratioquadrics: an alternative model for superquadrics. The Visual Computer, **12**:420-428, 1996.

[16] B. Crespin, C. Blanc, and C. Schlick. Implicit sweep objects. Eurographics '96, **15**(3):165-174, 1996.

[17] C. Grimm. Implicit generalized cylinders using profile curves. Proc. of Implicit Surfaces '99, September 1999.

[18] A. Pasko, V. Adzhiev, A. Sourin, and V. Savchenko. Function representation in geometric modeling: concepts, implementation and applications. The Visual Computer, **8**(2):429-446, 1995.

[19] B.Wyvill, A. Guy and E. Galin. Extending the CSG tree – warping, blending and Boolean operations in an implicit surface modeling system. Proc. Of Implicit Surfaces '98, pp. 128-136, 1998.

[20] L. Barthe, V. Gaildrat and R. Caubet. Combining implicit surfaces with soft blending in a CSG tree. CSG '98 Proceedings, pp. 17-31, 1998.

[21] E. Ferley, M-P. Cani and J-D. Gascuel. Practical Volumetric Sculpting. Proc. of Implicit Surfaces '99, September 1999.

[22] D. Dekkers, K. van Overveld, and R. Golsteijn. Combining CSG modeling with soft blending using Lipschitz-based implicit surfaces. Technical Report, Eindhoven University of Technology, Computer Graphic Group, 1997.

[23] C. de Boor. A practical guide to spline. Applied Mathematical Sciences, **27**:156-162, 1978.