Relational Roles and Qua-individuals

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Abstract

In this paper we present and compare two different approaches to the notion of "relational role" and connected issues. Both approaches are based on the introduction of the notion of "qua-individual" and on the reification of relations. We use a classical problem in the literature on roles, namely the counting problem, as testbed, and we show how the two approaches deal with it in a similar way. We nevertheless show that they are to be distinguished, examining several aspects emerging from their relation with the notion of trope.

Introduction

In the conceptual modeling and philosophy literature, the classical approach for modeling roles is in terms of anti-rigid (or dynamic) and relationally dependent unary predicates. For example, take the role $Student$ that is subsumed by the kind $Person$, i.e. all students are persons. $Student$ is anti-rigid because persons are only contingently students, for example a person can be a student only during a short period of his life. Additionally, $Student$ is relationally dependent because, for a person to be $Student$ it requires the existence of another entity, namely a certain $University$ in which this person is $Enrolled$. In this classical approach, a person *plays* the role $Student$ if he is an instance of the predicate $Student$. Similar cases can be made for roles such as $Father$, $Husband$, $Employee$, and so on.

In an extensive study presented in (Steimann 2000), this view of roles as anti-rigid and relationally dependent predicates is shown to be supported by the vast majority of approaches in the conceptual modeling and object-modeling literature. Nonetheless, this approach suffers of some problems (see Steimann, *ibid*), among which the "counting problem" (Gupta 1980) is one of the most difficult to solve. The approaches presented in (Wieringa, de Jonge, & Spruit 1995) and (Loebe 2003) are the best known alternatives that depart from this traditional view. These approaches, and in particular the second one, are motivated also by a genuine different philosophical position about the ontological nature of properties. In any case, both approaches propose a separation of role and kind hierarchies refusing the idea that roles are subsumed by kinds. In particular, students *existentially specifically depend* on persons but they are not persons (i.e. the instances of $Student$ obey a different principle of identity than the instances of $Person$). In this case, both the predicates $Student$ and $Person$ are rigid, and the *playing* relation is analyzed in a complex way: person p *plays* the role $Student$ because there exists an instance of $Student$, 'p's studentness', that depends on p.

In this paper, we present two approaches that explicitly represent the anti-rigid and relational nature of roles. These approaches are extensions of the ones introduced, respectively, in (Guizzardi 2005; Guizzardi, Wagner, & Herre 2004) and (Masolo *et al.* 2004). We will compare the expressive power of the two approaches with respect to the solution they provide for the counting problem and the way they handle the notion of specialization between roles.

Relational roles

The main focus of this paper is on *relational roles* (more specifically on relational roles of endurants[1]) that we consider here as dynamic, anti-rigid, and relationally dependent properties.

The first two aspects (dynamism and anti-rigidity) regard, respectively, the temporal and modal nature of the relation between roles and their *players* (see (Guarino & Welty 2002) and (Masolo *et al.* 2004) for an extensive explanation of these notions). Entities can play a role only during a specific time interval, or only in a specific possible world (or set of possible worlds). For instance, a person could be a student for only two years, and even in the case he is a student for his whole life, it is not necessary for him, i.e. persons (humans beings) are not necessarily students.

Regarding the third aspect, intuitively, a property is relationally dependent when it depends – via a *pattern of relationships* – on additional "external" properties[2]. We adopt a generalization of the notion of *definitional dependence* introduced by Kit Fine (Fine 1995): a property α is definitionally dependent on a property β if, necessarily, any *definition* of α ineliminably involves β. In particular, we consider roles that can be defined on the basis of a relation whose arguments are

[1]Endurants, also called continuants, objects, etc. are usually opposed to perdurants, also called processes, events, etc. Endurants can be seen as entities that persist being *wholly* present at any time they are present, e.g. a car, Aristotle, a law, the K2, some gold, etc.

[2]The notion of "external property" is quite problematic. Roughly speaking, notions like part, constituent, and quality typically identify things that are internal to other things.

characterized by specific properties. Let us consider, for example, the role of 'being a student' defined as: "a student is a person enrolled in a university". In this case, 'being a student' is defined on the basis of 'being enrolled in', 'being a person', and 'being a university'. Formally, considering the previous properties as predicates, this definition can be formulated as:

$$Student(x) \triangleq Person(x) \wedge \exists y(Enr(x, y) \wedge University(y))$$

More generally, we will focus on roles defined on the basis of a n-ary relation Rel defined on predicates $P_1, ..., P_n$:

$$Rel(x_1, \dots, x_n) \rightarrow (P_1(x_1) \wedge \dots \wedge P_n(x_n))$$

(S) $$R_m(x_m) \triangleq \exists x_1, \dots, x_{m-1}, x_{m+1}, \dots, x_n (Rel(x_1, \dots, x_n))$$

Given a specific relation Rel and following the schema (S), it is possible to define n different predicates R_m. For example, in the case of the relation $Enr(x, y) \rightarrow (Person(x) \wedge University(y))$, the predicate $EnrollingUni$ can be defined as:

$$EnrollingUni(x) \triangleq \exists y(Enr(y, x))$$

$EnrollingUni$ has exactly the "same logical form" as $Student$, but this does not imply that $EnrollingUni$ is a role. Let us assume a theory containing an axiom stating that, necessarily, universities enroll at least one student, i.e., when a university looses all its students, it ceases to be a university. In this theory, 'being an enrolling university' is a rigid property of universities, and therefore it cannot be a role (assuming $University$ as rigid). In addition, the two predicates $EnrollingUni$ and $University$ coincide from an extensional point of view (since all universities are enrolling universities) and they cannot be distinguished by means of the theory. In this case, the predicate $EnrollingUni$ seems "redundant" with respect to the predicate $University$ because they are provably equivalent.

From a methodological point of view, in order to have "economical" and clearer representations, given a relation Rel, we will introduce only roles (defined, on the basis of Rel, following the schema (S)) in the models.

First approach

We consider here a slight modification of the approach that was introduced in (Guizzardi 2005; Guizzardi, Wagner, & Herre 2004) with the purpose of harmonizing the two alternative views of roles present in the conceptual modeling literature and evoked in the introduction, namely, (i) the widespread view in which roles are anti-rigid and relationally dependent unary predicates whose instances are the players; and (ii) the one proposed initially by (Wieringa, de Jonge, & Spruit 1995) in which roles are rigid types whose instances are adjunct entities (Steimann 2000) that existentially depend on their players but are disjoint from them. As demonstrated in (Guizzardi 2005), being able to successfully harmonize these two hitherto competing views is a benefit of this approach from a conceptual modeling point of view.

The main idea followed by this approach concerns the representation of *relations* as sets of complex *tropes* called *relators*. Trope theory (see (Armstrong 1989; Daly 1997) for a review) is a philosophical theory that explains the sharing of a property by two entities in terms of *exact resemblance* between their tropes: a and b are both red because of 'a's redness' and 'b's redness', two distinct tropes (individual properties) that *inhere*, respectively in a and b, and resemble each other. The case of relations is similar to the one of properties: a and b are in the relation R because of the existence of a complex *relational* trope (called here *relator*) inherent both in a and b. The exact resemblance partitions the domain of tropes/relators in equivalence classes: two tropes/relators exactly resemble each other if and only if they belong to the same equivalence class. Therefore, from the representational point of view (but not from the philosophical one), it may be convenient to consider these equivalence classes as primitive and recover resemblance from them. In particular, in order to be compatible with the main conceptual modeling languages, we will consider primitive unary predicates corresponding to these equivalence classes of resembling tropes/relators.

The case of relational properties defined following the schema (S) deserves a deeper analysis. Let us take the properties of 'being a student' and 'being an enrolling university' defined by the 'being enrolled in' relation as in the previous section. If a and b are in the enrollment relation, then a is a student and b an enrolling university, i.e., intuitively, a is a student and b an enrolling university *just because* they are in the enrollment relation. From the perspective of the trope theory, if an enrollment relator exists, then both a student trope and an enrolling university trope exist and the opposite is also true because of the relational nature of 'being a student' and 'being an enrolling university'. Therefore the tropes and the relator *mutually specifically existentially depend* on each other. Now, let us suppose that b enrolls (at least) two students, a and a'. While, in general, the particularization of the property 'being an enrolling university' to b presupposes only one trope ('b's being an enrolling university'), in this case, there exist two (more specific) tropes: 'b's enrolling a' and 'b's enrolling a''. Intuitively, both the tropes "support" the predication of the property 'being an enrolling university' on the same university b, i.e. the property 'being an enrolling university' is a disjunction of more specific properties like 'being a university enrolling person a', 'being a university enrolling person a'', etc.[3] The same is true for the property 'being a student': it can be seen as a disjunction of properties 'being a student of university b', 'being a student of university b'', etc.[4] Therefore one could question the possibility to determine what are the tropes that support the properties 'being a student' and 'being an enrolling university'. In the example, 'a's being enrolled in

[3] This topic is related to the one of saturated roles that we will discuss at the end of this paper.

[4] Note that this means that a person that changes university remains a student even though there is a "substitution" of tropes (e.g. 'a's being enrolled in university b' is substituted by 'a's being enrolled in university b'').

$$
\begin{tikzcd}
Person & & & & University \\
Student \arrow[u, "isa"] & {}^{enr}StudTp \arrow[l, "i"'] \arrow[r, leftrightarrow, "eSD"] & Enr \arrow[r, leftrightarrow, "eSD"] & {}^{enr}UniTp \arrow[r, "i"] & EnrollingUni \arrow[u, "isa"'] \\
\mathtt{luc} \arrow[u, dashed, "inst"] & \mathtt{luc}_{\overrightarrow{enr}}\mathtt{untn} \arrow[u, dashed, "inst"] \arrow[l, dashed, "i"'] \arrow[r, dashed, leftrightarrow, "eSD"] & \mathtt{enr}_{\mathtt{luc,untn}} \arrow[u, dashed, "inst"] \arrow[r, dashed, leftrightarrow, "eSD"] & \mathtt{untn}_{\overleftarrow{enr}}\mathtt{luc} \arrow[u, dashed, "inst"] \arrow[r, dashed, "i"] & \mathtt{untn} \arrow[u, dashed, "inst"]
\end{tikzcd}
$$

Figure 1: The enrollment relation (Enr) defines two roles: $Student$ and $EnrollingUni$.

$$
\begin{tikzcd}
Person & & & & University \\
Student \arrow[u, "isa"] & {}^{enr}StudTp \arrow[l, "i"'] \arrow[r, leftrightarrow, "eSD"] & Enr \arrow[r, leftrightarrow, "eSD"] & {}^{enr}UniTp \arrow[ur, "i"] & \\
\mathtt{luc} \arrow[u, dashed, "inst"] & \mathtt{luc}_{\overrightarrow{enr}}\mathtt{untn} \arrow[u, dashed, "inst"] \arrow[l, dashed, "i"'] \arrow[r, dashed, leftrightarrow, "eSD"] & \mathtt{enr}_{\mathtt{luc,untn}} \arrow[u, dashed, "inst"] \arrow[r, dashed, leftrightarrow, "eSD"] & \mathtt{untn}_{\overleftarrow{enr}}\mathtt{luc} \arrow[u, dashed, "inst"] \arrow[r, dashed, "i"] & \mathtt{untn} \arrow[uu, dashed, "inst"]
\end{tikzcd}
$$

Figure 2: The enrollment relation (Enr) defines only one role: $Student$.

b' and 'a's being enrolled in b'' both support 'being a student' but they do not exactly resemble each other because they support different properties. Therefore a more general notion of resemblance, an "inexact" resemblance, collecting all the tropes supporting 'being a student', is needed. For relational properties this inexact resemblance can be defined as follows: "two tropes *inexactly resemble* each other if and only if they depend on *exactly* resembling *relators* and they inhere in entities of the same kind (belonging to the same predicate on which the defining relation is defined)".[5] Coming back to the example, 'a's being enrolled in b' inexactly resembles 'a's being enrolled in b'' because they both depend on an enrollment relator and both inhere in persons. Analogously for 'b's enrolling a' and 'b's enrolling a'' and the property 'being an enrolling university'.

As already said, in this approach, the equivalence classes of (inexact and exact) tropes are represented by predicates. In the previous example, three predicates are needed: one for the enrollment relators, and two for the tropes supporting 'being a student' and 'being an enrolling university'. Mutual dependences between these predicates are needed. Roles are represented as predicates on entities in which supporting tropes inhere. For example, since 'being a student' is a role, a predicate $Student$ (subsumed by $Person$) is introduced. At least one trope supporting, in the sense explained above, 'being a student' inheres in every instance of $Student$.

The student example can be completely represented as in figure 1 (where $EnrollingUni$ is considered as a role) or figure 2 (where $EnrollingUni$ is not considered as a role)[6] where the following conventions are assumed:

- properties are in italics, capitalized;
- individuals are in lower case type font;
- the tropes corresponding to the assertions $Rel(\mathtt{a}, \mathtt{b})$ are $\mathtt{a}_{\overrightarrow{rel}}\mathtt{b}$ and $\mathtt{b}_{\overleftarrow{rel}}\mathtt{a}$, and the relator is $\mathtt{rel}_{\mathtt{a,b}}$;
- relations between properties are labeled arrows: $U_1 \xrightarrow{R} U_2$ stands for $U_1(x) \rightarrow \exists y(U_2(y) \wedge R(x, y))$;
- relations between individuals are dashed labeled arrows: $\mathtt{a} \overset{R}{\dashrightarrow} \mathtt{b}$ stands for $R(\mathtt{a}, \mathtt{b})$;
- the *ISA* relation between properties and the *instance-of* relation between an individual and a property are respectively labelled by isa and $inst$;
- the *inherence* and the *specific existential dependence* relations are respectively labelled by i and eSD.

We can state the previous assumptions in a more formal way: given an n-ary relation Rel^* such that $Rel^*(x_1, \ldots, x_n) \rightarrow (P_1(x_1) \wedge \ldots \wedge P_n(x_n))$, there exist a set of *relators* Rel, n sets of *tropes*[7] $T_1, \ldots, T_n$, and m ($m \leq n$) sets, called "roles", R_j with $j \in I \subseteq \{1, \ldots, n\}$ and $\|I\| = m$ such that:

$$
\begin{array}{l}
R_j(x) \rightarrow P_j(x) \text{ for each } j \in I \\
T_j(x) \rightarrow \exists! y(R_j(y) \wedge i(x, y)) \text{ for each } j \in I \\
T_j(x) \rightarrow \exists! y(P_j(y) \wedge i(x, y)) \text{ for each } j \in \{1, \ldots, n\} \setminus I \\
T_j(x) \rightarrow \exists! y(Rel(y) \wedge eSD(x, y)) \\
\hfill \text{for each } j \text{ s.t. } 1 \leq j \leq n
\end{array}
$$

[5]This assumes that all the arguments of the relation involve different predicates of kind. Otherwise we need to explicitly consider different relations between the tropes and the relators.

[6]Note that existence of the trope $\mathtt{untn}_{\overleftarrow{enr}}\mathtt{luc}$ supporting the property 'being an enrolling university', is independent from the fact that this property is considered as a role or not; $\mathtt{untn}_{\overleftarrow{enr}}\mathtt{luc}$ exists in any case when the relator $\mathtt{enr}_{\mathtt{luc,untn}}$ exists.

[7]Note that, from a technical point of view, these n sets of tropes are necessary, otherwise it would not be possible to express the difference between $Rel^*(a, b)$ and $Rel^*(b, a)$ when $P_1 = P_2$.

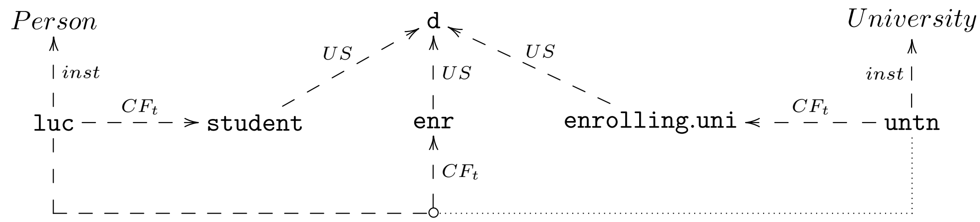

Figure 3: Formalizing the student example using the second approach.

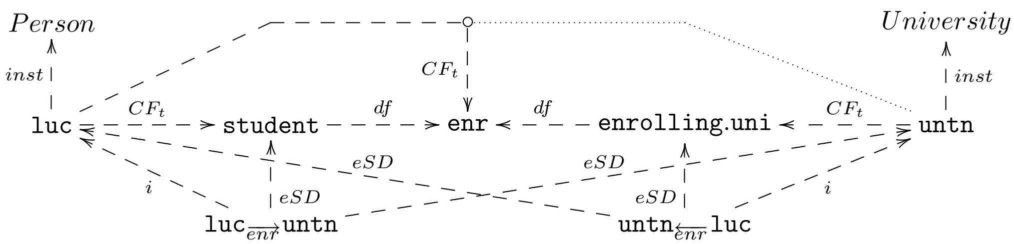

Figure 4: Introducing "qua-individuals" in the formalization of the student example.

$$Rel(x) \to \exists! y_1, \ldots, y_n(T_1(y_1) \land \ldots \land T_n(y_n) \land eSD(x, y_1) \land \ldots \land eSD(x, y_n))$$

Second approach

We consider now an extension of the approach introduced in (Masolo *et al.* 2004). The novelty of the approach consisted in reifying in the domain of discourse (of a first order theory) social concepts, among which roles, and the social conventions or contexts that define them. The extension put forward in the present paper is based on the application of this general strategy also to *relations*, while in the original proposal only the reification of properties and their definitions was considered.

This approach is based on a clear distinction between:

- the properties and relations in the *ground ontology*, represented as predicates and therefore assumed as static, rigid, extensional, and not explicitly defined or linked to a social context (e.g., the primitive predicates of the theory);
- the properties (called "concepts") and relations reified at the object level, that are not necessarily static, rigid, and extensional and for which it is possible to explicitly describe some aspects of the social contexts that define them (called "descriptions").

Concepts are *defined* (DF) or *used* (US) by descriptions and they *classify* (CF) other individuals: $DF(x, y)$ stands for "the *concept* x is defined by the *description* y"; $US(x, y)$ stands for "the *concept* x is used by the *description* y", i.e. it could be defined by y or simply "imported" in it; $CF(x, y, t)$ stands for "at the *time* t, the *individual* x is classified by the *concept* y", i.e., "at the *time* t, the *individual* x satisfies all the constraints stated in the description of the *concept* y".

To extend this framework to take into account the reification of n-ary relations, we need at least to:

(i) introduce a classification relation for each arity considered, i.e., if a n-ary reified relation r is considered in the domain of quantification[8], then the $(n + 2)$-ary predicate CF is introduced, where $CF(x_1, \ldots, x_n, r, t)$ stands for "at the *time* t, the *individuals* $x_1, \ldots, x_n$ are classified by the *relation* r";

(ii) extend the primitives DF and US to the reification of predicates in general, i.e., concepts and relations.

Assuming that $Person$ and $University$ are in the ground ontology (and are therefore predicates), the previous student example can be represented as on figure 3 (where an arrow labeled with CF_t between a and cn stands for $CF(\mathtt{a}, \mathtt{cn}, t)$ and a complex arrow labeled with CF_t linking a and b to rel stands for $CF(\mathtt{a}, \mathtt{b}, \mathtt{rel}, t)$). If 'being an enrolling university' is in fact considered neither as a role nor as a concept,[9] the entity enrolling.uni is simply dropped from this picture along with its links.

The fact that student and enrolling.uni are concepts defined on the basis of the same relation enr is represented by the fact that student, enrolling.uni, and enr are used in the same description d. In general, this is not enough because the direct links between the relation and the concepts it defines is lost. Let us consider, for example, the case in which a single description defines (or uses) two binary relations and four concepts that classify entities of the same kind. In this case it is not possible to determine which concepts are defined by (depend on) a specific relation. To avoid this problem, a link between a relation and the concepts it defines is necessary. We thus introduce the predicate df, with $df(x, y)$ standing for "the (relational) *concept* x is defined by the *relation* y". Clearly, in order to define a rela-

[8] Only finite arities are considered.

[9] See (Masolo *et al.* 2004) for the characterization of the concepts that are roles.

tional concept x, a description needs to use the relation y by which x is defined: $(DF(x,d) \wedge df(x,y)) \rightarrow US(y,d)$.

In (Masolo *et al.* 2004) the authors proposed a refinement of the theory taking into account the so called "qua-individuals" in order to handle some classical problems (see below for more details on the counting problem). Qua-individuals exist when an entity is classified by a "saturated"[10] role, but they have different identity criteria from the classified entity. They are existentially dependent on both the classified entity and the role under which the entity is classified. As for tropes in the first approach, in the case of a *relational role*, the qua-individual actually depends on all the entities jointly classified by the relation defining the role. The distinction between inherence (i) and specific existential dependence (eSD) is thus needed to express the fact that the qua-individual is relative to only one of these. Taking now qua-individuals into consideration, our student example can be represented as on figure 4 (where 'Luc-qua-student' and 'UnTN-qua-enrolling.uni' are respectively called $\mathtt{luc}_{\overrightarrow{enr}}\mathtt{untn}$ and $\mathtt{untn}_{\overleftarrow{enr}}\mathtt{luc}$ to make the link with figure 1). The difference between $\mathtt{luc}_{\overrightarrow{enr}}\mathtt{untn}$ and $\mathtt{untn}_{\overleftarrow{enr}}\mathtt{luc}$ is given by the fact that the first is inherent in luc while the second in untn. Note that in this approach the trope relator $\mathtt{enr}_{\mathtt{luc},\mathtt{untn}}$ is not necessary.

The counting problem

As evoked briefly in the introduction, a classical puzzle for roles is the counting problem (Gupta 1980), which can be formulated in the following way:[11]

- Alitalia served one million passengers in 2004
- Every passenger is a person
- Ergo, Alitalia served one million persons in 2004

If a given person flew several times Alitalia in 2004, which is more than likely, the conclusion is false. This example has been throughly discussed, paying particular attention to the relationships between the identity criteria for passengers and persons. Here we will consider only a specific aspect made explicit by this example: to count passengers we cannot just count persons. So, what do we count? We'll examine first how the two approaches described above handle this problem.

Let us consider 'passenger' as a relational role defined in terms of the binary relation 'flies' (Fl), such that $Fl(x,y) \rightarrow (Person(x) \wedge Airline(y))$ and $Passenger(x) \triangleq \exists y(Fl(x,y))$. With these hypotheses, let us try to represent the situation in which (during 2004) Luc flew twice Alitalia and once KLM, and Sam flew once Alitalia; for this example, the two approaches respectively yield the representations depicted on figures 5 and 6.

This example involves only two persons (Luc and Sam) while Alitalia served three passengers and KLM one, which makes a total of four passengers. Both approaches correctly represent only two persons. In both cases, in order to count passengers we don't count persons, but the entities $\mathtt{person.x}_{\overrightarrow{fl}}\mathtt{airline.y}$, that can be considered to represent what is called "qua-individuals" in the literature. There are exactly four such entities in both approaches, as required. The apparently unique solution to the counting problem that makes use of qua-individuals needs some clarification and a deeper analysis that we will tackle in the following sections.

Let's just remark for the moment that in the first approach, three entities (one relator and two other tropes) are needed for each passenger, and only one in the second. As far as statements are concerned, the first approach requires, for each passenger, three instance statements, two inherence ones, and two existential specific dependence ones. The second approach needs for each passenger two classification statements (including a relational one, e.g., $CF(\mathtt{luc},\mathtt{alit},\mathtt{fl},t_1)$), one inherence and two existential specific dependence statements. As far as universals are concerned, the first approach introduces four predicates ($Passenger$, ${}^{fl}PassTp$, Fl, and ${}^{fl}AirlTp$) and the second approach, one concept (passenger) and one reified relation (fl). On the whole, we can observe that the second approach is somewhat representationally lest costly than the first.

Qua-individuals and time

In the previous example, Alitalia served Luc twice. In the two approaches this is represented by the existence of two different qua-individuals both inherent in luc and both existentially dependent on alit: $\mathtt{luc}_{\overrightarrow{fl}}\mathtt{alit}_1$ and $\mathtt{luc}_{\overrightarrow{fl}}\mathtt{alit}_2$. But these two entities seem to be "generated" by only one fact: $Fl(\mathtt{luc},\mathtt{alit})$. Therefore one could wonder, how come two different entities for the same fact? What makes the difference between the two? In fact, in the two approaches, the two qua-individuals are related to all the others entities present in the models exactly in the same way.

The problem is related to the meaning of the expression "Alitalia served Luc twice". This expression seems to reflect the fact that $Fl(\mathtt{luc},\mathtt{alit})$ "holds" twice, i.e. an additional parameter seems to be missing in Fl.

A first solution could consider the flight number as the missing parameter. In this case "twice" means "in two different flights". However, flight numbers are usually still types, not individuals, in the sense that a given flight number of a given airline typically flies several times a week. A second solution refers to a temporal parameter, i.e. "twice" means "at two different times". With this solution, the two entities $\mathtt{luc}_{\overrightarrow{fl}}\mathtt{alit}_1$ and $\mathtt{luc}_{\overrightarrow{fl}}\mathtt{alit}_2$ refer, respectively, to two different situations: $Fl(\mathtt{luc},\mathtt{alit},t_1)$ and $Fl(\mathtt{luc},\mathtt{alit},t_2)$, where t_1 and t_2 are two disjoint intervals or instants of time. Therefore, the difference between $\mathtt{luc}_{\overrightarrow{fl}}\mathtt{alit}_1$ and $\mathtt{luc}_{\overrightarrow{fl}}\mathtt{alit}_2$ is temporal.

It is of course possible to consider time just as another parameter, and indeed consider Fl as a ternary relation. Nevertheless, the two approaches choose to take time into account using a different strategy: $\mathtt{luc}_{\overrightarrow{fl}}\mathtt{alit}_1$ and $\mathtt{luc}_{\overrightarrow{fl}}\mathtt{alit}_2$ are both endurants, therefore they are in time and they have a

[10]See the last section for a discussion about saturated roles.

[11]The classical example refers to passengers, as expressed here, although a similar argument could be made based on the student example we've just seen, considering that the same person could be enrolled in different universities.

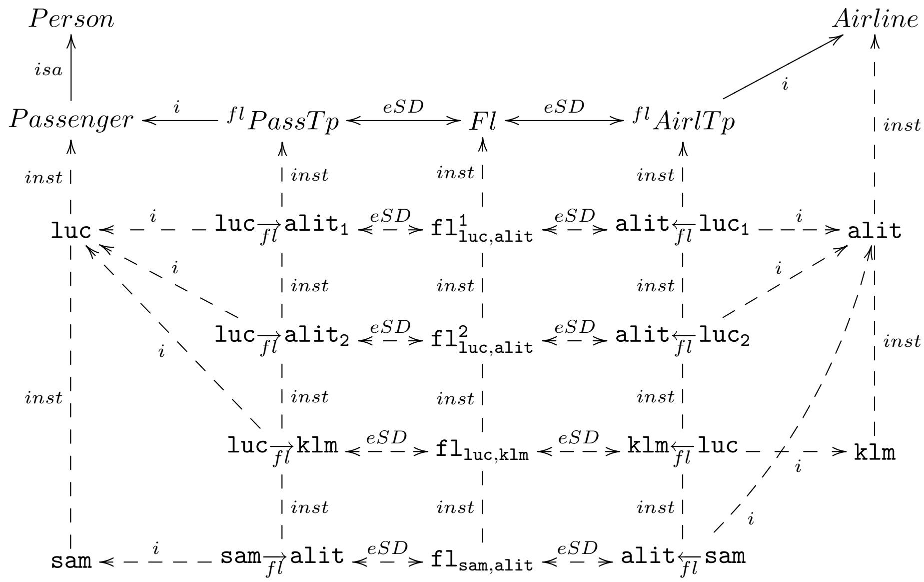

Figure 5: The passenger example represented using the first approach.

temporal extension (a life). The situation $Fl(\texttt{luc}, \texttt{alit}, t_1)$ is therefore represented by simply establishing that the temporal extension of $\texttt{luc}\underset{fl}{\rightarrow}\texttt{alit}_1$ is t_1 and similarly for $Fl(\texttt{luc}, \texttt{alit}, t_2)$. For readability, the temporal extensions of endurants are not explicitly represented on figures, but the two approaches do consider them in the complete model.

A further refinement would bring into the picture the events, i.e., the perdurants, that generate the facts. It can indeed be held that specific events are associated to roles. For instance, in the passenger example, each event of Luc checking in (or perhaps Luc boarding) generates one Fl fact and one passenger. The passenger entities, that is, the qua-individuals $\texttt{luc}\underset{fl}{\rightarrow}\texttt{alit}_1$, $\texttt{luc}\underset{fl}{\rightarrow}\texttt{alit}_2$ etc., are therefore existentially dependent on such generating events. Similarly, presidents are dependent on elections or nominations, students are dependent on enrollment events, etc. It could be argued that counting qua-individuals could thus be reduced to counting such generating events. Note though that this must be distinguished with a solution to the counting problem sometimes proposed in the literature, which does not consider qua-individuals at all, and simply counts events in which “standard” individuals (e.g., persons) are involved. In fact, such an approach doesn’t propose to count the specific events associated to roles, but the events actually described in the numerically quantified sentence. For a sentence like “In 2004, Alitalia carried a million passengers”, it makes no difference, as it can be assumed that carrying events are in one-to-one correspondence with checking-in events. But for a sentence like “In 2004, the Italian Prime Minister participated in thirty industrial meetings”, it does. Only three nomination events brought Berlusconi to play the Prime Minister role, and accordingly, there are only three “Prime Ministers Berlusconi” (as there are G.W. Bush-1 and G.W. Bush-2), but there have been thirty participation events in 2004. And to count these thirty participation events, it is not possible to count how many times the person Berlusconi participated in an industrial meeting, since he may have participated to additional industrial meetings, not in quality of Italian Prime Minister, but, say, as the President of the Mediaset company. We need to count the industrial meetings in which ‘Berlusconi-qua-Prime Minister’ participated. This last counting example shows that we cannot do without qua-individuals, and that in addition, qua-individuals do participate as such in events. Nonetheless, we have more to say about the latter issue in the next section.

Tropes vs. qua-individuals

In both the approaches examined in this paper, we described as “qua-individuals” the entities denoted by $a\underset{rel}{\rightarrow}b$. And in both approaches, these entities solve the counting problem in a very similar way. However, there is a basic assumption in the first approach that such entities are *tropes*, while in the second one no such assumption is taken. In order to fully understand whether this assumption makes a difference or not, and whether these entities are actually of a different nature in both approaches, we need to examine from an ontological perspective tropes and qua-individuals. The notion of trope, as used in philosophy, has already been briefly sketched. The reason we have limited ourselves to a succinct discussion of tropes is due to the fact that, compared to qua-individuals, tropes have been much more investigated in the philosophical literature, and from a more general point of view. In what follows, we take a look at the existing literature on qua-individuals.

The notion of qua-individual is ancient and comes at least from Aristotle.[12] For example in *On Interpretation* he says

[12] See for example (Szabó 2003) and, for a more historical and deep account (Baek 1982).

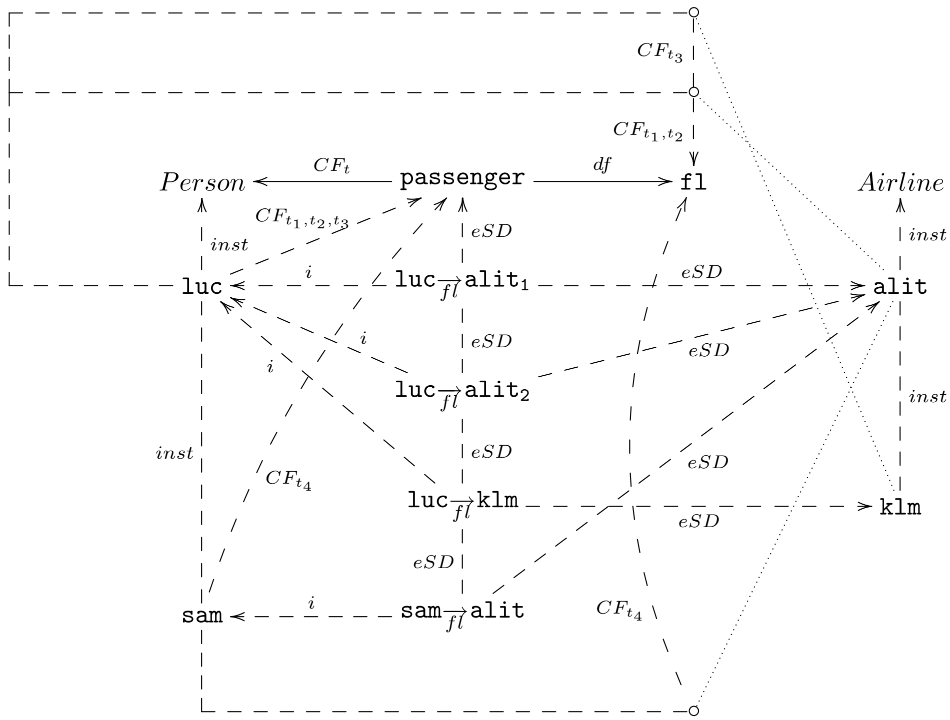

Figure 6: The passenger example represented using the second approach.

that someone might be good *qua* cobbler without being good.[13] This problem, that was called by medieval philosophers the problem of *reduplicatio* or the problem of *qualification* (Poli 1998), was also faced by Leibniz, when he formulated the identity principle known as "Leibniz rule", and Brentano (Angelelli 1967). More recently, in contemporary philosophy qua-individuals are introduced in order to solve some problems related to the theory of constitution (Fine 1982) and the theory of action (see (Anscombe 1979) and (Fine 1982)). Consider the classical example of the statue (called Goliath) and the lump of matter that constitutes it, which involves identity problems in an extensional view of constitution, as stated by Kit Fine in (Fine 1982). According to Fine, a theory of constitution equipped with a theory of qua-individuals (called qua-objects in his account) solves the identity problem, saying that "the statue may be identified with that matter under the description of having Goliath shape", or with that matter qua-Goliath-shaped.

It has been said that the qua-individual should be regarded "as some sort of amalgam of the given object but wearing the property on its face"(Fine 1982). In general qua-individuals are considered in the context of expressions like "x qua y". Expressions of this kind are known in the literature as "qua phrases" (Moltmann 1997). Let us take as example, the situation in which Ciampi is the President of the Italian Republic. A sentence involving a qua phrase could be: "Ciampi *qua* President of Italy dissolved the chambers".

The first approach we have seen holds that qua-individuals can be seen as tropes inhering in their bearers. Under this perspective, "Ciampi qua President of Italy dissolved the chambers" is analyzed as "Ciampi dissolved the chambers *because* of his 'Presidentness-of-Italy"'. The 'Presidentness-of-Italy of Ciampi' is the sum (or bundle) of the individual properties, the tropes, that Ciampi has by virtue of being president, i.e., this sum is in some sense only *part* of him. Therefore, with the trope view on qua-individuals, what participate to events are necessarily the bearers, e.g., Ciampi, and not the (bundles of) tropes, e.g., the 'Presidentness-of-Italy of Ciampi'. Qua-individuals analyzed as tropes do not participate in any event. Nevertheless, we have seen in the previous section that to hold that qua-individuals actually participate in events would be very useful to account for some counting examples. And an analysis of Fine's view on qua-individuals could be that qua-individuals are individuals *plus* some property — just as the statue is taken to be matter-plus-shape, rather that some part of it, as under the trope view. Let's consider yet another example from the Italian constitution: "The president may dissolve one or both chambers after having consulted their speakers". With this rule in mind, when Ciampi dissolves the chambers *qua* President of Italy, it is natural to hold that it is the qua-individual 'Ciampi-qua-President of Italy', and not simply Ciampi, who performs the action. The trope view of qua-individuals would thus appear to have a limited expressivity.

However, in an alternative interpretation of the constitutional rule according to the trope view, "the president" can refer to whomever instantiates the role 'president' under those circumstances, namely, Ciampi. A possibility then

[13] *On Interpretation*, 20, b35-7.

to interpret Ciampi's participation in an event as the president of Italy can be modelled as a ternary participation relation $Pc(Ciampi, e_1, c_1)$ in which the third argument c_1 is the bundle of tropes 'Ciampi-qua-president', which qualifies Ciampi's participation in event e_1. Intuitively, this formula can be interpreted as Ciampi participates in the event e_1 because of his 'Presidentness-of-Italy' and as the president of Italy, i.e., with all the properties constituting the corresponding qua individual (e.g., rights, obligations and powers afforded by his 'Presidentness-of-Italy'). We have previously discussed the example in which the same individual Berlusconi participates in meetings as the Italian Prime Minister or as the President of Mediaset. Consider now a situation in which he participates "twice" to the same meeting, as both Italian Prime Minister and President of Mediaset. In the trope view, this situation can be modeled as $Pc(Berlusconi, m_1, b_1)$ in which b_1 is the bundle of tropes 'Belusconi-qua-Italian Prime Minister' and $Pc(Berlusconi, m_1, b_2)$, in which b_2 is 'Berlusconi-qua-President of Mediaset'. The two statements represent the modes of participation of Berlusconi in the same meeting. In summary, in the first approach we have a qualified ternary participation relation with the bundle of tropes as an additional argument, which qualifies the mode an individual participates in a certain event. In the second approach, we have directly the qua-individual, i.e., the individual plus some individual properties, who participates in an event.

Let's now go back to the passenger example. Intuitively, with respect to persons, passengers have *additional* properties. For example passengers, and not persons, have the attribute 'flight number', while all the properties of persons (weight, mood…) seem to apply to passengers. With the trope view, a qua-individual is the sum of the individual properties that the bearer has because of its involvement in the underlying relation. "Properties" of qua-individuals are thus simply individual properties of their bearers that are part of the bundle making up the qua-individuals. So it is possible to state that $\mathtt{luc}\underset{fl}{\rightarrow}\mathtt{alit}_1$ has the attribute 'flight number' because Luc, when he is a passenger, has this attribute (changing in time). But if we suppose that a person can be passenger of two different airlines at the same time, as, for example, in the case of a shared flight, time is not enough to disambiguate between passengers. The passenger of, say, Alitalia, could have a different flight number from the passenger of, say, KLM, even though in both cases we are talking of the same person at the same time. This situation is unproblematic in the second approach, since these additional properties, i.e., properties that the "bare" individuals do not have, can be attributed to the qua-individuals. For instance, in this case, one flight number can be attributed to $\mathtt{luc}\underset{fl}{\rightarrow}\mathtt{alit}_1$ and another one to $\mathtt{luc}\underset{fl}{\rightarrow}\mathtt{klm}_1$. On the other hand, this case seems at first to clash with the trope view of the first approach. However, as we have seen, the tropes that compose the qua-individual are all extrinsic tropes, i.e., tropes that inhere in one object (its bearer) but which are existentially dependent on some other objects independent from the bearer. For example, in this case, the flight number that Luc has because he is flying this specific Alitalia flight is an attribute of Luc but it also depends on the existence of this specific Alitalia flight. Likewise for the other attribute flight number that Luc has because he is flying a specific KLM flight. Therefore, instead of having two incompatible flight number attributes, Luc has in fact two relational attributes 'Alitalia-flight-number' and 'KLM-flight-number'. It is important to emphasize that since in the trope view the qua-individual is the sum of tropes that Luc has because he is a passenger of a particular airline, in this example, Luc would have different tropes in relation with the different airlines, regardless if these happen to share the same flight. For instance, Luc as a passenger of Alitalia could have different rights than as a passenger of KLM.

There are other classical examples about individuals having conflicting properties qua different roles, for instance the famous "Nixon qua quaker is a pacifist, while Nixon qua republican is not". Again, in the second approach, we can easily draw a consistent picture in which Nixon himself is either a pacifist or not (or neither of the two if we consider that people do not need to take a decision on all issues), while 'Nixon-qua-quaker' is pacifist and 'Nixon-qua-republican' is not, which means for instance that 'Nixon-qua-quaker' participates in anti-war-actions while 'Nixon-qua-republican' participates in pro-war-actions. In the trope view, in contrast, it seems at first that this example could only be regarded as a logical contradiction. However, a solution to this problem according to that approach can be provided using again extra arguments. In a sentence such as "Nixon qua quaker is a pacifist" the property of 'being pacifist' can be considered a relational one. That is to say, Nixon is a pacifist in this case because he is related to (member of, part of) the community of Quakers. Likewise, Nixon has the property of 'being pro-war' because he is related to the Republican party. Nixon qua himself can still bear or not the intrinsic property of 'being pacifist' (or 'being pro-war'). Thus, in this view, we can have the case that Nixon is at same time 'pacifist qua quaker', 'pro-war qua republican' and, for instance, 'pacifist qua himself'. From a representational point of view, analogously to the case of participation aforementioned, we can have that $\textit{Pacifist}(N, \textit{N-qua-quaker}) \wedge \neg\textit{Pacifist}(N, \textit{N-qua-republican}) \wedge \textit{Pacifist}(N, N)$. We are aware that this solution still leaves open a question regarding the argument structure of predicates. That is to say, how can one know a priori which predicates should allow for an additional qua-individual argument? This issue is matter of future investigation.

Let us summarize the two views. Depending on the view, the entity $\mathtt{luc}\underset{fl}{\rightarrow}\mathtt{alit}_1$ either is:

- a "genuine" entity, that bears all the properties of Luc plus other additional properties coming from the Fl relation he holds with Alitalia; the relation between Luc and $\mathtt{luc}\underset{fl}{\rightarrow}\mathtt{alit}_1$ can be described as a peculiar sort of *constitution*, characterized by the fact that the constituted entity specifically depends on the constituent;[14] or
- a sum of "qualities" of Luc, that he acquires by enter-

[14] In the ordinary relation of constitution this dependence is generic, not specific.

ing in the Fl relation with Alitalia. In this second case, $\texttt{luc}_{\overrightarrow{fl}}\texttt{alit}_1$ is a bundle of tropes of Luc, and the relation between $\texttt{luc}_{\overrightarrow{fl}}\texttt{alit}_1$ and Luc is the *inherence* relation (as in the case of the 'redness of a', which is just a specific aspect of a).

Saturated roles

Apart from the nature of qua-individuals, the two approaches further differ on what predicates can be considered as roles. Let us consider the three predicates: $Passenger$, $PassengerofAlitalia$, and $PassengerofKLM$. Intuitively, all these unary predicates are relationally dependent and anti-rigid, therefore they are good candidate for roles; let us assume they are roles. We can represent the fact that Luc is a passenger of Alitalia and of KLM introducing two different qua-individuals both dependent on Luc, but one dependent on Alitalia, and the other one on KLM. This is exactly what is represented, using either of the two approaches, on figures 5 and 6. So, in both cases there is actually *no* qua-individual 'Luc-qua-passenger', only the qua-individuals 'Luc-qua-Alitalia passenger(1/2)' and 'Luc-qua-KLM-passenger'. The generation of a qua-individual is done only when all the arguments involved in the relation 'flies' are fixed.

But what about the roles 'Passenger of Alitalia' and 'Passenger of KLM'? On figures 5 and 6, only the role 'Passenger' is represented. Is it possible to introduce the other two roles and what would be the relation between these two additional roles and the role 'Passenger'? Intuitively, on the basis of the binary relation 'flies' (Fl), these three roles could be defined as:

$$Passenger(x) \triangleq \exists y(Fl(x,y))$$
$$PassAlit(x) \triangleq Fl(x, \texttt{alit})$$
$$PassKlm(x) \triangleq Fl(x, \texttt{klm})$$

i.e., in order to define the predicates $PassAlit$ and $PassKlm$ it is necessary to fix the second argument, choosing a specific individual. In the case of n-ary relations, we can have roles of different levels of *saturation*, depending on how many arguments are fixed.

In (Masolo *et al.* 2004), such different levels of saturation of roles were considered, introducing the additional primitive relation of *specialization* (SP) between roles. Instead of using a primitive, with the second approach we can actually define such a relation between roles. For roles defined on binary relations, we have:

$$\begin{aligned} SP(r_1,r_2) \triangleq\ & SB(r_1,r_2) \wedge \\ & \exists rel(df(r_1,rel) \ \wedge \ df(r_2,rel) \ \wedge \\ & \quad \exists! y(\forall x,t(CF(x,r_1,t) \rightarrow CF(x,y,rel,t)) \vee \\ & \quad \quad \forall x,t(CF(x,r_1,t) \rightarrow CF(y,x,rel,t)))) \end{aligned}$$

where SB is the subconcept relation extensionally defined:

$$SB(r_1,r_2) \triangleq \forall x,t(CF(x,r_1,t) \rightarrow CF(x,r_2,t))$$

Using the second approach we can therefore introduce the two additional roles pass.alit and pass.klm linked by a SP relation to the role passenger (see figure 7). It must be noted that a given qua-individual, e.g., $\texttt{luc}_{\overrightarrow{fl}}\texttt{klm}$, exemplifies *both* the role passenger and the role pass.klm; in some sense, qua-individuals are generated by the more saturated roles, while the less saturated ones inherit their qua-individuals from roles that specialize them.

This corresponds exactly to the observation we made much earlier about the student example in the first approach, that the same trope supports different properties, i.e. in that example, 'being a student' and 'being a student of university b'. However, introducing roles at various levels of saturation in the first approach is not straightforward. In fact, this approach, used in conceptual modeling, models only the level of universals and does not consider the level of instances (although we pictured it on figures 1, 2 and 5), and accordingly, roles are always at the lowest degree of saturation. Considering the addition of an instance level, a definition similar to the previous one could be attempted also in this case. But since this amounts to assume that different roles can be founded on the same relation, with the same relators instantiating it, a special mechanism would be needed for saturated roles so as to select only the relevant subset of relators in which one of the arguments is fixed.

Conclusion and future work

In this paper we have analyzed and compared two different approaches to relational roles. The major similarity we found in them is related to the introduction of particular entities, which are called qua-individuals and that have shown to be very useful in the treatment of some typical puzzles related to roles, like the counting problem. This problem has proven very hard to solve by using only the notion of role, while the introduction of these new entities provides a straightforward solution. Furthermore, in the literature on conceptual modeling and on philosophy it is easy to find approaches based on roles either as anti-rigid and dynamic properties or as rigid types of "adjunct entities", but these are some of the first efforts which try to conjugate these two elements, thus harmonizing in a single framework two different notions of role present in the literature.

Another important advantage that is common to both approaches comes from the deep analysis dedicated to the relational aspect of roles; this relational aspect is extended also to qua-individuals and this is something entirely new, as far as we know. This analysis allows a more fine-grained description of what qua-entities are, since it makes explicit the dependence of roles not only on the entities that instantiate the role, but also to other "extrinsic" entities, namely relators, in the first approach, or concepts and reified relations in the second one, that in some sense hint at the context in which the role has been assigned to its players.

Even though they are very close in many respects, we nonetheless showed that the two approaches are not equivalent with respect to the ontological nature of qua-individuals. For the first one qua-individuals are (bundles of) tropes inherent in the role players, while for the second one qua-individuals are genuine entities with additional properties with respect to the role players. As genuine entities, the second can participate in events, while tropes don't and need to

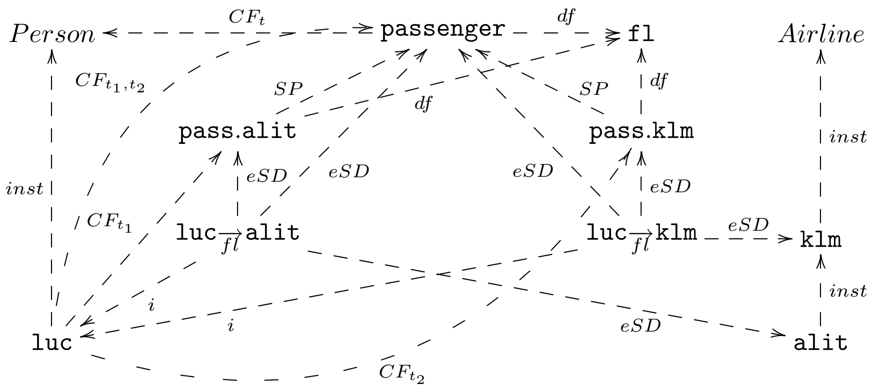

Figure 7: The passenger example with the two roles that specialize `passenger` in the second approach.

be introduced as extra arguments in predicates.

The second approach has the merit of being able, thanks to the introduction of a relation called *specialization*, to create a sort of "hierarchy" among roles based on levels of generality, such that more specific roles "fill the variables" that are unfilled in more generic ones. In this way, it is possible to have an explicit characterization of the relations holding among roles that are one the specialization of the other, or roles that are specializations of the same general role.

In this paper, due to the relational nature of roles, we have focused our investigation on relational qua-individuals, i.e., entities which primarily possess relational properties. Examples exist in the literature of qua-individuals that possess primarily intrinsic properties (e.g., 'John-qua-adult' or 'John-qua-male'). In future work, we intend to extend the discussion promoted here to countenance other types of qua-individuals as well as to investigate their interrelations.

Finally, in the present work we have just hinted at the fact that there are events that generate qua-individuals or, in other words, are their foundations. The analysis of the relations holding among qua-individuals and their foundations is a very promising topic to be analyzed in some depth.

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